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ABSTRACT

This teaching guide accompanies the film series "Challenge of the Unknown," which focuses on problem solving in the real world. The guide is designed to bridge the gap between the concepts presented in the films and their classroom applications, by answering teachers' questions and needs and providing a variety of possible activities. Seven chapters, each corresponding to one of the films, concern orienteering, information, restatement, outcomes, management, estimation, and argument. Chapters contain sections, each developing differing aspects of the film. The first part of each chapter is a preview defining and explaining the problem-solving ideas covered. A fact sheet, a classroom discussion situation, a brief rationale, and a group of short exercises for getting started are included in this part. The second section, on problem creating, provides sample student-teacher dialogues, problem ideas from the film, and student problems. In the next section, the focus is on material in the films themselves, with answers to some questions and attention to some areas in which curiosity might be channeled in the classrooms. Each chapter closes with a selection of exercises related to the skill being addressed in the film. A bibliography is included. (MNS)

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THE CHALLENGE OF THE UNKNOWN

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THE CHALLENGE OF THE UNKNOWN
Math Matrix

	Angles	Circular figures	Data collection, analysis	Computers	Equations	Exponents, powers, large numbers	Fractions, decimals, percents	Graphs, coordinates, functions	Logic, reasoning	Maps, navigation, polar coordinates	Measurement	Models, drawing, blueprints	Physical sciences	Probability, odds	Projective geometry	Ratios, d/rt	Rate and proportion	Rectangular figures	Scale	Sequences, ordering	Solid geometry	Statistics, error, sampling	Symmetry, patterns	Triangular figures
SITUATION																								
Orienteering	▼									●	▼					▼								
Wind Prospecting						▼				●	▼					●								
Sun Sculpture	▼										▼	▼			●								●	
Comedy: Revolving Door		●														▼						●		
Exercises	●	▼	▼		▼	▼		▼		●					●	●			▼				▼	
INFORMATION																								
Ice Climbing			▼								▼		5											
Anasazi Indians			●					●			▼													
Speed Skiing			●	▼			●	●			▼	▼				▼	▼							
Fish Finding			▼	▼						●	▼		6			▼								
Comedy: Lineup			▼																					
Exercises			●	▼		▼	▼	●		▼						●	▼			▼			▼	
RESTATEMENT																								
Dinosaurs					●	▼					▼	●				●								
Locks											▼	●												
Okefenokee Swamp				▼							▼	●						▼						
Ski Bus	▼										▼	●										●		
Comedy: Wallpaper											▼	●	3,4			▼								
Exercises				▼	●	●		●	▼			●		▼		▼		●				▼		
OUTCOMES																						●		
Vegreville Egg				▼							▼													
Cathedral							▼				▼											●	●	
Demolition											▼		4			▼					●			
Comedy: Sandwich															●		▼		▼		●			
Exercises		▼		▼									4					▼		●	●		▼	
MANAGEMENT																								
Canoe											▼					▼				●	▼		●	
Comedy: Outing					▼															●				
Exercises												▼				▼	▼			●	▼		●	
ESTIMATION																								
Sharks			●				▼				▼											●		
China Census			●	▼		●	▼				▼					●								
Avalanche Forecast			▼								▼			●								▼		
Comedy: Bakery															▼									
Exercises			●					▼						●								●		
ARGUMENT																								
Boomerang	▼										▼		2											
Sun Daggers	▼	▼							●	▼	▼													
Blimp				▼							▼		1,2								●			
Stonehenge									●		▼		2											
Comedy: Piano									▼		▼													
Exercises									●	▼			1,2,3			▼	▼	▼		▼			▼	▼

● = Principal concept
▼ = Supplemental concept

1. air temperature and density
2. lift and drag
3. force, levers, inclined planes
4. vectors
5. water temperature and pressure
6. speed of sound

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THE CHALLENGE OF THE UNKNOWN

TEACHING GUIDE

(to accompany **THE CHALLENGE OF THE UNKNOWN** Film Series)

W. W. Norton & Company
New York London

Teaching Guide

THE CHALLENGE OF THE UNKNOWN

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1 2 3 4 5 6 7 8 9 0

CONTENTS

CONTRIBUTING TEACHERS ..... *vi*
FOREWORD by Harold R. Jacobs ..... *vii*
PREFACE ..... *ix*
HOW TO USE THIS BOOK ..... *xi*

SITUATION: *Where Am I?* 1
Fact Sheet
Why SITUATION? Teaching SITUATION
SITUATION in the Classroom
Behind the Scenes: SITUATION on Film
Getting Started: Thinking SITUATION
Stalking a SITUATION Problem: Making Up Your Own Problems
Following Through: Problem Ideas from the Film
On the Lighter Side A Comedy of Errors

The Lost and Found Art of Orienteering
Background
Math-Eye VIEW
Annotated Bibliography

Wind Prospecting: Where the Wind Blows
Background
Math-Eye VIEW
Annotated Bibliography

Sun Sculpture: Minneapolis in Motion Around the Sun
Background
Math-Eye VIEW
Annotated Bibliography

Exercises and Class Activities

INFORMATION: *What Do I Know?* 37
Ice Climbing: Information on Ice
The Anasazi—What They Left Behind
Speed Skiing: Going for the Limit
Fishing with Sonar

RESTATEMENT: *How Do I See It?* 79
A Case for a Dinosaur Detective
New Keys to Old Locks
The Okefenokee Swamp: Unraveling the Mysteries
Ski Jumping: Best Way to Fly? Take the Bus

OUTCOMES: *How Do I Get There?* 123
Hatching a Giant Egg on a Computer
Building a Cathedral: A Gothic Tale
Demolition: Bringing Down the House

MANAGEMENT: *What Do I Do Next?* 161
Canoe Building: A Balancing Act

ESTIMATION: *Am I Close?* 193
Sharks: One If by Air. . .Two If by Sea
China Census: Everyone Counts
Avalanche: Snow Job

ARGUMENT: *Am I Right?* 231
Boomerangs: Many Happy Returns
Sun Daggers: A Little Light Reading
The Very Model of a Modern-Day Dirigible
The Master Builders of Stonehenge

PHOTO CREDITS AND PERMISSIONS 272
BIBLIOGRAPHY 275

A matrix-type index of math topics appears on the inside front and back covers.

*Please note that the films may be shown, and the ***Teaching Guide*** chapters used, in whatever order best suits the needs of the individual teacher.

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FOREWORD

One of the "My Turn" columns in *Newsweek* magazine several years ago had the title, "Tips for Teens." Included among the tips was the advice: "Stand firm in your refusal to remain conscious during algebra. In real life, I assure you, there is no such thing as algebra."

Although the column was intended to be humorous, these remarks refer to a question about mathematics that can sometimes create serious problems in teaching it. The question concerns the importance of the subject.

Morris Kline, in his book *Mathematics and the Physical World* (Crowell, 1959), wrote: "Certainly the lesson history teaches, and the reason for the great emphasis placed on mathematics today, is that mathematics provides the supreme plan for the understanding and mastery of nature." A student whose experiences in mathematics have been limited to routine lessons in arithmetic or algebra or geometry has no way of knowing or appreciating this. The contrived word problems that typically fill algebra books cause many perceptive students to wonder about the worth of what they are studying. A geometry course in which no attempt is made to show some of the many remarkable problems that have been solved with the methods of the subject (Eratosthenes' determination of the size of the earth is one of these) can easily cause students to question the value of geometry.

The most exciting aspect of **THE CHALLENGE OF THE UNKNOWN** project is that it provides teachers with materials that answer many of these basic questions about the worth of mathematics in a very real and convincing way. Certainly, the films produced in the series provide a marvelous spectrum of examples of mathematics being used to solve problems of genuine significance. Although the specific problems are not ones likely to be encountered by most people, their diversity dramatically demonstrates the value of knowing mathematics.

To merely view the films will not result in the learning that comes from practicing the problem-solving techniques and applying the mathematical principles which are illustrated. The purpose of this *Teaching Guide* is to help in accomplishing these goals. In addition to the essays dealing with the content of the individual films and the ways in which mathematics is used in them, this *Guide* provides a wide variety of exercises of varying difficulty which have been carefully chosen to give students the opportunity to learn how to become better problem solvers.

In *Mathematical Discovery* (Wiley, 1965), George Polya distinguishes between four types of problems. The first type, which he calls "one rule under your nose," is the sort of problem that can be solved by mechanically applying a rule or imitating an example. Textbook problems that provide practice with a particular procedure are of this type; such exercises provide no experience in real problem solving and are not included in this *Guide*.

Polya's second and third categories—"application with some choice" and "choice of a combination"—fit most of the problems in this *Guide*. To solve them, the student must use some judgment as to what rule or combination of rules should be applied. Some examples of the fourth category—"approaching research level"—are also included. An additional resource in the *Teaching Guide* that can be especially useful is the material in each chapter devoted to helping students invent their own problems, thus giving students direct experience in identifying the components of problems before being asked to solve them.

As a full-time classroom teacher, I am excited about being able to motivate my students with the films and to help them become better problem solvers through the resources provided in the films and this *Guide*. For me, mathematics will become an even more exciting subject to teach with the support of **THE CHALLENGE OF THE UNKNOWN**.

—Harold R. Jacobs
Grant High School
Van Nuys, CA
(Author of *Mathematics: A Human Endeavor; Elementary Algebra; Geometry)*

PREFACE

THE CHALLENGE OF THE UNKNOWN is an introduction to the *fun* of mathematics and problem solving. Students may come to look at homework problems as punishment, but most young people (and adults, too) look forward to spending hours on a challenging puzzle, a brain teaser, a mystery novel or even Rubik's Cube. What we set out to do with **CHALLENGE** was to tap some of that enthusiasm and use it to help provide students a foundation of interest in solving problems, particularly mathematical problems.

We think of **CHALLENGE** as a field trip, a trek outside the walls of the math classroom to the "real world," where mathematics proves to be a handy tool in helping to solve problems as varied as blowing "down" buildings, building a sun sculpture, counting lemon sharks or predicting avalanches. Mathematics is the terrain that **CHALLENGE** explores; problem solving is our guide on this trip.

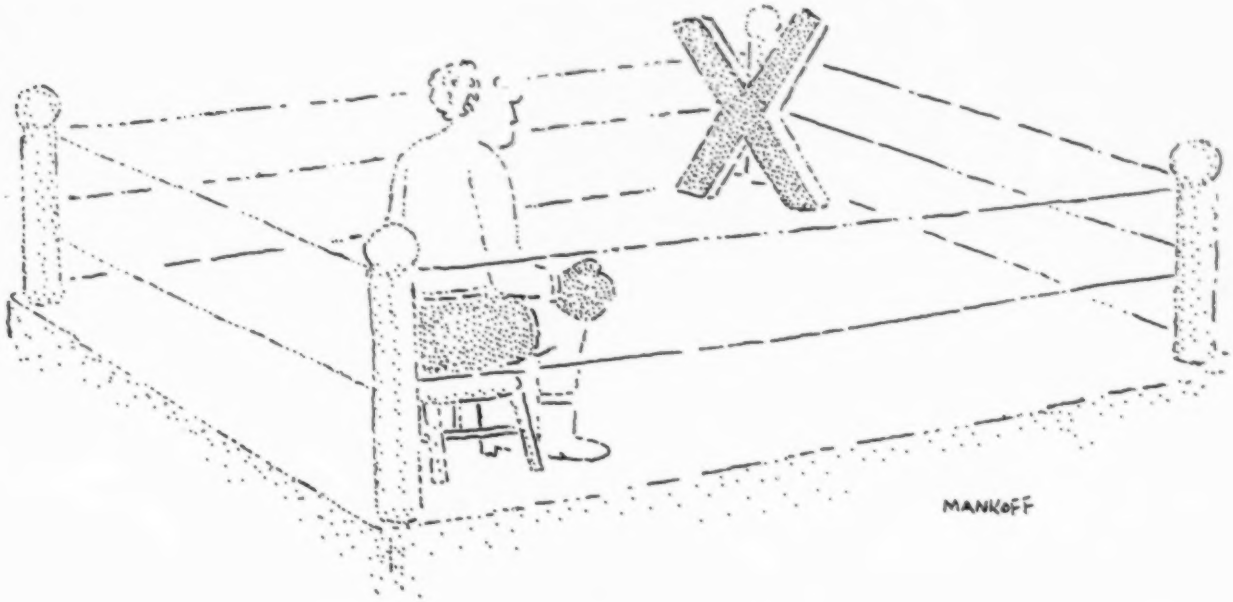
Focusing on problem solving allows us the necessary flexibility to reach our student audiences. **CHALLENGE** will be seen by math and science classes at many levels. We cannot hope to catch all students at the same moment of comprehension, but, through the lens of problem solving, we can catch them at many levels and expose them to basic mathematical ideas and concepts which they can explore with their teachers at the appropriate level.

The mathematical and problem-solving content of **THE CHALLENGE OF THE UNKNOWN** was set by a board of 27 distinguished advisors and many classroom teachers who volunteered their time to let us know in what ways a series on mathematics and problem solving could enhance their curricula and help to enliven their classrooms. Here are just some of their responses:

- *Math is thinking, not just operations.* Most students are capable in the basic mechanics of math, but have difficulty choosing the right operations.
- *Word problems trip up many students.* "Even good students have difficulty decoding word problems."
- *Students tend to use only one math "tool" for every job,* whether it is the right one or not. They invest as little thought and energy as they can. If a formula worked the last time, they will use it again this time and hope for the best.
- *"Students don't solve problems, they finish them."* They don't stop to think, analyze, appreciate and understand.

For these students, math is not a subject to study and enjoy, but merely a "school survival skill."

After long hours of research and consultation with our advisors, we established a goal for **CHALLENGE:** to break down students' negative stereotypes about mathematics and to offer a new way for them to think about math, which was at once useful and enjoyable. We wanted to remind students that problems—anxiety, suspense, the unknown and all—are like puzzles and games. *They can be fun.* The films provide real-life examples of



people solving problems and using math (as unselfconsciously as writers use grammar) to make their jobs easier.

Many students believe that math is only relevant for mathematicians, scientists, accountants and engineers, working at dull jobs in sterile environments. The films show another side of mathematics. They show artists, craftspeople, builders, athletes, and even scientists and engineers, using mathematics to tackle unusual problems in exciting environments. They are people very much *in* and *of* the world, exploring its secrets in intriguing ways and inspiring others with their energy and insights.

There is a lighter side to the **CHALLENGE** series as well. For too many students, math is deadly serious. Especially for math-anxious students, laughter can restore a sense of proportion and allow them to learn from other's—and their own—inevitable mistakes.

It was agreed by all that a comprehensive teaching tool should tackle both sides of a subject—the negative as well as the positive. Somehow, the films should show students that there are ways *not* to tackle a problem, but they should do this in a manner consistent with the positive approach of the documentaries. Dramatically, it was felt that the films would benefit from a contrast in mood that would add another dimension to the problem-solving theme and would appeal to a hard-to-please student audience. Enter, Bill Irwin, actor and mime extraordinaire!

Bill Irwin, the first active performing artist to win a prestigious MacArthur Foundation award, brings laughter into the classroom as the star of the short comedy segments which knit together the documentary portions of **THE CHALLENGE OF THE UNKNOWN**. Irwin and a repertory company of actors express the humor that emerges when someone "bullies" a problem, pushing at it without thinking through a strategy or goal.

Bill Irwin

Where the films show people solving real-world problems, the **CHALLENGE *Teaching Guide*** challenges students to make up their own. Devising and solving problems of their own will not only give students practice using problem-solving techniques, but it will also allow them to use math as the rest of us do—as an efficient and valuable tool, one that doesn't make work, but saves work. By helping students create their own problems, it is hoped that students will more fully understand the inner workings of problems and, once finding themselves inside the problem-making *process*, to be able to identify and trace familiar routes to their own solutions.

This ***Teaching Guide*** is designed to bridge the gap between the concepts presented in the films and their classroom applications. The ***Guide*** is there to answer the question that many students will ask after seeing the films: "What does this have to do with math?" In the films, the math is woven into the stories; in the ***Guide***, the math is featured close up. Each film story has a section devoted to its math applications.

Again, directed by advisors and classroom teachers, we have tried to anticipate the needs of real classrooms. We've included articles on the mathematical and problem-solving concepts in each segment in each film. There are background articles for teachers, as well as exercises, word problems and classroom activities. There is material to use *before* and *after* viewing the films. And all of this we've tried to make as accessible as possible to the busy teacher. Each chapter includes a brief fact sheet on the individual films. A matrix-type index on the inside front and back covers of the ***Guide*** provides teachers with an easy guide to the math concepts in the individual film segments. As much as possible, the ***Guide*** maintains the format and liveliness of the films. It is full of photographs and other visuals in an attempt to keep the spirit of the films alive.

Above all, the ***Teaching Guide*** was designed, like the films, to be fun—to make math and problem solving enjoyable for teacher and student alike.

HOW TO USE THIS BOOK

This book is a catalogue. Use it the way you would use any other catalogue. You can read it, skim it, browse through it, shop it for help, use it any way you want; it's designed to help you explore and make the most of **THE CHALLENGE OF THE UNKNOWN** film series for your students.

This book is also a toolbox; it gives you specific tools to help your students develop problem-solving skills and to unlock the creative potential that many math textbooks ignore.

This book is also a "defensive manual." Once your students have seen **THE CHALLENGE OF THE UNKNOWN**, they will undoubtedly have a lot of questions. In this book you'll find the material you need to respond to a broad range of questions and to initiate discussion as you choose.

And, most importantly, this book is an "offensive manual," helping you to get your students involved with the problem-creation, hence problem-solving, process.

The ***Teaching Guide*** has been designed to be as flexible as possible, and to adapt itself to the way *you* want to use it. Following is a brief description of its contents.

Chapter Organization

Each of the seven main chapters in the ***Teaching Guide*** corresponds to one of the films in **THE CHALLENGE OF THE UNKNOWN** series. Chapters are broken down into sections, each of which focuses on a different aspect of the film. Some sections are for you, the teacher, alone, to give you the materials you need to understand and teach the problem-solving concepts and to discuss the films with your class. Others, including the exercise sections at the end of each chapter, are for your students, while a third group can help you and your students explore problem creation together.

PreVIEW

The first part of each chapter defines and explains in detail problem-solving ideas covered in the chapter and parallel film. What are they? Why are they important? What can they do for your students? *How do you teach them?* The section consists of the following components.

1. A ***Fact Sheet*** at the beginning of each chapter gives a capsule summary of the elements of the film, including math and problem-solving concepts discussed or suggested by the material in each film. In ***How to Spot. . .***, students are also introduced to some of the signposts to look for in identifying particular types of problems.

2. In ***"SITUATION" in the Classroom***, some of the basic mathematical and problem-solving components are introduced, with an eye to how they might be explored in classroom discussion.

3. ***Behind the Scenes*** gives a brief rationale for how each of the documentary and comedy segments was selected for inclusion in the individual **CHALLENGE** films.

4. The ***Getting Started*** section provides you with a group of short exercises designed to introduce students to the problem-solving topic *before* viewing the films.

Problem Creating

Next in the lineup is a close look at problem creation. What is a good problem? How do you know what skills to apply? *How can students be helped to see the value of making up their own problems and do just that.*

1. Sample ***Teacher-Student Dialogues***, dealing with problem creation, illustrate how the process might work. (Remember that these dialogues *are* fictional. Take them as signposts pointing in general directions, rather than as road maps detailing specific routes.)

2. ***Following Through: Problem Ideas from the Film*** provides suggestions for creating problems based more directly on ideas and situations in the films.

3. A number of ***Student Problems*** were generated during the review process of this guide. In each chapter you'll find one or two samples of how individual students took the information provided by the films and made up problems of their own.

VIEWpoint

Next, the chapter focuses on the material in the films themselves. One of the main purposes of the series is to stimulate curiosity and encourage exploration. This section is intended to provide some answers and indicate some areas in which this curiosity might be channeled in the math classroom.

1. ***On The Lighter Side*** focuses on the comedy segment in each film, combining discussion of the problem-solving meat behind the comedy with a few light, humorous exercises based on it.

2. ***Background*** articles flesh out the documentary stories. After seeing the films, students might have questions that fall well outside the scope of your normal curriculum. They might want to know, for instance, how fast a speed skier can go; how sonar works; how marine biologists can estimate the shark population in a given area; why a boomerang returns. This section gives you the material to address all of these questions and many others besides. It also provides the resources (in an ***Annotated Bibliography*** for each documentary film segment) to direct students in their own research of the topics presented.

3. The ***Math-Eye VIEW*** sections with accompanying exercises cover, in some depth, at least one aspect of the mathematics in each of the 22 documentary film segments. Because the film series is intended for use across a wide spectrum of age and ability levels, you'll find that these sections run from simple to more advanced mathematics. Because the philosophy of the films is to avoid overt explanation of mathematics in favor of showing people who actively use it, you'll find that some of the math concepts are only implied in the films. You'll have to use some judgment here, to decide which math concepts are appropriate for discussion with your grade level. (The ***Math Matrix*** on the inside front and back covers of this book provides you with the data you need in an easy-to-use format, to find out what the math concepts are and where to find them.)

Exercises

Each chapter closes with a selection of exercises relating to the skill being addressed in the film. The exercises, culled from many sources, cover a wide range of skills. Some of them you'll probably have seen before in different contexts. They're meant to be fun, and many of them are exercises in pure problem solving rather than computation. (***Student Exercises*** are labeled as such and appear alone on the page so they may be copied for classroom use.) **CHALLENGE** ***Exercises***, interspersed among the main exercises and throughout the ***Math-Eye VIEW*** sections, are more difficult; some of these problems may even tax the teacher! They are intended, however, as their label suggests, not to confound, but to *challenge* and be food for thought and discussion in the math classroom.

Where appropriate, exercises are accompanied by ***Teacher's Notes*** (on the left-hand facing pages to the Student Exercises), which explain how a particular problem is solved and place it within the context of a particular problem-solving skill. Answers are provided selectively.

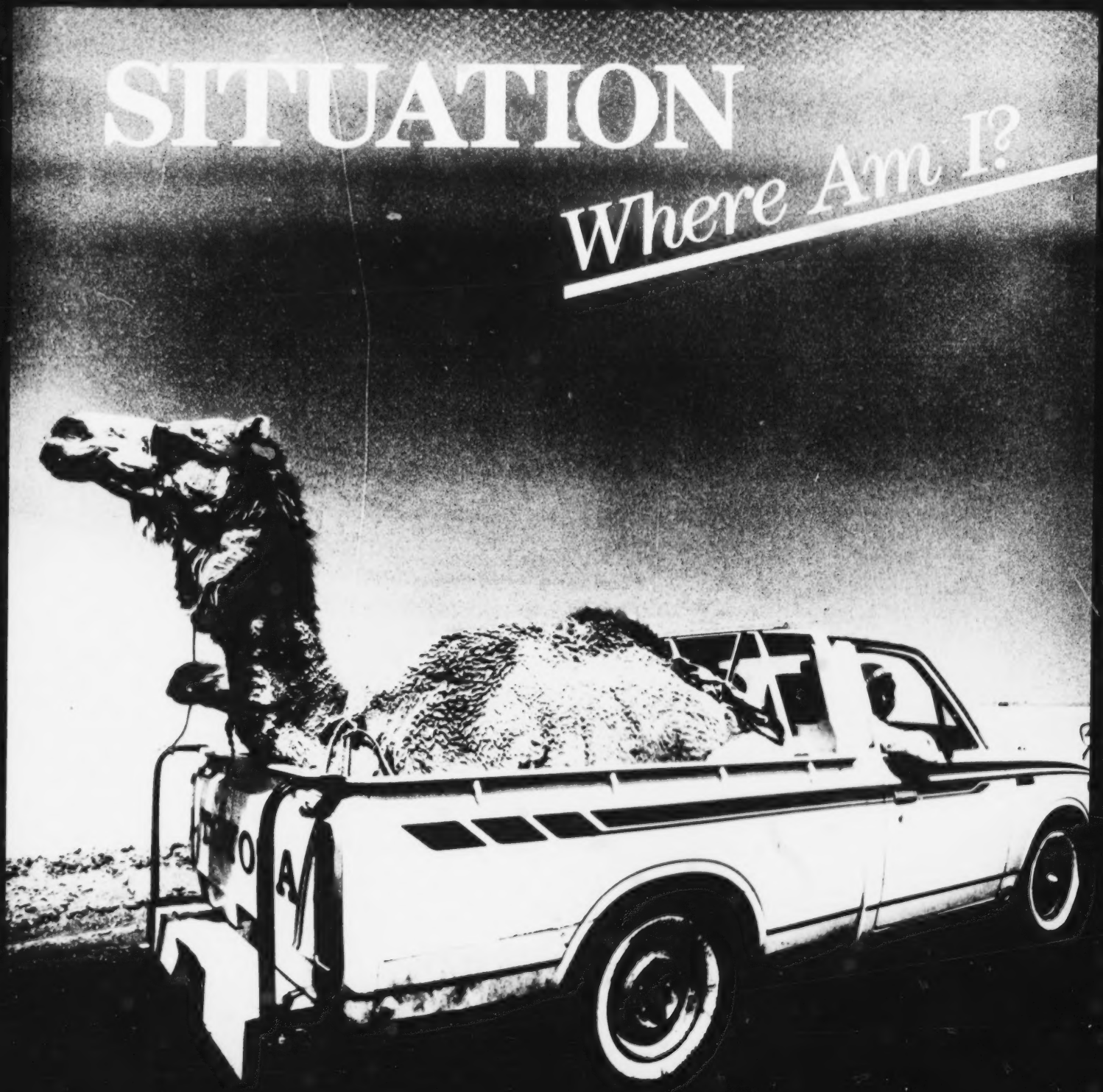
A Final Note

Perhaps the best advice we can give you on the use of this ***Teaching Guide***, and **THE CHALLENGE OF THE UNKNOWN** series as a whole, is to have fun with them. Play with them. Use the book at different levels in different areas.

Fit **THE CHALLENGE OF THE UNKNOWN** into your curriculum where you think it appropriate; use it as an adjunct where you think it worthwhile. The film series and this book belong to you.

SITUATION

Where Am I?



Fact Sheet

SITUATION: *Where Am I?*

Running Time: 18:01 minutes

Documentary Segments (3):

1. ***ORIENTEERING:*** **On your mark, get set...Where am I?** Orienteering is a cross-country race, where competitors have to find the way, guided only by map and compass. ... 14
2. ***WIND PROSPECTING:*** The relentless winds of California's Tehachapi Mountains may prove to be a reliable source of power for Los Angeles. **But where is the best place to put up a wind turbine?** ... 20
3. ***SUN SCULPTURE:*** **Can you picture the earth turning?** This sculpture can show you the earth in motion, and it's all done with mirrors. ... 24

Comedy Segment: ... 12

4. ***REVOLVING DOOR:*** "Buzhnikov, the Doorman" at Hotel Le Crimm is stumped by the revolving door. **Can he figure out a way to get the baggage into the hotel and stop going around in circles?**

Math Concepts:

ORIENTEERING: Scale, map-reading, angles, ratio and proportion, distance rate time.

WIND PROSPECTING: Pythagorean theorem, sine, distance rate time, sampling, mapping.

SUN SCULPTURE: Mirror symmetry, patterns, projective geometry, models, scale drawing.

Problem-Solving Concepts:

Context or problem environment; identifying components; recognition.

HOW TO SPOT A SITUATION PROBLEM

You can tell it's a situation problem if...

- ***The problem is the unknown.*** *In many problems, the destination or the information is unknown. In a situation problem, the challenge is to find and pin down the problem.*
- ***You haven't seen a problem quite like this before.*** *Understanding the nature or context of the problem is the key to solving it.*
- ***You need to measure, fathom or lay out the problem.*** *A situation problem needs to have a frame placed around it, the terrain in question needs to be mapped and delineated. The tools of measurement are important to working out a problem situation. Navigation and survey problems, for example, are situation problems.*
- ***You spend more time formulating and defining the problem than you do executing the solution process.***

X=

Throughout the chapter, the X = sign indicates pages containing exercises and problems for classroom use and discussion.

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SITUATION

Where Am I?

Problem solving doesn't begin with a problem; it begins with a situation. Long before a problem can be defined, and a solution process formulated and executed, the problem must be identified. It must take shape in the mind of the problem solver, teased out of the surrounding landscape or situation. This may be the most demanding and crucial part of the problem-solving process.

Why **Situation?**

Most textbook problems come already defined. Real-world problems, however, are often found or are chosen from a problem environment that can obscure the problem and be confusing to the problem solver. A problem situation, for example, may arise as a feeling of dissatisfaction or a vague idea that something is wrong. It may also crop up as a general question, or statement. "Why can't I ever save any money?" "I really want a computer." "I'd love to redecorate my room." There's a lot of defining work that must be done before such problems can even begin to be solved. First, they must be seen as problems.

A case in point: An architect is hired to remodel a suite of offices. She doesn't begin by sitting down at the drafting table and drawing up a set of blueprints. Before she can start on her solution process, she must assess the problem situation: What kind of work goes on in these offices? How many people are employed there? How many will work there in a year or two? What are their work habits? What are the traffic patterns of people and material? What are the budget and time limits on the job? If she ignores the problem situation, the resulting design she proposes is likely to be unacceptable to her clients; she could create more problems rather than find a solution.

Teaching **Situation**

There are several aspects to identifying and assessing a problem situation:

Develop a clear picture of the problem situation. Before getting down to details, develop an overview of the whole terrain. Good problem solvers try to get a picture of the forest before they head off into the trees.

Recognize the situation's component parts. With a picture of the forest in mind, sketch in the landmarks. Examine the elements of the situation and their relation to each other.

Identify familiar patterns. In many cases, the elements may form patterns or remind you of other problems you've already solved. For instance, in tackling textbook problems, students might ask themselves, "Is this a mixture problem, a compound interest problem, a river-current problem?" They identify familiar problem types. Once they identify a familiar type, they can call on familiar solution strategies. *Not* recognizing a pattern alerts them that the problem is something new, and they may need to call on additional resources in solving it.

Establish or describe relevant goals, rules, conditions or limits. A goal points in the general direction of a solution; rules or limits place certain parts of the terrain out of bounds. By describing the conditions of the problem situation, the problem solver takes a bearing in preparation for heading off on the solution process. Without goals, limits and initial bearings, the problem solver might stay stuck in the situation with no way to get out. ■

Situation in the Classroom

In order to define and solve their problems, Rebecca Frawley, Eliza Dixon and Dale Eldred, the characters in the three documentary film segments in SITUATION, each rely on a variety of mathematical concepts.

Maps/Models/Blueprints/Scale Drawings: When Rebecca Frawley asks herself, "Where Am I?" she turns to a topographical map. Dale Eldred uses blueprints, models, maps and scale drawings to plan his sun sculptures and their place within the environment. Precise measurements and careful scaling allow him to see his sculptures in many different forms *before* committing himself to one path. Eliza Dizon maps the wind.

Reading a map or blueprint gives students practice in scaling, measurement and ratio and proportion. Making a scale drawing or model gives them a chance to exercise their judgment in selecting relevant information to be included. Graphing data or reading a graph can give students the opportunity to "see" relationships between algebra and geometry.

Angles: In building his sculptures, Dale Eldred calculates and fixes the angles of his mirrors so they reflect light in precise positions. Eldred's sculptures rely on the findings of the famous Greek mathematician, Euclid, who first reported that when a ray of light strikes a mirror it is reflected back at the same angle at which it struck the mirror—i.e., the angle of incidence equals the angle of reflection.

Triangles: Students ask, "What good is the Pythagorean theorem, anyway? When am I ever going to use it?" Eliza Dixon answers that question. When she's gathering data on the end of a kite string, the Pythagorean theorem tells her the height of her kite above the ground—necessary information for her calculations of wind speed.

Word Problems: Students often have trouble solving word problems because they cannot form an image of what's going on. They grope around for "the right formula" to plug in the numbers they have been given and keep their fingers crossed. Many teachers report that distance/rate/time problems are particularly difficult for students. Orienteering is a word problem on the move. When Rebecca Frawley is orienteering, she thinks in terms of distance/rate/time. ■

SNAFU: "Situation normal all fouled up." Umbrellas are fine protection against rain, but a gust of wind can turn the situation inside out.

Behind the Scenes: SITUATION On Film

How do you film situation? It's difficult and risky enough to film a problem in the process of being solved. But how do you film a story of a problem being identified?

This was our challenge in filming SITUATION: to show how a problem solver defines a problem, how he or she chooses or finds one particular problem among a multitude of possibilities.

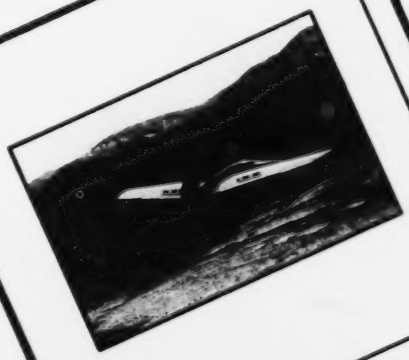
Orienteering: One of the very first stories we pinned down for the CHALLENGE series was orienteering, a footrace in which calculations count as much as athletic ability. When orienteers jog out of the starting gate with map and compass in hand, they take their first look at their problem: a map of the terrain marked with control points. They get their bearings and set their course. From there on, it's a contest of mind over mileage.

CHALLENGE was there at the 1983 O'Ringen meet in Sweden, the "world series" of orienteering. We followed American competitor Rebecca Frawley as she assessed the situation and put her problem-solving skills to the test. Just to finish requires a grasp of the subtleties of map scale, topographical maps and compass reading—to say nothing of maintaining speed and stamina . . . and a sense of humor.

Wind Prospecting: The powerful winds of the Tehachapi Mountains sweep across the landscape in southern California, making the land almost useless for farming or grazing. Most people consider these winds destructive; others have come to see them as a natural resource. Wind prospector Eliza Dixon is mapping the invisible landscape of the wind in the Tehachapis, trying to determine if there's wind power enough to generate electricity for the homes and businesses of southern California. As she takes wind speed readings, we explore the concepts of distance/rate/time, and data collection and measurement. And where's the math in flying a kite? The Pythagorean theorem, of course!

Sculpture: Where's Minneapolis? One answer is eastern Minnesota; another, latitude 93° longitude 45°. Artist Dale Eldred's answer is evident in his sun sculptures. The dancing light and colors created by his mirrors and diffraction panels show where Minneapolis is in relation to the sun at any given moment—a constantly changing situation which makes for a constantly changing work of art. The story explores this paradox: the math concepts of angles and measurements, mapping and modeling employed in the service of art.

Revolving Door: At Hotel Le Crimm, when "Buzhnikov the Doorman" asks himself "Where am I?" the answer is, "going around in circles!" He knows that something's wrong: The baggage never gets into the hotel lobby. But what exactly is the problem? Careful "analyzation" of the situation doesn't help, until he realizes this is an automatic door. The problem isn't the door, however, it's the doorman. Sometimes problems contain not only their own solution, but create new problems to solve. Perhaps Buzhnikov should advertise for his new job in "Situations Wanted." ■



GETTING STARTED: **Thinking SITUATION**

It sounds deceptively simple to say that when students first encounter a problem they must begin by understanding it. This step is too often ignored by problem solvers, who jump into a situation without first taking the time to recognize or define exactly what the problem is.

For starters, try the following exercises with your students *before* screening *SITUATION: Where Am I?* They are designed to sharpen observation skills and emphasize the need to check out the situation before solving the problem.

1. Ask students to describe the following "situations":

- Do the doors of your school open in or out?
- What color is the inside of your school's auditorium?
- Where was the sun in the sky when you went to school this morning? Where is it on a morning in early July?
- Stand up and turn to face the direction of your home.
- Which shoe do you put on first?
- When you cross your arms, which arm is on top?

Identifying or diagnosing the underlying problem or problems in a situation can be a problem in itself. When the television breaks, the problem is not that the picture is wavy, but *why* this symptom occurs. A good repair person would want to know the *whole situation*—how old the set is, what repairs have been done in the past, what the reception is normally like, whether the television has an antenna or cable, whether it has been moved recently to a different part of the house, etc.

2. Ask the class to describe what they would want to know in order to identify these problems:

- a lamp that won't light
- an alarm clock that rings an hour too early
- a shirt that doesn't fit
- a paper airplane that won't fly
- a dog who sneezes all the time
- a bad grade in social studies

Seeing the situation means getting an *overview* of the problem, sometimes from more than one perspective.

3. The drawing below can be interpreted as at least three different three-dimensional shapes. See if your class can describe all three.

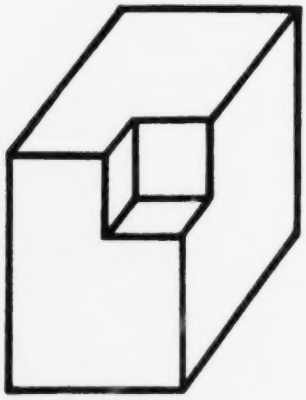


Fig. 1

For Openers

After viewing SITUATION, ask your class the following questions to help them get a feel for the topic:

- What is a problem situation? What makes it important?
- What are the kinds of problem situations you face in daily life? How do you analyze them?
- How might your problem-solving approach depend on the situation?
- In what ways is situation particularly important to the characters in the film?
  1. Situation is extremely important in the sport of orienteering. What do contestants like Rebecca Frawley do to make the most of it?
  2. What is the situation that wind prospector Eliza Dixon works with? On what does she base her decisions?
  3. How does Dale Eldred use a situation to create a work of art? How does his art depend on the situation?
  4. In what situation does "Buzhnikov the Doorman" find himself? Does he deal with it successfully?

"Before you attempt a solution to a problem, avoid getting lost in detail, go over the elements of the problem rapidly several times until a pattern or total picture emerges. Try to get the picture of a forest before you get lost in the trees."

Moshe F. Rubinstein, *Patterns of Problem Solving*

Stalking a **Situation** Problem

Making Up Your Own Problems

The following is a hypothetical conversation between teacher and students. It is intended to suggest ways to encourage students to make connections between problem-solving concepts in the film, their own experience and their knowledge of mathematics.

"

♦ ***Now that we've seen the SITUATION film, let's talk about how situation plays a role in our own problems. First, let's review what situation is. The first story in the film is about orienteering. Why do you think that's a situation story?***

Well, the film's also called "Where Am I?" and that's what orienteering is all about—finding where you are both on the map and in the woods.

♦ ***Can you expand on that?***

The problem in orienteering is figuring out the best route. But before you can do that, you have to know where you are, what the area is like, what obstacles are ahead, stuff like that.

♦ ***So you're saying the specific problem is navigating from one point to the next, but in order to do that well, you have to assess the general situation and get an idea of what you're up against. What about Eliza Dixon, the wind prospector? How would you describe the situation in her story?***

I think the situation is the mountains and the winds. That's what's there. Then, by investigating the situation further—taking measurements of the wind—she determines if the situation is right for a wind farm.

♦ ***This is the key to this story, I think. Most people don't give the environment of a problem a second thought, but she sees a problem to be solved there. What about the artist Dale Eldred? How does he take advantage of the situation?***

He's like Eliza Dixon in one way. She looks at the mountains and the wind and sees a resource; he looks at the sun and a location, Minneapolis, and sees the possibilities of using a city as the background for his art.

♦ ***If I wanted to describe Minneapolis, I would say it was a large midwestern city in the United States. But how would Dale Eldred describe it? What terms would he use?***

He talks mostly about sunlight and shadow, where the sun falls and where the shadow line is.

♦ ***Excellent. Two people faced with the same situation will probably define two entirely different problems for themselves.***

I don't understand.

♦ ***Here's an example. Pretend you and I live in the same neighborhood and come to school every day on the bus. But we both hate the bus. It's always late and crowded. That's the situation. So we both decide to try and find another way of getting to school. What do you do?***

Can I ride my bike?

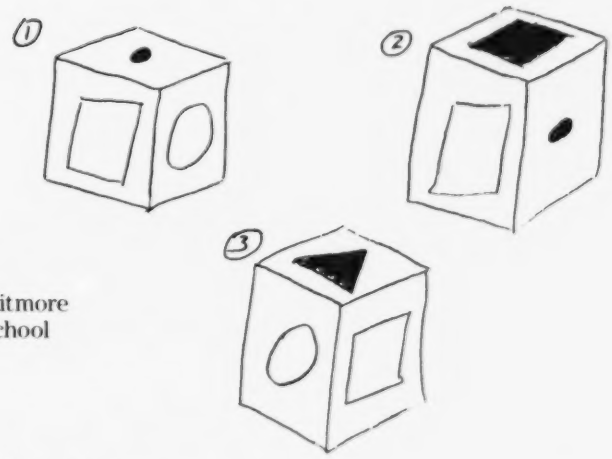
♦ ***I don't know. We'll say it's 2 miles each way.***

Then I'll ride my bike and save my bus money.

♦ ***What about the winter?***

Well, even if I have to take the bus sometimes, I'm ahead.

question: Which face of the cube is on the bottom of the last diagram?



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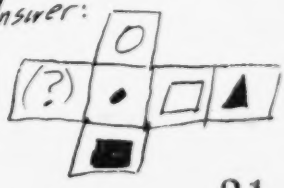
Answer:



Using the above diagrams, the dot is found to be on the bottom.

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♦ ***That's true. But I don't have a bike. That solution isn't right for me. I'm going to buy a car. That means my problem is more complicated. I'll have to figure out how much I can afford to pay and what upkeep will cost. All of these factors will influence my decision. But do you see how the same situation creates different problems? Now somebody give me another example. Name a situation—something you want to change or fix or plan for.***

I'd like to change my room. I hate the color and the old furniture.

♦ ***Okay, that's a situation. Why don't you change it?***

My parents say it's too expensive.

♦ ***Okay. Let's make a list of the parts of the situation:***

EXISTING SITUATION	DESIRED GOALS	LIMITS
Wrong color paint	Paint room	Parents
		Money
Old furniture	New furniture	Time

♦ ***How expensive is repainting your room going to be?***

I don't know.

♦ ***You haven't gotten very far with this problem, have you? We'll help you out. And we'll show you how you can use math to solve your problem. Now, how big is your room? If you don't know, estimate.***

About 20 feet by 20 feet.

♦ ***And how high is the ceiling?***

6 or 7 feet.

♦ ***I doubt that. If it were 6 feet high, some peoples' heads would hit the ceiling. How about 8 feet?***

That sounds right.

♦ ***Now everybody, let's calculate the wall and ceiling area of the room.***

I get 1040 square feet.

♦ ***How did you get that?***

You need first to find the area of each wall—that's 8 feet by 20 feet times 4 walls, or 640 square feet. Then, you need to add the area of the ceiling, which is 20 feet by 20 feet, or 400 square feet. The total area to be painted is 640 square feet, plus 400 square feet, or 1040 square feet.

♦ ***What else would you need to know to figure out how much paint will cost?***

You'll need to know how much paint it will take to paint the room. I think paint stores can give you that information.

♦ ***What are some of the other problems that this situation—or other situations you might think of—give rise to? And how about solutions?***

"What is the unknown? What are the data? What is the condition? Have you seen it before? Or have you seen the same problem in a slightly different form?"

G. Polya, *How to Solve It*

Following Through: Problem Ideas from the Film

In SITUATION, Dale Eldred, Eliza Dixon and Rebecca Frawley work on solving three specific problems: designing a sun sculpture; surveying available wind power; and competing in an orienteering meet. Within each segment is additional material for your class to work with in formulating and solving other problems as well.

Orienteering: The sport of orienteering is a combination treasure hunt and cross-country race.

- Devise an orienteering treasure hunt for the class. The hunt begins when they open their first clue, consisting of a compass heading and a measure of distance. When they reach their destination, they will find a new heading and distance measure.

- Map reading is crucial to orienteering. Divide students into teams and give each team a pair of identical topographical maps. One map of each pair should be marked with a route with several checkpoints. The object of the exercise is to have the student with the marked map verbally guide the other student over the course. To make it even more challenging, the student with the marked map may not refer to place names, only to directions, distances or features of the land.

- Rebecca Frawley uses a system called "pace counting" to estimate how far she has come and how much further she needs to go to the next control point. Frawley estimates she jogs about 44 paces to go 100 meters.

  Have your students estimate the number of paces they take for a given distance. (An easy way to do this is to mark off 10 meters and have each student count the number of steps or jogging paces they take.) By multiplying that number by 10, they can then find out how many steps they take in 100 meters. Using this information, students can describe the route from school to home, or between other points in the community, using compass bearings (see box on "How to Use an Orienteering Compass," pg. 16) and estimated distances based on pace counting.

"When we struggle with a hard problem, we may start off in a hundred wrong directions, explore numerous blind alleys, miss critical clues, and only slowly come to recognize what's important about the problem and what is not. As we make progress, we may make important changes in our representation, detect faulty assumptions or discover useful detours."

John R. Hayes, *The Complete Problem Solver*

Wind Prospecting: In the Tehachapi Mountains, Eliza Dixon collects data on wind speed in order to determine available wind power.

- Devise a method for conducting a wind survey at school. Decide how, where, and for how long to take readings to get fairly accurate results.

- When Eliza Dixon looks at the Tehachapi Mountains, she sees a possible resource for wind power. Then she collects data to test her case.

  Have students examine their school, neighborhood or community. What resources do they see there that others might miss? Perhaps enough students are interested in seeing a popular movie to qualify for a discount group rate at a local theatre. Challenge your students' ingenuity to devise a survey to discover the hidden resources in your classroom.

Sun Sculpture: Dale Eldred began planning his Minneapolis sculpture a year in advance, walking the site and observing the many angles that the sun's rays made with the surfaces of the city.

- Have your class make their own sun sculpture. They will have to decide where to exhibit it and what materials to use to capture and reflect the sun's rays. Like Dale Eldred, they might want to use mirrors to design more complicated and interesting effects.

- While Dale Eldred uses the interaction of the sun and the façades of a city to create some of his works of art, other artists use the interaction of the *air* on objects in the environment. Such sculptures are called *mobiles*.

  Have your students design a mobile of found objects. They will have to consider a number of factors including how large the objects should be; what to use to connect the objects; how to balance objects of different weights and sizes; and how to hang or exhibit the mobile.

On the Lighter Side

If you try to solve a problem without understanding it, you'll end up going around in circles, like Buzhnikov the Doorman at Hotel Le Crimm. He's got the pieces of his problem down pat—greet the guests and carry their luggage inside—but he can't make the whole thing work. He's concentrating on details and forgetting the big picture.

Then presto, he finds a magic solution—the automatic door. Life doesn't often work that way, but even the hardest workers and best problem solvers sometimes dream of magic solutions. We can laugh at his foibles—and our own.

A Comedy of Errors: The Revolving Door

"I am being doormat at hotel. You know, man with hat. I am saying, 'Good morning,' and carry baggages. I am trying my best to be a doormat, but this is not good door. This is not like any door I have in Buzhnia where is my home."

"Well, I was making analyzation of my own to see with this door how I go in, how I do my responsibilities, but it is very difficult."

The Ins and Outs of Hotels

One situation can breed lots of different problems.

- Draw an aerial view of the revolving door. There are 4 partitions in the door, and each door is 4 feet wide.
  - —At what angle are the partitions to each other?
  - —If there were 6 partitions, at what angle would they be set?
  - —What is the circumference of the circle described by the outer edge of the partitions as the door revolves?
- A 20-member brass band, each with one instrument, is playing tonight at a club across town from Hotel Le Crimm. They will have to travel by cars to the concert. Each car can carry 4 bandmembers and 3 instruments or 3 bandmembers and 5 instruments. How many cars will they need to carry the whole band crosstown?
- Hotel Le Crimm has 100 rooms. It employs 7 maids at $4.50/hour; 3 cooks at $6.00/hour; 2 bellhops and a doorman at $4.25/hour; a deskclerk at $5.75/hour; and the manager at $10.00/hour.

"Door is causing many mishaps and many hotel guests are being accidented. Many peoples are not blaming door but blaming Buzhnikov."

"This time in the evening, I realize this is good door. This is automatic door. Take bag, put in, make spin, go automatic . . . . Still, to my surprise still after I am finished with job, Mr. Hotel Manager say, 'Mr. Buzhnikov, we have automatic door. We don't need you for doorman.' "

Answer the following questions. For each, make a note of what information in the problem is irrelevant to the question being asked.

—How much per hour does the hotel pay in wages?
—How many employees are there?
—What is their average salary?
—How much does the hotel pay out weekly to its entire staff for a 40-hour week?

■ Buzhnikov has finally figured out how to work the automatic door. His next task is to get the 20 band instruments into the hotel. He sets the door going at 10 revolutions per minute and begins to put the luggage in the door.

—If he could feed one piece of luggage into each of the four sections of the door as it went by, how long would it take Buzhnikov to get all the luggage into the hotel?
—If he can feed a piece of luggage into the door once every revolution, how long would it take?

The Lost and Found Art of Orienteering

"Sometimes you'll have to go the longer way, but it will take the shortest time."

It's part cross-country race, part treasure hunt. This is one way to sum up the sport of *orienteering*. "All" it takes to compete are map and compass, the skills to use them, stamina and the ability to keep track of where you are and where you're going in a constantly changing situation—maybe not simple, but it can be a lot of fun.

Contestants compete over courses set according to their age, sex and experience. They walk, jog or run over the terrain trying to locate, as quickly as possible, a series of control points in a given order. Their tools are an orienteering compass, by which they set their course, and a topographical map, which shows in two dimensions the three-dimensional features of the landscape. The winner is the one who makes the best time after hitting his controls in the proper order.

Every summer in Sweden, thousands compete in the "world series" of orienteering, the 5-day O'Ringen meet. In 1983, the meet drew some 22,500 people, ages 8 to 80, from 25 countries. In such a large meet, there are many different courses for different divisions, each with its own control points. Competitors are divided into small groups, each of which starts at a different time; each group includes several levels of competitors, often headed in entirely different directions.

"In this forest today," explains Roland Offrell, organizer of the 1983 meet, "there are 250 controls—although any individual course consists of 7 to 10 controls."

Taking Control

At each control, the orienteer finds a hole-punch. He or she uses the punch to mark the scorecard with a special symbol, proof that she "took" that control point in the right order. Then it's time to reassess the situation and plot a course to the next control.

The orienteer's skills grow with experience. A skilled orienteer doesn't simply follow a compass bearing, but uses it in combination with the topographic map to plot the *best* course around such geographic obstacles as ravines, lakes or steep hills.

"Sometimes you'll have to go what seems the longer way—but it will take the shortest time," says 16-year-old American competitor Rebecca Frawley. She finds that those moments spent studying the map and doing the initial planning for the course are the most valuable. She sometimes draws lines on her map connecting her numbered controls so she can get an overview of the entire course. Next, she "skims" her map, so that she sees only large features like roads or sizable clearings. She then takes a mental "snapshot"—the outline of her course with these large features filled in, marking the way. This is the "big picture"—the situation—that an orienteer needs to hold in mind throughout the race.

Both speed and accuracy are important in orienteering. Offrell warns competitors to pace themselves from the start: "At the beginning, you can run very fast, but your orienteering capacity is not good enough to maintain that speed. You are too fresh—your brain is 50 meters behind you all the time. Then at the end of the course you have to put much more effort into just keeping your legs moving, and there's nothing left to concentrate on the orienteering. You make mistakes."

Pace Yourself

Frawley relies on "pace-counting" to tell her when she is getting close to a control point. "Before you go out," she advises, "figure out how many paces you take in 100 meters. Usually I have to jog about 44 paces to go 100 meters." During the race, she measures the distance to each control on the map and then counts her paces on the way.

But an experienced orienteer like Frawley is prepared to make mistakes, spot them and take action to correct them. This is her edge—being flexible enough to adjust to new situations as they arise. One rule of thumb for orienteers: If lost, don't try following anybody. He's probably not going your way; if he is, he may be lost, too. Instead, stop and reconsider the situation. ■

END OF THE ROAD: Orienteering contestants cross the finish line at the 1983 O'Ringen meet. The 5-day competition drew 22,500 people ranging in age from 8 to 80 from 25 countries.

WHERE AM I? Rebecca Frawley pauses to get her bearings. In the cross-country race she will pursue her own individual course, hitting 7 to 10 out of the 250 checkpoints hidden in the woods. Her only tools: a map, a compass and her wits.

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Getting Your Bearings: How to Use an Orienteering Compass

Orienteers use a special compass* (see Fig. 1) to set their course from control point to control point. An orienteering compass has three main parts: 1. a *base plate* with a "direction of travel" arrow on it; 2. a clear *movable housing* with red orienteering lines and an arrow outline; and 3. a *red magnetic needle* spinning freely inside the compass housing.

It's important to take a map bearing *before* you set out. First, place the compass on the map. Line up the direction of travel arrows on the base plate with your desired *line of travel* (the line between your present location and your destination).

Next, turn the compass housing until the orienteering lines on its transparent bottom are parallel with the meridian lines on the map. The marker on the dial (sometimes marked with an "N") will point to magnetic north (MN) as marked on the map.

Then, remove the compass from the map. Without moving the housing, turn the entire compass horizontally until the red magnetic needle points to north (or O°), as set by the marks on the clear dial. The needle will be aligned with the red orienteering lines of the compass.

Now walk in the direction pointed to by the red direction of travel arrow on the compass base plate.

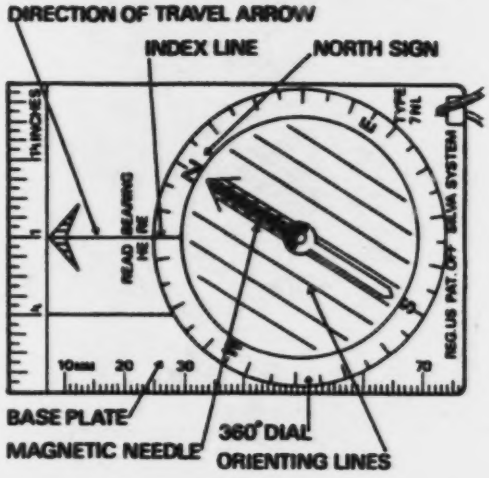


Fig. 1

*Orienteering compasses are available at many camping or sporting goods stores or through local orienteering clubs.

All About Map Scaling

To be accurate, a map must show everything on it in the same relative size. *Scale*, or the relationship between distance on the map and distance on the ground, is usually given as a fraction or ratio. The first number of the ratio is distance on the map, and is always 1. The second number is distance on the ground, and is set by the map maker. The larger the second number, the smaller the scale.

A scale of 1:10,000, for instance, means that one unit of measurement on the map (usually inches or centimeters) represents 10,000 of the same units on the ground.

At a scale of 1:316,800, 1 inch on the map represents 5 miles (316,800 inches) on the ground, and an area 100 miles square can be mapped on a sheet less than 2 feet square. Change the scale to 1:63,360 (1 inch = 1 mile, or 63,360 inches) and you'd need a map more than 8 feet square to cover the same area.

Some people, such as urban planners, need very large-scale maps for precise location of houses, power lines and streets. Maps of this sort are commonly scaled at 1:600 (1 inch = 50 feet); buildings, roads, railroad tracks and other features usually represented on maps by symbols can be easily drawn to scale.

To determine distances on a map, measure or estimate the distance between the two points you are interested in. Then multiply by the unit of measurement each map unit represents. For instance, if the distance on the map is 2 inches, and the scale is 1 inch = 5 miles, then the actual distance represented is 2 × 5, or 10 miles.

Fig. 1

The following exercises should give students some practice in scale and mapping.

Exercises:

1. Draw a map of the classroom. Start by measuring the length and width, noting the overall shape and the placement of objects in the classroom. Decide how large and/or detailed you want the representation to be (it's helpful to use graph paper) and develop your own scale.

What symbols did you use to show special features on your map (windows, doors, desks, etc.)? How did you choose them? What scale did you use? Why?

Try mapping the classroom in relation to other classrooms and the rest of the school. Then, try mapping the school in relation to the street and town it's located in. How does scale change in mapping larger areas?

2. Set a scale and draw a map representing the following series of directions (again, it's probably helpful to use graph paper):

- Start at Point S and walk 20 steps due north to point 1.
- Turn 45° to the east and walk 15 steps to Point 2.
- Turn to the right 65° and walk 30 steps to Point 3.
- Turn to the right 70° and walk 10 steps to Point 4.
- Turn right 30° and walk 28 steps to Point 5.
- Now go back to Point S.

a. How many steps is it from Point 5 to Point S?
b. How many degrees was your last turn?

3. Now take a compass (see box on how to use an orienteering compass, page 16), pencil and paper to a local park or playground. Try to make up a route for yourself and draw a map of it.

Walk a fixed distance from your starting point. Turn a certain number of degrees (you decide; use the compass for measurements) and count out another number of steps.

Continue for as long as you wish. Set a scale and draw your route on paper. Use a protractor to make sure the angles you draw are the same as the ones you measured with the compass. You can test the accuracy of your map by leaving markers at each "reference" point and then having a classmate try to find all of the markers by following your map.

4. On a certain map, 1 cm represents 10 km. Find the actual distance between two points which are 3 cm apart on the map.

5. Answer Exercise 4 where the two points are 2.4 cm apart on the map.

6. Two towns are 38 km apart. If each town is represented by a point on a map whose scale is 1 cm = 20 km, find the distance on the map between the two points representing these towns.

7. An architect is drawing the blueprint of a house using the scale 1 cm = 4 m. The sketch of a room on the blueprint measures 2 cm by 3 cm. Find the actual dimensions of the room, the area of the room on the blueprint and the actual area of the room.

1:100,000-scale

Fig. 2

1:125,000-scale

Fig. 3

Three views of Mt. Rainier, Washington, each at a different scale. Fig. 1 is 1:24,000; Fig. 2 is 1:100,000 and Fig. 3 is 1:125,000.

Finding Your Way:
An Orienteering Mini-Course*

Orienteering is an enjoyable sport and an exercise in many problem-solving skills. This mini-course in orienteering will give your students some practice in all kinds of map reading and prepare interested students for a real orienteering course.

For further information about orienteering and orienteering courses available locally, contact:

United States Orienteering Federation
P.O. Box 1039
Ballwin MO 63011

Orienteering Services U.S.A. Division
Educational Promotional Branch
P.O. Box 1604
Binghamton NY 13902
607-724-0411

*Some of the following exercises were adapted from "How to Teach Map and Compass Skills," a pamphlet published by the National Science Teachers Association and available by writing them at 1742 Connecticut Avenue NW, Washington DC 20009. Ask for stock number: 471-14704 (Price: $0.60).

Exercise #1

More Maps

Bring to class a map of a region in your state that shows land features, roads and highways, key cities and towns and points of interest. With the map tacked to a bulletin board, familiarize yourself with map reading by asking yourself some basic questions: How far and in what direction would you have to travel between two cities? What would be the fastest route to take? What would be the most scenic? Traveling at an average speed of x kph, how long would it take? etc.

Then, try making up your own problems using the information on the map. Use scale, direction of travel, distance rate time and any other mathematical concepts that might be relevant.

Exercise #2

Profiles in Disguise

Try to visualize how different landscape features might be represented on a topographic map and vice versa.

1. Match the drawings **(a–d)** with the verbal descriptions **(1–4)** provided.

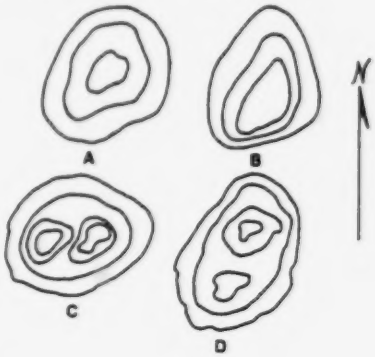


Fig. 1

1. a round hill with two summits
2. a steep southern side
3. gentle, uniform slopes
4. two summits—the northern one is higher

2. Match the land profiles **(a–d)** with the contour lines **(1–4)** provided.

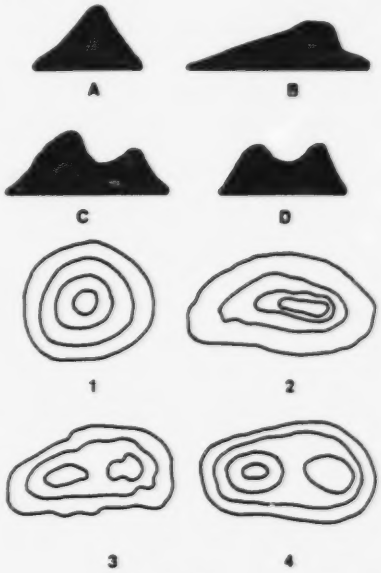


Fig. 2

Exercise #3

Point to Point

Before starting this exercise, you will need to identify the directions **N** and **S** and the scale of distance. The information, **"Point 6 185° for 70 m,"** is particularly helpful.

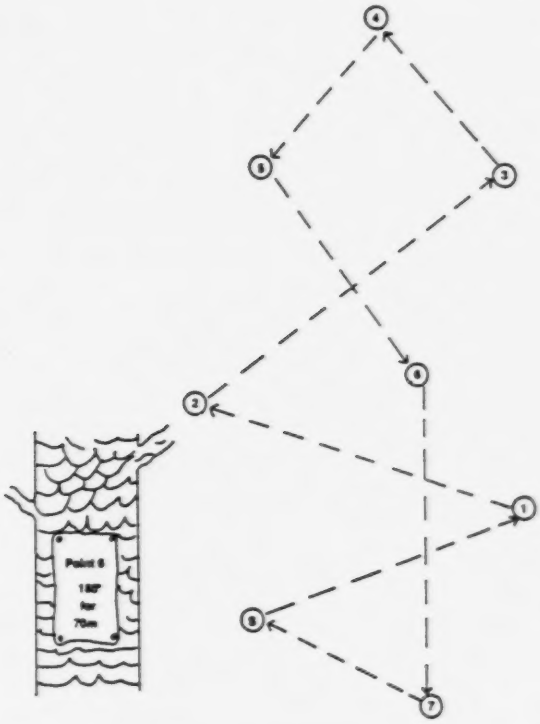


Fig. 3

Requirements: ruler, protractor.

1. How far is it from **Point S** to **Point 1** to **Point 2**?

2. If it takes you 2 minutes to go from **Point S** to **Point 1**, about how long will it take, traveling at the same speed, to go from **Point 1** to **Point 2**? How long will it take to go from **Points 2** to **3**? **3** to **4**? **7** to **S**?

3. What is the total distance of the course?

4. Using a protractor, measure the angles at **Points 1, 2, 5, 6, S.**

5. Calculate how long it would take to complete the course if you walked at an average speed of 7 kph.

Further Readings

Bengtsson, Hans, and Atkinson, George. *Orienteering for Sport and Pleasure.* Brattleboro, Vermont: Stephen Greene Press, 1977.

This 224-page book offers a thorough introduction to orienteering, including proper use of a map and compass, descriptions of the various types of meets, techniques and training, clothing and equipment and a brief history of the sport. It also offers a list of orienteering clubs and a sample map.

Henley, B.M. *Orienteering.* Wakefield, England: EP Publishing, 1978.

A clearly written and copiously illustrated description of orienteering. Lots of black-and-white photos, sample maps. An easy-to-follow, step-by-step beginner's walk guides the reader through a course.

Larkin, Robert F., and Schoenstein, Roger. "How to Teach Map and Compass Skills." Washington, D.C.: National Science Teachers Association, 1976.

These teaching notes provide exercises and activities to help students learn how to read a map and compass. The well-illustrated activitites include desk exercises and games (some of which are included here), as well as simple orienteering exercises that can be done in class and on the school grounds. (Available from NSTA, 1742 Connecticut Avenue NW, Washington DC 20009, Stock #471-14704, Price: $0.60).

Linthicum, Dave. *Orienteering Bibliography.* United States Orienteering Federation (USOF), September 1980.

This 12-page annotated bibliography gives a short description of every book and article written about orienteering in the past 30 years. Its subcategories include training and techniques, cartography, meet organization and films and filmstrips. An excellent resource for students and teachers seriously interested in the sport. (Available from USOF, P.O. Box 1039, Ballwin MO 63011.)

Where the Wind Blows

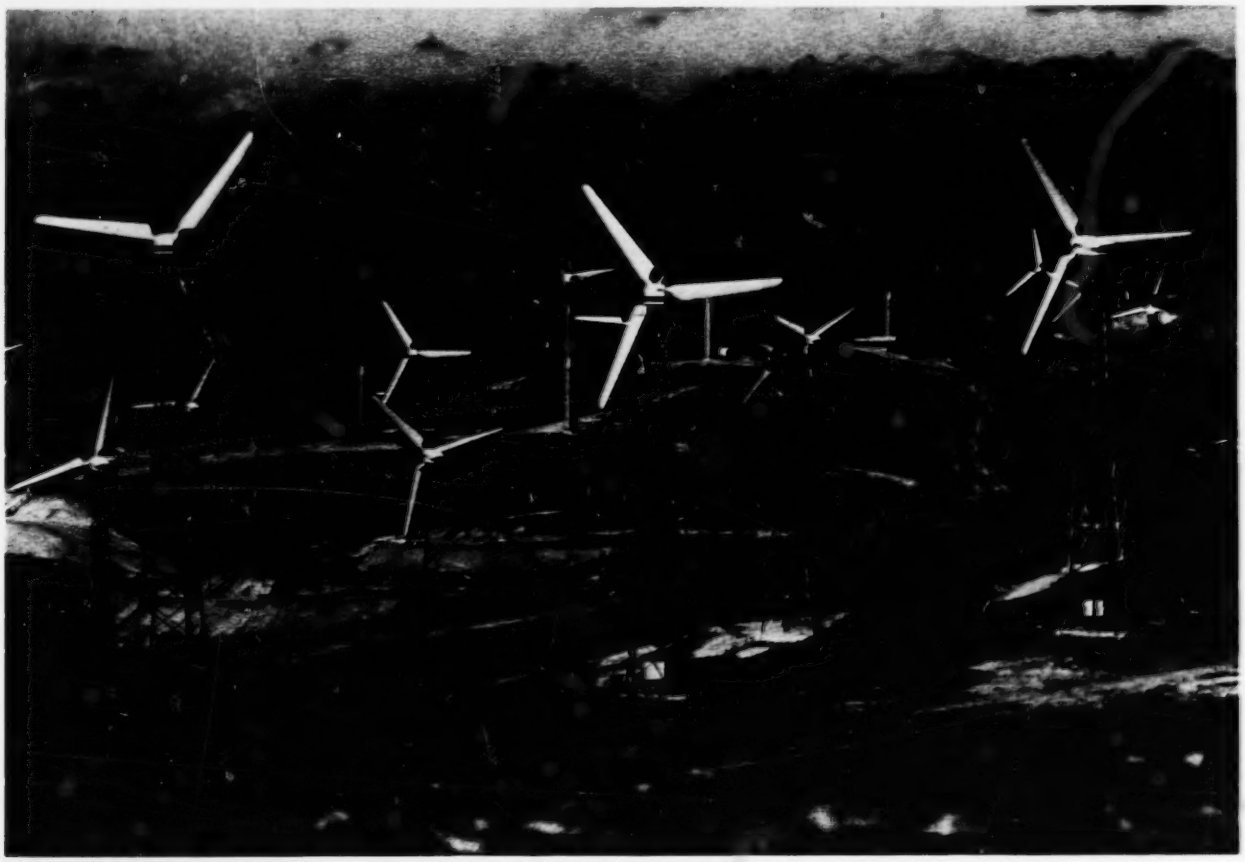
With rising costs of fuel and electricity, the old idea of using wind power as an alternative energy source is generating new interest . . . and new energy.

The same wind that propels sail planes aloft thousands of feet over the Tehachapi Mountains in southern California also provides a clean and environmentally safe source of energy to a growing number of households in southern California.

Wind power is by no means a new commodity; for thousands of years it has supplied power to pump water, grind wheat and move ships across oceans, among many other things. With rising costs of fuel and electricity, the old idea of using wind power as an alternative energy source is generating new interest.

The first requirement of wind energy is, naturally, wind—a strong steady wind to provide constant power. In the Tehachapi Mountains, the situation is right: Winds rise over the Pacific Ocean and race 200 miles across California's San Joaquin Valley. As they reach the Tehachapis they are funneled, at great speed, through a pass that leads to the hot, arid Mojave Desert. At this junction between valley and desert, geography and weather conditions create an ideal situation for wind power.

Once a general area has been identified, wind prospectors like Eliza Dixon locate the best sites for companies to place wind-powered generators or turbines. The first thing Dixon considers is the lay of the land. By examining trees and vegetation, for example, she can infer a great deal about the wind speeds and wind directions of a given area.



WIND POWER: Each one of these towering California turbines is situated on a spot specially chosen to make the most of the strong, steady Pacific wind over the Tehachapi Mountains—one of the windiest places on the continent.

Towering Records

"Once you have found a spot that you think will be fairly good," says Eliza Dixon, "you put up a tower and take data for a period of time." The tower is capped with an *anemometer*, an electrical device which measures the speed of the wind. This instrument consists of several cups that rotate around an axle. Wired to another device on the ground, it can record wind speeds at regular intervals, 24 hours a day. In this way, an average wind speed for the site can be determined.

A few feet from the anemometer, however, the wind situation may be different. To measure wind speed at different heights and locations, Dixon uses a weather kite connected to another device called a *dynamometer*. With a counter, she first measures the line to the kite, and then calculates, with the aid of a *clinometer**, the angle the kite string makes with the earth. With this information, she can then figure the exact height at which the kite is flying. (See page 22 for more information on her calculations.)

The dynamometer gives a reading of wind speed as the pull of the kite on its string. The amount of pull is registered through a sensitive spring mechanism within the dynamometer. This method enables Dixon to get a reading of the wind speeds at heights and locations not covered by the anemometer towers. By plotting all her findings, Dixon can then pinpoint exactly where the winds blow the strongest and most consistently.

When Dixon finds a site where steady winds average 12 miles per hour, she can recommend raising a wind turbine to generate electricity. At present, there are hundreds of turbines whirring away in the Tehachapi Mountains. The placement of each one was determined by taking a calculating look at the situation. ■

*An instrument used for measuring the angle of an incline.

TESTING THE WIND: Wind prospector Eliza Dixon prepares to erect a 30-foot anemometer tower, which will measure the speed of the wind at one particular spot and altitude. To judge a location's suitability for a wind farm, she will raise several of the towers and compare the readings over a period of time. To find the wind speed at different altitudes, she flies a kite with an attached dynamometer. If the wind averages over 12 mph along a broad front, it's a good place for a wind farm.

Further Readings

Farrell, John. "Reaping the Wind." *Westways*, November 1983.

The story of the founding of Zond Systems, the wind farm company featured in SITUATION. The article describes how and why this particular site was chosen, and how the wind farm was designed.

Park, Jack, and Obermeier, John. *Common Sense Wind Energy*. Andover, Mass.: Brick House Publishing, 1982.

This 75-page booklet, put out by the California Office of Appropriate Technology, tells you everything you want to know about wind power as an energy source. Good charts, tables and line drawings help keep explanations accessible to a general reader. (Available from Brick House Publishing Company, 34 Essex Street, Andover MA 08101.)

Spaulding, James. "Wind Prospecting." *Science 80*, March/April 1980.

An article on the development of modern wind prospecting. This article talks about how the method evolved and how it contributes to wind-farm development.

Go Fly a Kite

Eliza Dixon calculates the height of her kite using the basic formulas for right triangles. Imagine a huge right triangle with the distance between Dixon and the kite as the hypotenuse, an imaginary line from the kite to the ground as one side of the triangle and the distance from Dixon to the base of that imaginary line as the other.

In the film, Dixon mentions that the string length is about 37 yards and the angle between the kite string and the ground is 45°. What is the height of her kite?

(Note: First, correct for the sag of the string.
37 yards × 0.95 = 35.15 yards.
The hypotenuse, rounded off, is 35 yards.)

Here are two ways she can find the height of the kite:

Method 1: According to the Pythagorean theorem, since the angle where Dixon stands (A) is 45°, the imaginary triangle is an isosceles right triangle and the two sides a and b have equal length. (See Fig. 1.)

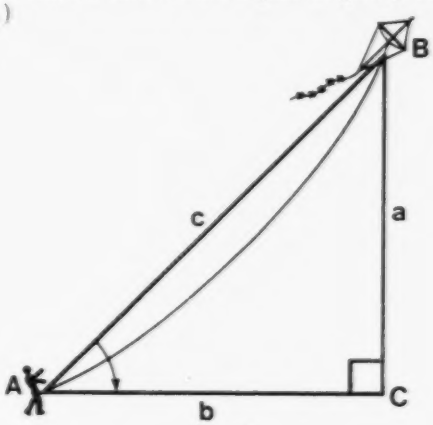


Fig. 1

Thus $c^2 = 2a^2$

or $(35)^2 = 2a^2$

$a^2 = 612.5$

$a = 24.75$ yds

The kite is about 25 yards above the ground at a point 25 yards from where Eliza Dixon is standing.

Method 2: This is the method Eliza Dixon uses. The angle A, between the string and the ground, is 45°. The length of the hypotenuse, c, is 35 yards.

The sine of the angle between the ground and the string equals the ratio of the opposite side to the hypotenuse.

Thus, sine $A = \frac{a}{c}$

or

$.707 = \frac{a}{35}$

$a = 35 \times .707$

$a = 24.7$ yds.

or about 25 yards.

Exercises:
In each of the following exercises, assume the ground is level and the kite string is perfectly taut.

1. A kite is tethered to a stake in the ground by a string 40 ft long, which makes an angle of 60° with the ground. How high is the kite (to the nearest foot)?

2. In Exercise 1 above, if the kite fell vertically to the ground, how far away from the stake would it land (to the nearest foot)?

3. A kite flying 20 ft above the ground is attached to a string 80 ft long. The string is being held by a person on the ground. Find the angle (to the nearest degree) between the string and the ground.

4. A kite flying 30 ft above the ground is attached to a string, which makes an angle of 52° with the ground. Find the length of the string (to the nearest foot).

Use methods 1 and 2 to work each of the following:

5. The angle where you stand is 45° and the kite string is 100 ft long. Find the height of the kite.

6. The angle where you stand is 45° and the height of the kite is 50 ft. Find the length of the string.

Answers:
1. 35 ft
2. 20 ft
3. 14°
4. 38 ft
5. $100\sqrt{2}$ or 71 ft
6. $50\sqrt{2}$ or 35 ft

X=

Measuring the Wind

The wind is air in motion. You can measure its speed using the familiar equation $d = r \times t$. This sort of direct measurement of the wind is applied by meteorologists with weather balloons.

Wind prospectors like Eliza Dixon want to determine the wind's speed at a particular location such as the site for a proposed wind turbine. Because the wind varies over time and distance, they need an instantaneous measure. So they use an *anemometer*. The anemometer converts the linear motion of the wind at a particular point into circular motion—rotation around an axis.

The anemometer's motion can be recorded in several ways: The rotations can be counted directly if the cups are spinning slowly enough. In practice, this can be difficult. For example, if the radius from the axis out to the cup is 1 foot, one revolution corresponds to 6.28 feet. In a 30-mph wind (44 feet per second), the cups would rotate about 7 times a second—too fast to count with the eye.

Alternatively, the anemometer might turn a small generator whose output of electricity can be recorded to give an average wind speed or an average of electrical potential. In high wind speed areas, this method is much more reliable.

Exercises:

1. Measure a course of 100 yd. Have one student race the course on a bicycle while another times him. With the distance and time determined, have them solve for rate.

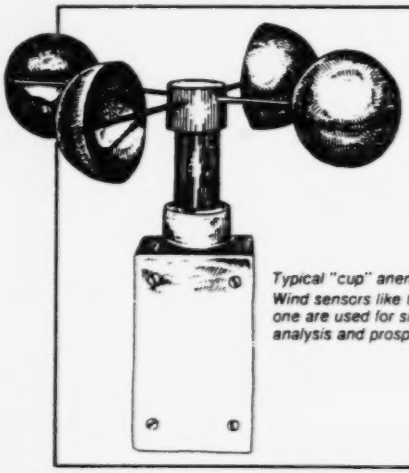
2. Find the circumference of a bicycle wheel whose diameter is 24 in. Express your answer in ft. Suggestion: First change 24 in to ft. Use $\pi = 3.14$.

3. How many revolutions of the wheels of a 24-in bicycle are needed to travel 50 yd?

4. A bicycle is traveling 15 mph. Express its speed in ft/sec. Express the speed in kph.

5. A bicycle with 24-in wheels is being ridden on a course so that the wheels revolve 20 times every 5 seconds. Calculate the speed of the bicycle in ft/sec.

Answers:
2. 6.28 ft
3. 24
4. 22 ft/sec; 25 kph
5. 25 ft/sec



Typical "cup" anemometer. Wind sensors like this one are used for site analysis and prospecting.

Wind Speed Cubed

Accurate wind-speed data are critical, according to Dixon, because *"energy production increases in proportion to the cube of wind speed."* In other words, if a 10-mph wind doubles its speed, the amount of energy produced will be increased not twice as much, but *eightfold*.

That is, if E represents the energy produced by a wind with speed S, then $E = kS^3$, where k is some constant of proportionality. If $S = 10$, $E = k(10)^3$, or $E = 1000k$. However, if S is doubled, $E = k(20)^3 = 8000k$, eight times the amount of energy produced if $S = 10$.

Finding the strongest wind at a site can make a big difference in the amount of energy the site will eventually produce.

Exercises:

1. How does the amount of energy produced change if the wind speed becomes $1\frac{1}{2}$ times greater?

2. If the wind speed triples, how is the amount of potential energy changed?

3. The windspeed today is half what it was yesterday. What is the relationship between the amount of energy produced today to that produced yesterday?

Answers:
1. The energy produced is $3\frac{3}{8}$ times greater.
2. The energy produced is 27 times greater.
3. The energy produced today is $\frac{1}{8}$ what it was yesterday.

Minneapolis in Motion Around the Sun

"You finish it at three in the morning. Everything's dark, so the piece is not working. You come back in the morning and you're as astonished as anyone else who comes to see it for the first time."

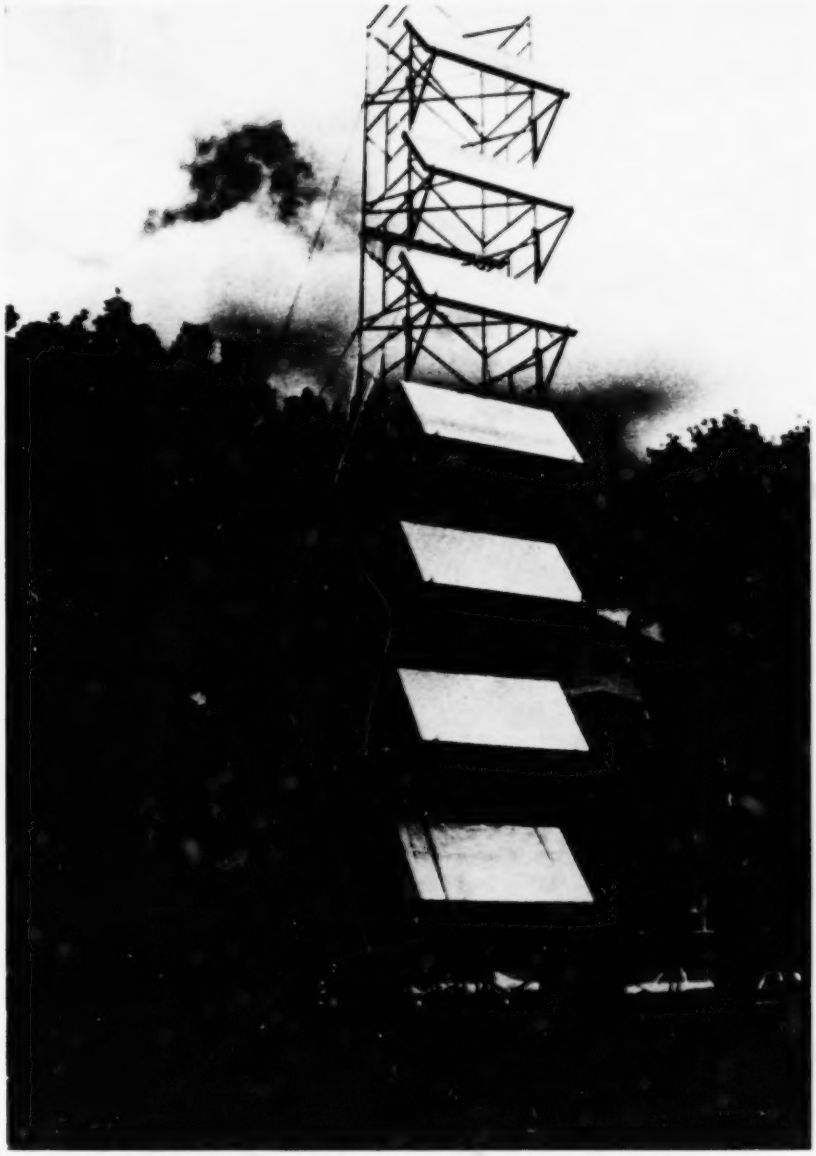
Sunlight is artist Dale Eldred's medium. With towers and panels and mirrors and the many façades of a city, his sun sculptures diffract and reflect sunlight, creating moving images of light and color.

His canvas can be huge, the size of an entire city. A single sculpture, like the one commissioned for the 1983 Centennial of the Minneapolis Institute of Art, may have interacting components scattered over several acres, and may be visible from miles away.

Eldred first became interested in sunlight as a medium during a visit to the Valley of the Queens in Egypt. The young architectural engineering graduate was struck by the way the guides illuminated even the furthest reaches of the pyramids. They did not use torches or lanterns, but mirrors, angled to bend sunlight around dark corners to be reflected onto other mirrors. This started Eldred thinking about the properties of reflected sunlight and the interplay of architecture and astronomy, of light and shadow.

Sunlight Plans

To control the play of sun and shadow, Eldred's sculptures depend on sophisticated planning, precise engineering and high technology materials. Eldred builds towers of racks that hold mirrors that reflect the sunlight onto the walls of buildings or billboard-size panels covered with fluorescent paint.



AN ARTIST'S PALETTE: Dale Eldred's tower of mirrors catches the sunlight and reflects it onto his "canvas," which might be a billboard, a wall or even an entire city.

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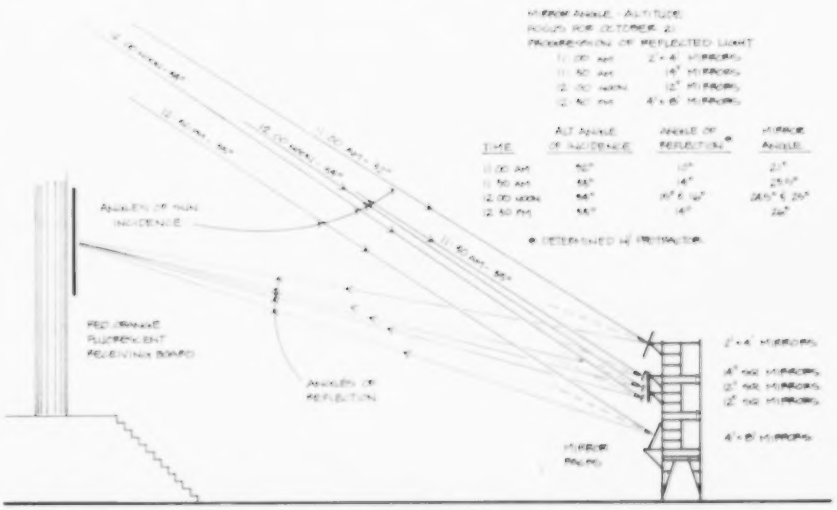
Other panels, covered with *prismatic diffraction grating* (PDG), a fine aluminum foil scored with grooves .00007 inches apart and sealed in clear plastic, diffract the sun's rays. When light hits PDG, it is broken up by the grooves, separating into the reds, greens and blues of the optical spectrum. (If you've ever angled a phonograph record against the light, you might have seen the effect.)

Each of Eldred's works is a combined product of his vision and the specific environment. Eldred begins planning one of his solar exhibits by checking out the situation: walking around the site to see what opportunities the terrain and the architecture offer. He then spends days mapping the sun's path, noting its altitude and angle during the time of year his sculpture will be "working," measuring and recording the length of the shadows the sun will cast.



GETTING A FIX ON THE SUN: Eldred's first job is to plot the sun's position day by day, hour by hour. Then, calculating angles of incidence and reflection, he fixes the position of his mirrors (below, with assistant) in order to reflect the sunlight into the patterns he wants. This drawing was made to determine the position of mirrors casting light onto the giant red-orange fluorescent board on the front of the Minneapolis Art Institute. The focus is figured for midday October 21, 1983, roughly the midpoint of the exhibit, which ran from September 15 to November 21.

Modeling Shadows

From there, Eldred drafts working drawings and then makes a scale model of the sculpture and site. By placing the model into a *heliodrome*—a scale model of the sun and earth—he can determine the exact action and interplay of the sun on the buildings and solar structures.

Finally, the models become real: The diffraction panels, mirrors, reflector boards and other pieces are built at Eldred's studio in Kansas City, Missouri, and trucked to the site for assembly. A crew of student volunteers erects the sculptures using cranes and forklift trucks. Installation is often not complete until long after dark. As Robbie Lord, an engineer who collaborates with Eldred, puts it, "You finish it at three in the morning. Everything's dark, so the piece is not working. You come back in the morning and you're as astonished as anyone else who comes to see it for the first time."

Situation Within Situation Within Situation

In Minneapolis, Eldred worked with a problem situation within a problem situation within a problem situation. His sculpture was designed to focus on the Minneapolis Institute of Art, situated in the city of Minneapolis and to highlight the changing angle of the sun during the fall of the year. Eldred turned the Art Institute into an enormous sundial of sorts, setting up towers of mirrors and diffraction panels all around the Institute grounds. Altogether, the towers, mirrors and panels recorded the sun's passage through the sky throughout the day.

In the morning, the mirrors and diffraction panels on the sides of the Institute that faced the rising sun would begin to light up. By noon, the bank of mirrors facing the north side of the Institute would be bouncing a row of jewel-like lights on a red fluorescent board across the front of the Institute. In the afternoon, other mirrors and diffraction panels on the sides of the museum, the ones facing the setting sun, would come into play.

Simultaneously, Eldred was working on a larger canvas: the entire city of Minneapolis. On the roof of a 50-story office tower, an 80- by 14-foot diffraction panel flamed with changing colors as the day progressed. At noon, 8 mirrors atop another downtown tower focused an intense burst of white light that could be seen by anyone standing on the front steps of the Institute about four miles away. And in the middle of Lake Calhoun, a hundred different diffraction sticks anchored in the water bobbed and dipped, reflecting continually changing colors throughout the day.

Equinox Art

For many people in Minneapolis, the 1983 autumnal equinox and the Minneapolis Institute of Art's centennial were made memorable because Eldred's art allowed them to see sunlight in a new way. As the sun moved in its daily arc through the sky, the light and prismatic colors of the sun sculptures moved and changed—an arresting, daily reminder of the earth's motion around the sun. ■

A MOMENT CAPTURED: One moment in the life of Eldred's work, different from the last moment and from the next, each one a reflection of its time.

Mirror, Mirror . . .

Dale Eldred begins creating his art by making careful observations of the sun's movement and how it plays on different surfaces of the city of Minneapolis throughout the day. These observations involve making accurate measurements of angles, lines and distances. With this data, Eldred is able to make predictions about where to place mirrors which will reflect and diffract sunlight in the patterns that he wants to create. These pattern transformations are what make his work so dynamic.

Geometry is concerned with both static patterns and transforming patterns. To give students practice in recognizing patterns in transformation, try the following exercises.

Exercises:

1. *You will need a small rectangular mirror.* Pivot the mirror on the line in Figure 1 to duplicate Figures **a–d**. In each case, draw a dotted line to show where the mirror must be placed.

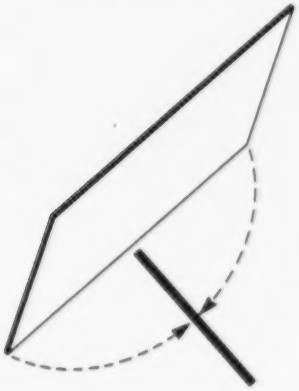
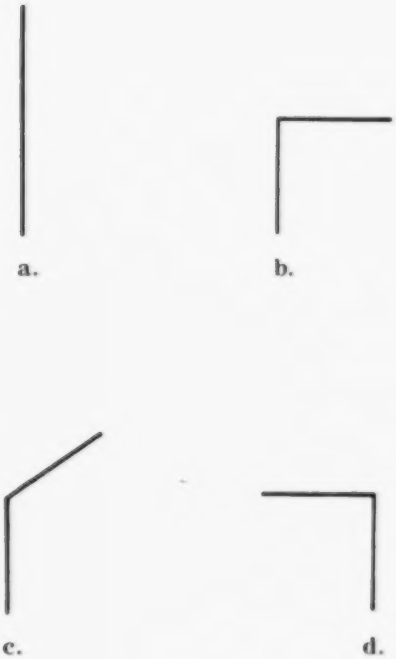


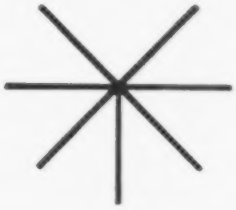
Fig. 1



2. *You will need two identical rectangular mirrors, hinged together with tape, as in Figure 2.* Rotate the mirrors in such a way on the solid black line in Figure 2 as to duplicate Figures **a–d**. In each case, draw dotted lines to show where the hinged mirror must be placed.

Fig. 2

a.



b.

c.

d.

Projective geometry principles are used by artists, architects and engineers to make drawings appear three dimensional. This is called *perspective drawing*, and it works because of the brain's ability to transform what is seen into understandable shapes. We do this not just by recognizing patterns but by recognizing relationships between patterns as well.

3. Imagine that the rectangle in Figure 3 is a table top and that you have to photograph it from different angles.

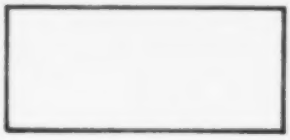


Fig. 3

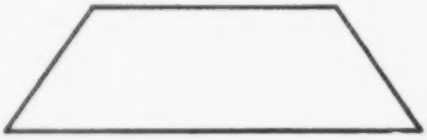


Fig. 4

Fig. 5

a. Where would you have to be standing to get a picture of the table shaped as Figure 4?
b. Where would you have to be standing to get a picture of the table shaped as Figure 5?
c. What would the table shape look like if you took the picture from the table's corner several feet away?

4. Another everyday illustration of how projective geometry works can be done by using a piece of cardboard (cut in the shape shown in Fig. 6) and a candle or flashlight. Set up the candle and the cardboard against a bare wall or large piece of white paper as in Figure 7. Practice changing the shape of the shadow cast by the light of the candle (or flashlight) on the card by changing the position of the card.

Fig. 6

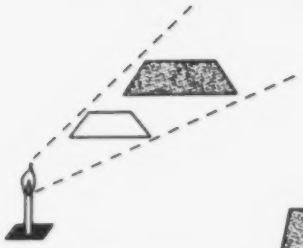


Fig. 7

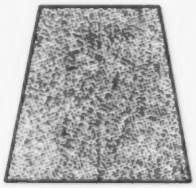


Fig. 8

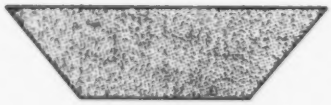


Fig. 9

a. How would you hold the cardboard relative to the candle and wall (or paper) to cast a shadow as in Figure 8?
b. How would you hold the cardboard to cast a shadow as in Figure 9?
c. Without using the cardboard, predict how you would hold it to cast a rectangular shadow. Now try it out with the cardboard.

These shadow problems illustrate a basic principle of geometric proof: The way you know that one shape is a transformation of another shape is by recognizing primitive (simple) features such as parallel lines or angles, and using those features to construct a more complex whole.

Further Readings

Coe, Ralph T. "Eldred: The Nelson Gallery and Atkins Museum."

A beautifully photographed booklet describing the extensive environmental/sun sculpture which artist Dale Eldred installed at the Nelson-Atkins Museum and its environs in 1979. Explains how Eldred conceived the piece and how its individual components work with the sun. (Available from the Nelson-Atkins Museum of Art, Kansas City MO.)

Jespersen, James, and Fitz-Randolph, Jane. *From Sundials to Atomic Clocks: Understanding Time and Frequency.* National Bureau of Standards, 1977.

This book offers an excellent introduction to time and time-keeping, beginning with telling time with the sun and the rotation of the earth and ending with atomic clocks (some 170 pages later). Amusing cartoons and graphics. (Available from Superintendent of Documents, U.S. Government Printing Office, Washington DC 20402.)

Maisel, Jay (photographer). "Sunlight Art," *Life*, October 1979.

An interview with Dale Eldred about the various influences on his work, accompanied by striking photos of his exhibit at the Nelson-Atkins Museum of Art in Kansas City.

Stein, Kathleen. "Solar Chronometry." *Omni*, May 1981.

Beautiful photographs and some explanation of how Eldred's sun sculptures work.

Teacher's Notes

(see opposite page)

Crossnumber Puzzle

This is a tough one! You might want to give your students a hint or two. For instance: In 3 across, what is the *only* number that, when multiplied by itself five times, results in a two-digit number?

Answer:

1 2	**2** 1	**3** 3	**4** 2	**5** 4
6 1	4	3	**7** 3	6
8	**8** 4	3	**9** 2	**10** 4
11 4	**12** 9	**13** 1	2	0
8	**14** 6	4	**15** 5	6

a = 2; b = 3; c = 5; d = 7; e = 8;
f = 12; g = 11

It's Only Logical

Sometimes, trial and error is the key to unlocking a situation. The matrix approach to this problem helps structure trials and minimize errors. You might suggest that students use a matrix-type of chart to help them put the pieces of this problem in order. Squares are "x'd" out as they are ruled out as possible positions. The *bah* can't be least. The *ick* is second, so it is written in and the rest of its row and column are crossed out. And so on.

smallest ⟶ largest

yuk		X			
ugh		X			
bah	X	X			
ick	X	ick	X	X	X
glok		X			

Fig. 1.

Or students might write the names on slips of paper and move them around until they find an order that satisfies all the conditions of the problem.

Answer:
From smallest to largest, the order is *ugh, ick, bah, yuk, glok*.

X=

43

Crossnumber Puzzle

Here's a familiar situation with a twist. Use your knowledge of how to solve crossword puzzles to solve this crossnumber puzzle. Fill in the numeric value that fits each clue below. All are whole numbers. Each letter has only one value for all the clues.

1	2	3	4	5
6			7	
	8		9	10
11	12	13		
	14		15	

Across		Down	
1. bd	**11.** d^2	**2.** f^2	**9.** $(b + f)^2$
3. a^5	**13.** acf	**3.** bg	**10.** ce
6. 13g	**14.** e^2	**4.** $a^4 + d$	**11.** abe
7. $(ab)^2$	**15.** a^3d	**5.** $d^2 - b$	**12.** ef
8. $48b^2$		**6.** ab^2	**13.** ad

a = _____; b = _____; c = _____;
d = _____; e = _____; f = _____;
g = _____.

It's Only Logical

1. A *bah* is not the least of the whole group.

2. Only one thing is less than an *ick*.

3. A *yuk* is more than an *ugh*.

4. A *yuk* is not the greatest.

5. A *glok* is more than a *bah*.

6. More than one thing is greater than a *bah*.

Put the *yuk*, *ugh*, *bah*, *ick* and *glok* in order from smallest to largest.

"Understanding the problem—an awareness of the problem situation that stimulates the person to generate a statement of the problem in writing, orally or merely in thought."

Marilyn N. Suydam, *Problem Solving in School Mathematics*

Teacher's Notes

(see opposite page)

Catwalk

"Where are they," indeed? The essence of a situation problem, made more difficult by the density of extraneous information it contains. To begin with, it may be helpful for students to cross out all such extraneous information before beginning to solve the problem.

There are several ways to solve it. Here are two.

1. Keep an imaginary set of coordinates, x and y. (0,0) is the location of Alison's house. When she and Zeus travel north, add the number of blocks to the y coordinate; when they travel south, subtract the number of blocks from the y coordinate. Do the same for the x coordinate: Add the number of blocks they travel east, and subtract the number of blocks they travel west. They start out heading 3 blocks north, so their location is now (0,3). Two blocks east puts them at (2,3), and when they arrive at the grocer's, 4 blocks south, they are at (2, – 1).

From there they travel as follows:

5 blocks west	(– 3, – 1)
2 blocks south	(– 3, – 3)
7 blocks east	(4, – 3)
8 blocks north	(4, 5)
4 blocks west	(0, 5)

Alison and Zeus end up 5 blocks due north of home.

2. Draw a map on graph paper, with one square equaling one block. Fill in the squares on the graph paper after establishing which square is home base and which direction on the graph paper is north, south, east and west.

Students might come up with their own systems for plotting the situation.

Class Project:
As a follow-up to this exercise, divide the class into teams and have each team try to stump the others with a "route scenario" of their own.

Advanced students might want to try a similar scenario using three dimensions rather than two.

Over Hill, Over Dale

A careful analysis of the situation depicted in the graph shows that "4" is the correct answer. The graph shows the walkers covering 5 km in the first hour; over the span of the second hour they remained at a constant distance of 5 km from their starting point; then they covered the original 5 km back home in the third and fourth hours.

"The process of finding a solution has two major parts: 1) representing the gap—that is, understanding the nature of the problem, and 2) searching for a means to cross it. . . . If people fail to understand the nature of the gap (problem) they may well set off in the wrong direction to search for a solution."

John R. Hayes, *The Complete Problem Solver*

Catwalk

It was a beautiful day and Alison decided to take her cat Zeus out for a walk. As long as she was going out, she thought she would run all the errands she had been putting off all week. First she headed for the drugstore, which was 3 blocks north of her house, to buy some shampoo and toothpaste. From there, she went 2 blocks east to the dry cleaner's to pick up the skirt they'd been holding for her for 2 weeks. It was 4 blocks south to the grocer's, where she bought some yogurt for herself and some milk for Zeus, and then 5 blocks west to the hardware store to look for a can of Pale Lemon paint.

Two blocks south of the hardware store was the stationery store, where she bought a magazine and a box of paperclips. She then went 7 blocks east to stop at the post office to mail a package to her brother in California. Eight blocks north of the post office, Zeus began to get tired and looked at her imploringly. "Just one more stop," she said, "and then we'll go home for your catnap." They walked 4 blocks west to the pet store, where she got some catnip for Zeus as a reward for his patience.

Where are they? And how do they get home?

Over Hill, Over Dale

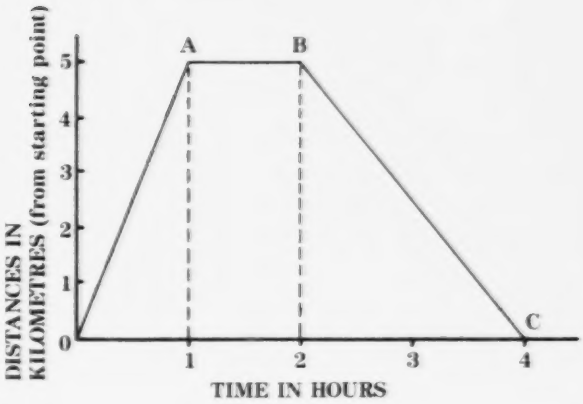


Fig. 1

Figure 1 describes the progress of a country walk. Which of the following describes the situation accurately?

1. A hill is 5 km high; it took 1 hour to get to the top and another hour to walk to the other side. It took 2 hours to get down the hill from the other side.

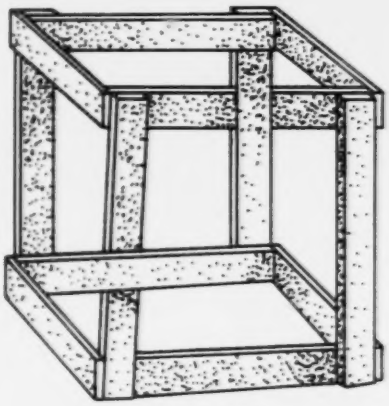
2. The walk lasts 4 hours. In the first hour they walk for 5 km, building up speed as they go. For the next hour the pace is fast and they walk another 5 km, but gradually over the next 2 hours the pace gets slower and slower until they stop.

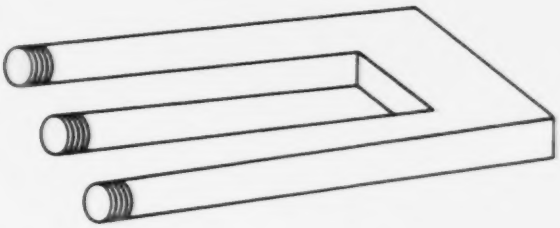
3. In 1 hour they walked 5 km; the second hour the pace was the same, but then it dropped to a rate of $2^1/_2$ kpm and by hour 4 they weren't walking anywhere at all.

4. The walkers covered 5 km in the first hour, rested for an hour and then walked back half as fast.

Seeing is Believing

It's often tempting to try to take in the whole situation at a glance, especially if it looks familiar. But watch out! Things are not always what they seem.





Appearances Aren't Everything

You can't always trust your eyes to give you the true picture of a situation. In Figure 1, the lower figure *appears* larger than the upper although both are the same size.

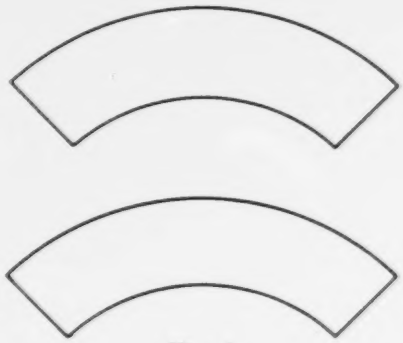


Fig. 1

Figure 2 shows how the total situation can influence the way you see the details. The figure lying on the striped background *is* an exact circle.

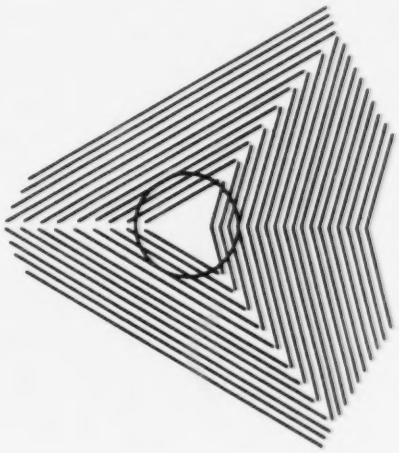


Fig. 2

A Thought for Your Pennies

Find the "real" penny below.

This problem points out the importance of being observant. It also shows how too many details can obscure the "telling" detail.

Scaling the Heights

You're the prop designer for a science-fiction movie about a 6-foot-tall man who shrinks to a height of 2 inches. The man can't shrink; you have to create the illusion that he does by enlarging the objects in his world.

For one scene, you must design the following props, all scaled to make the man appear smaller: scissors, pencil, paper bag, hairbrush, soda can, piece of pie, orange, book, typewriter.

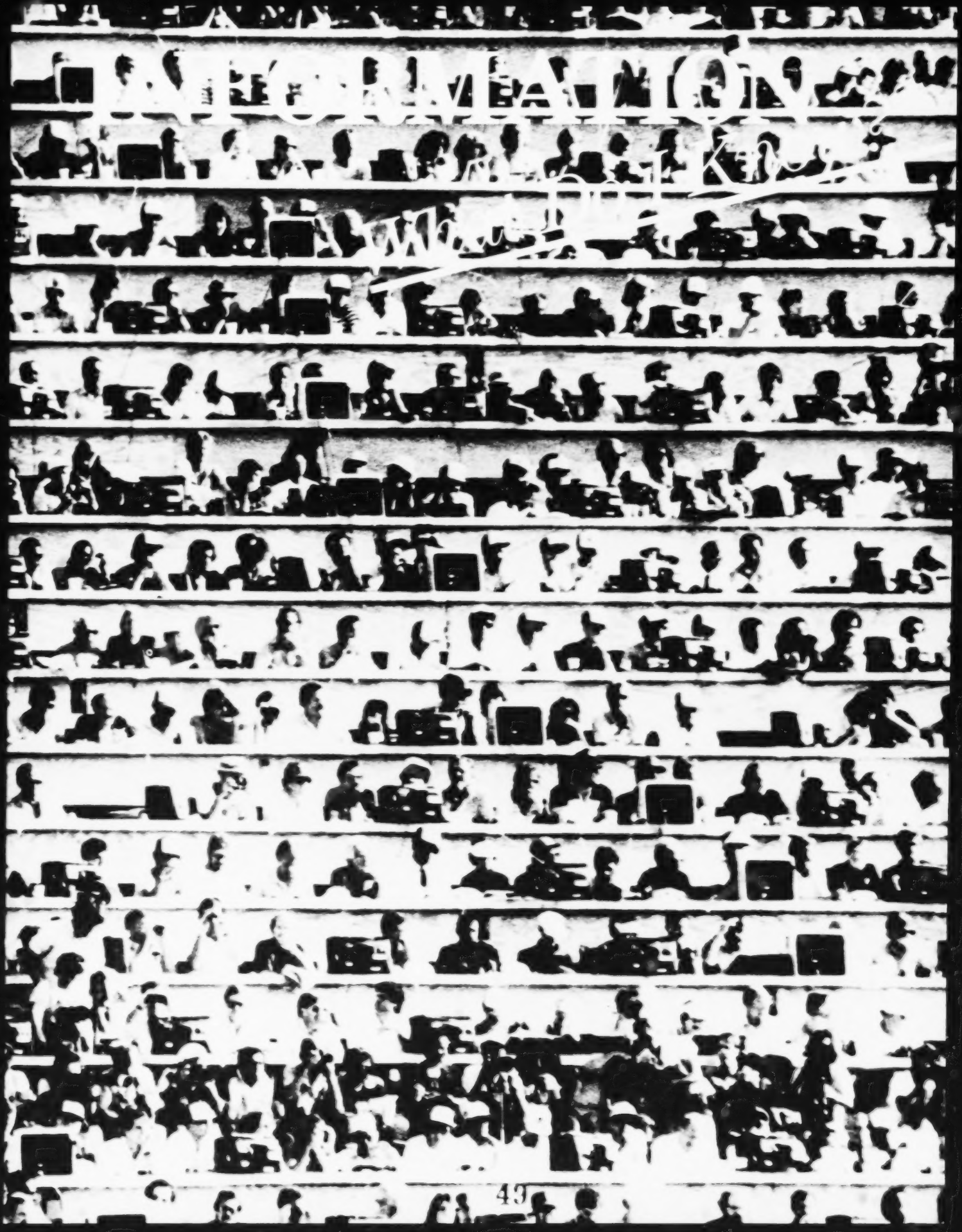
What will be the scale of the props? What will be their dimensions (based on your measurements of the real objects)?

Think of other common objects that might appear in such a movie and calculate the sizes the props would have to be.

Can you think of objects that could not be shown in full? For example, large objects such as armchairs, tables, refrigerators and automobiles could not be shown. Why not?

"Recognizing patterns is an important skill that young students are encouraged to develop . . . pattern recognition becomes more of an active search than a passive observation."

The Ohio Department of Education, *Problem Solving . . . A Basic Mathematics Goal*



Fact Sheet

INFORMATION: *What Do I Know?*

Running Time: 21:08 minutes

Documentary Segments (4):

1. ***ICE CLIMBING:*** Ice climbing is not just a matter of physical prowess, but also of expertise in gauging climbing conditions. **How do ice climbers know when to risk a climb and when to walk away?** ... 52
2. ***ANASAZI INDIANS:*** A thousand years ago the Anasazi Indians flourished in the American Southwest; then they disappeared mysteriously. **How much of their story can be decoded from the ruins they left behind?** ... 56
3. ***SPEED SKIING:*** Barreling straight down a mountainside, speed skiers reach speeds of up to 130 miles per hour. **How fast can they go—and live to tell about it?** ... 62
4. ***FISH FINDING:*** Sonar solved one problem for Norway's fishermen, but created another. **Now that sonar can lead fishermen to the fish with pinpoint accuracy, can it also be used to limit the number of fish caught?** ... 68

Comedy Segment:

5. ***THE LINEUP:*** "Bill the Hoofer" thinks he's auditioning for a big part, but he's blundered into a police lineup instead. **Can he now present the facts to convince the police they have the wrong man?** ... 50

Math Concepts:

ICE CLIMBING: Temperature and pressure, Celsius and Fahrenheit conversion, sequencing, measurement.

ANASAZI INDIANS: Mapping, Cartesian coordinates, three-dimensional graphing, volume, measurement (time and distance).

SPEED SKIING: Graphing, slope and acceleration, distance/rate/time, mathematical modeling, velocity, acceleration of gravity, exponents.

FISH FINDING: Distance/rate/time, navigation, percentages.

Problem-Solving Concepts:

Data collection; analysis; patterns; research; assessing quality of information (identifying relevant, irrelevant and missing data).

HOW TO SPOT AN INFORMATION PROBLEM

You know it's an INFORMATION problem if . . .

- ***There is data missing****—something you need to know which is essential to the solution process;*
- ***There is an array of data that needs sorting or sifting to make sense of it****—in order to reach a decision or arrive at a solution;*
- ***The key to the lock is some piece of data.*** *You must have it; the problem cannot proceed without it;*
- ***The answer is in the form of new information.*** *Some information problems ask only that new information results from a calculation or ordering of existing data.*

X=

Throughout the chapter, the X = sign indicates pages containing exercises and problems for classroom use and discussion.

PACKED GALLERY: Hordes of reporters covering the 1984 Summer Olympics made Los Angeles for a time the information capital of the world.

50

BEST COPY AVAILABLE

INFORMATION

What Do I Know?

Every day we have to contend with an avalanche of information—computers, books, newspapers, radio and television, magazines, memos, circulars, brochures, numbers, addresses—all of which demand attention to one degree or another. Only by developing skills in gathering, sorting and interpreting information can we save ourselves from being buried alive.

Why Information?

Mathematics provides the problem solver with powerful tools to sort and analyze information. Formulas, graphs, and equations are among the many mathematical tools that can be employed to break down information that might not easily lend itself to analysis in any other way. In many cases, mathematics provides the problem solver with the means to simplify a problem so that patterns and relationships can be identified and a solution found more readily.

Where mathematics can serve to *simplify* an information-rich environment, computers and other products of this technological age allow you to collect, assemble and analyze data in quantities and with a precision that dazzles the mind.

In whatever form it is used, *quantification* is one of the most basic and powerful tools at your disposal. It allows you, quite literally, to compare apples and oranges—at least in numerical terms such as weight, density, nutritional value and caloric content.

Teaching Information

Here are some of the rules of the road for problem-solving detectives who need to sort through and manage information.

Too much information can obscure a problem. Weed out extraneous information until all that remains is essential to finding a solution. For each piece of information, ask the question: Is this leading me toward or away from a solution?

Too little information can hold up the problem-solving process. Pinpoint as closely as possible what additional information is needed and how best to find it.

Test information for reliability. Don't take it for granted that all information comes from a valid and viable source. Test its pedigree. Who says it's true, anyway?

Information loses meaning out of context. A single clue abstracted from the scene of a crime loses value. *Where* a piece of information comes from in relation to others is itself valuable information.

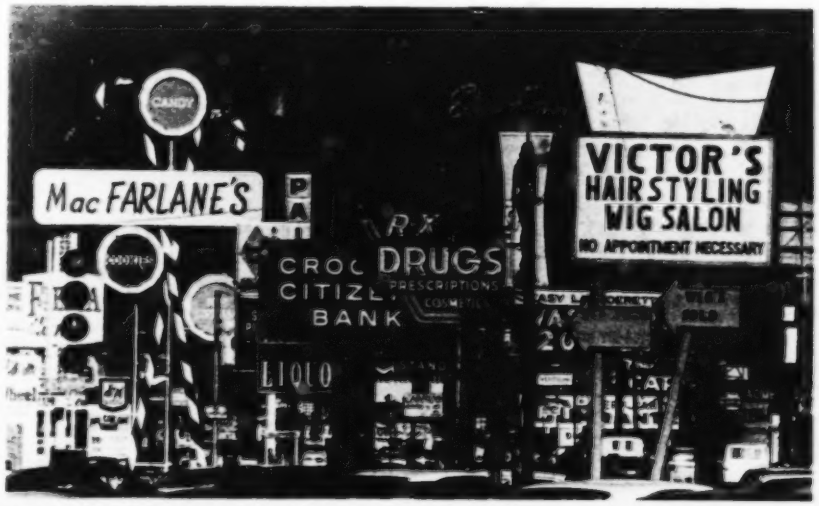
Marshall facts. With the needed information in hand, analysis can begin. When the pieces of the puzzle are assembled, patterns emerge which point in the direction of a solution. If difficulties arise, the problem solver must try to put the information in some other form that might make it easier to see a pattern—a table or a graph or a chart of some sort, for instance. Does any one piece of data stand out? Does that fact, or any other, invalidate the rest? Check the sources again. Is the analysis persuasive? Will it convince others? ■

Information in the Classroom

There's little room in the busy math curriculum for teaching information skills. Students are reminded to cross out extraneous information in word problems to get to the core of the relationship at hand, but by and large the classroom is an artificial world of "perfect" information. Students can expect that each and every exercise or problem will present them with all the relevant information to complete the task. There are, however, several areas in the math curriculum that can allow students to tackle information in a broader sense.

Graphing: Graphs, charts and tables are excellent ways not only to display information visually, but also to explore and analyze it. Even a task as simple as labeling the axes of a graph assumes that a relationship or function is suggested by the data—and spotted by the problem solver. Converting a chart or table into a graph or vice versa allows students to get a feel for the meaning of information and develop strengths in spotting trends or patterns. Taking a graph and converting it to a "story," a paragraph describing the data, can help even math-anxious students get a start in elementary "data analyses."

Mapping and Map Coordinates: Descartes' discovery—of locating points in a plane by describing their distance from two perpendicular lines—was an important advance in mathematics. Coordinate geometry also allows geometric figures to be related to equations, thus combining algebra and geometry.



INFORMATION OVERLOAD: How much information is too much?

With practice, students should become "at home" in the world of coordinate geometry. To give them practice, you might want to label a desk in the middle of the center row of the classroom as the *origin* with coordinates (0,0). All desks in that row can then be given y-axis coordinates and all desks in a row across from the (0,0) desk can be given x-axis coordinates. Using this scheme, each desk in the room can be identified by a pair of coordinates. Then, call roll by coordinates.

Sequences: Sequences, such as 2,4,6,8 or 1,1,2,3,5,8 for instance, common in textbook exercises or standardized tests, are patterns suggested by given information, which students must decode. Ranging from simple arithmetic progressions to two- or even three-variable sequences, they often baffle students, perhaps because students focus on "the right answer" rather than on the underlying code—the pattern of information.

Patterns: Patterns are an ever-present theme in mathematics from the most elementary to the most advanced level. They are tools with which we build order out of information. If students can discern a pattern behind the specifics of a problem or exercise, they can use the tool to fill in missing information. ■

52 *"Knowledge is of two kinds: We know a subject ourselves, or we know where we can find information upon it."*

Samuel Johnson

Behind the Scenes: INFORMATION On Film

The word "information" calls to mind images of libraries or computer terminals, bits and bytes and encyclopedias. All are worthy subjects of study, but hardly material for an exciting film. In the INFORMATION film we wanted to show that information counts, that it can save lives and livelihoods, that it can make an athlete perform better and that even the smallest piece of data can open up new vistas of understanding.

Ice Climbing: In the world of filmmaking, like the world of problem solving, things don't always go smoothly. On our first day of shooting, we were trying to capture on film the team effort of ice climbing. Instead, we filmed the spectacular collapse of a large spire of ice at the very moment that the climbers were discussing how unsafe it seemed. We returned to shoot another climb, giving us two kinds of success stories—the decision not to climb based upon the climbers' assessment of unsafe conditions and the conquering of another pinnacle of ice under better—and safer—conditions. This segment shows the importance of accurate data collection and assessment, especially when lives hang in the balance.

Anasazi Indians: Prehistoric archaeology is a rich field for demonstrating the value of information. Without written records, it is the artifacts, the "trash" of a dead civilization that supply the information with which we must reconstruct the past. Archaeologist Randy Morrison of the National Park Service took us on a tour of the Mesa Verde and Chaco Canyon ruins of the Anasazi Indians. He provides a superb commentary on the value of information, the importance of context and how, in some cases, unsophisticated data-collection techniques of the past have made data gathering and analysis difficult, if not impossible, for today's archaeologists.

Speed Skiing: In the past decade, athletes have teamed up with computer scientists, engineers, doctors and mathematicians to improve their performance by trying to understand it better. In the 1983 International Speed Skiing Championship at Velocity Peak, Colorado, we found an ideal team: Engineer Paul Hamilton was gathering data for his computer model of speed skiing, while skier Joyce Loeffler prepared for her second competition. Hamilton's mathematical model explores the limits of this sport and what skiers can do to approach them safely.

Fish Finding: Because the certainty of mathematics has made it possible to navigate the unmarked oceans, we knew we wanted to take to the sea for at least one segment in the CHALLENGE series. In the rough waters of the North Sea, we joined a Norwegian fishing expedition to show how sonar is used to find fish. Sonar has proved so successful, however, that fishermen must worry about catching too many fish. Fortunately, sonar can be used to solve that problem, too.

The Lineup: In the comedy segment, "Bill the Hoofer" has a one-track mind. His desire to do well in his big audition blinds him to the signs that he's taken a wrong turn. By mistaking the context, he misreads all the information. Once he sees his error, he has to perform magic to prove his innocence. ■

GETTING STARTED: **Thinking INFORMATION**

The exercises in this chapter are designed to help students explore the role of information in problem solving and to develop skills in information gathering, interpreting, evaluating and analyzing. Before showing your students the INFORMATION film, you might have them do the following exercises to give them a "feel" for information.

1. It's a rare occasion when you have all the information you need to solve a problem. When something's missing, how do you find it?

It depends what you're looking for. If you want to know the reigning dates of Henry VIII, the encyclopedia will give you the answer. But if you want to know what happened during that reign, you might look to a general history of England—or, for more specific information, to a biography. You could also talk to a historian.

Ask your students:

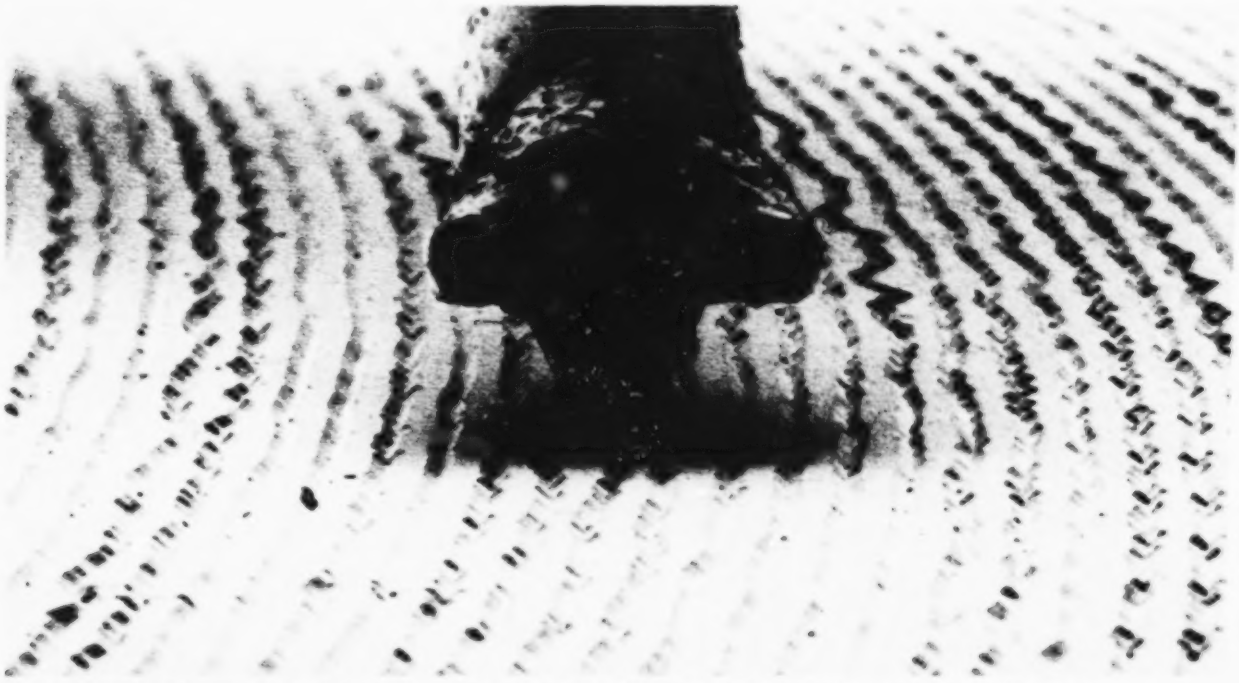
- What sources of information do they use during a typical day?
- What do they learn from them?
- How do they use the information?
- Where do they look for the information they need?

In the following problems, students must pinpoint what information is *missing*.

2. Ask your students what additional information is needed to solve these problems:

- Jill wants to buy a bicycle. She has saved $35. How much more does she need to buy the bike?
- The Allen family went on an extended camping trip this past summer. They estimated that their expenses would run about $45 per day. How much did the entire trip cost them?
- Alice Powderly, the school's fastest runner, was clocked at an average speed of 15 kph. What was her winning time in the race?

You can continue this exercise with word problems from a math textbook. Have one group of students remove a piece of information and see if another group of students can discover what's missing.



IT'S IN THE GROOVE: A laterally vibrating diamond stylus picks up the information encoded in the craggy grooves of a phonograph record. Transduced by a cartridge and passed through an amplifier, the result is music to the ears.

Too much information can be just as confusing and frustrating as too little.

3. Have your class cross out all information unnecessary for solving these problems:

- John's Aunt Mary, who has been baking bread for 30 years, uses two 3-lb sacks of enriched flour to make eight 1-lb loaves of rye bread. She bakes the loaves in baking tins that measure 8 in by 6 in by 4 in. Each loaf must bake for 45 minutes in a 375° oven. How many 1-lb loaves can she make with five 3-lb sacks of flour?
- An artist was hired to design a new sculpture for a local university. The piece was to be a solid pyramid made out of hollow steel balls, each measuring 1 m in diameter. The square base of the pyramid was to be 20 balls on a side. It was to be erected in the center of a one-acre-square piece of land. The balls were to be alternately painted with the two school colors: black and red. How many steel balls did the artist need to build the pyramid?

As a follow-up to these exercises, you might ask one group of students to *camouflage* word problems from their textbooks and have another group of students try to unearth the salient information. "Dressing up" problems can be a useful and fun exercise, but such problems can also be tricky.

4. Ask your students to solve the following trick problem as a good example of how easy it is for the obvious to get hidden behind extraneous information.

- You are a taxi driver. Your cab is yellow and black and has been in use for seven years. One of its windshields is broken and the carburetor needs adjusting. The gas tank holds 20 gallons of gas, but at the moment is only 3/4 full. How old is the taxi driver?

For Openers

Here are some questions for your students after they've seen the INFORMATION film:

- How is information important in solving problems?
- In your daily life, what kinds of information are important to you? What kinds of information are important to your parents? How and why might they differ? In what areas might they be the same?
- What kinds of information do the people in the film use to solve their problems?
  1. How is information at the root of "Bill the Hoofer's" problem in the comedy segment?
  2. How do ice climbers depend upon information to keep their footing on ever-changing spires of ice?
  3. There is little information about the long-lost Anasazi Indians except the ruins and relics they left behind. Is that unusual? How do we usually learn about people who lived centuries ago?
  4. How does Paul Hamilton, the engineer in the segment about speed skiing, use information? To what end?
  5. In the segment about Norwegian fishermen, what does the film mean when it says their problem is "too much information"?

Stalking an **Information** Problem

Making Up Your Own Problems

The following is a hypothetical conversation between teacher and students. It is intended to suggest ways to encourage students to make connections between problem-solving concepts in the film, their own experience and their knowledge of mathematics.

"

- ***Let's use this time to make up an information problem. I'm going to start by asking—what is an information problem?***

  When you're missing information and you need to find it.

- ***Here's a good example: a research problem. Is there a problem like that in the film?***

  The archaeologist who is studying the Anasazi Indians. There are no written records. He has to "dig up" the information he needs.

- ***Right, that's still research even though he isn't able to turn to people who might remember the Anasazi or to books where their history might have been written down. But isn't there more to Morrison's information problem?***

  He has to figure out a way to sort out what's important from what's not and keep it all straight.

- ***How do you suppose he does that?***

  I don't understand.

- ***Well, you've already used two important phrases that describe part of what's involved: "sorting out what's important" and "keeping it all straight." Sorting is another word for "classifying" and is an important process to understand when dealing with information. What do we mean when we use the words "sort" or "classify"? Anyone?***

  Doesn't it mean putting things together because they're similar in some way?

- ***That's right. And what kinds of things can be classified?***

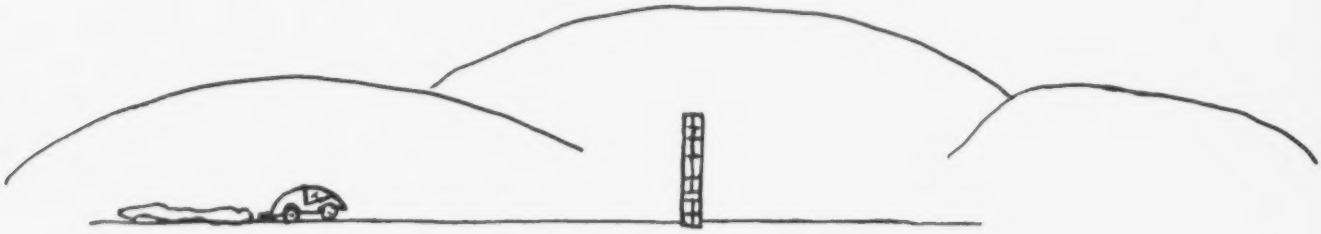
  Almost anything. Like clothes in a store that are arranged according to what they are: "pants," "shirts," "socks," etc.

- ***Can they be sorted another way?***

  Well, clothes can also be arranged by size, color or style. Sometimes they're even arranged by more than one of these at the same time. For instance, size 8 blue jeans are kept on a different shelf than size 8 corduroys.

- ***Why do you think that storeowners do that?***

  Probably so shoppers can find what they are looking for more easily. . .and also to keep the store organized and looking better.



If a man named Scott Fussel is cruising along a highway at a constant speed of 63.8 mph for 17.73 miles of road which has 1 inch of snow on the road and doesn't make a single turn and suddenly hits an 11' tall, 1' thick brick wall, with his 1985 Porsche, what speed would he be moving at when he comes through the wall – is driving on the other side?

Mike S. Clark
Howard Bishop Middle School
Gainesville, FLA

♦ ***Where else is classifying or sorting used? And this time, let's try to think of places where it's used in school.***

Well, the food in the cafeteria is always arranged by "type" of food. For instance, the trays are always at the head of the line; then you can get your main dish and pick from a couple of different selections; then all the desserts and beverages are together.

♦ ***Good. Anyone else?***

How about the library? All the books on history, for instance, are in one section; all the books on sports in another. Each book has a different number and even books on baseball are located according to their numbers on a different shelf from books on basketball. And if you can't find what you want on the shelf, you can look in the catalog by subject or author or title.

♦ ***Right, in libraries we see a good example of classifying or cataloging information. When archaeologists catalog the artifacts they find at a site, they use a system based on coordinates. Every object found is given two numbers which will locate that object on an x and y axis according to where it was found. We'll be doing some exercises with coordinates later on, but for now let's try to develop a classroom exercise that will help us understand classification better. Does anyone have any ideas?***

What if we were to divide up into teams and each team went about the classroom collecting a series of objects. Those different objects are then presented to the other teams whose job it is to divide those objects into "sets" of objects based upon some "common trait" they share. Then the team which had originally collected those objects has to guess what that common trait is.

♦ ***Let's see if it will work. It might be a good way to focus on the different ways that information presents itself. We can always change the rules if we find that it doesn't do what we want it to do.***

"

"The solutions of many problems in algebra and mathematics in general consist of transformations of the given information so as to make the solution, which is obscure, become transparent in a new form of representation."

Moshe F. Rubinstein, *Patterns of Problem Solving*

Following Through: Problem Ideas from the Film

The stories in the INFORMATION film provide background for devising exercises and a chance for you and your students to formulate some interesting problems. Here are a few examples:

Ice Climbing: Ice climbers organize their climbs in stages, called *pitches*. Often they do not climb straight up, but may have to take a longer route to avoid climbing hazards.

- This is a diagram of an ice climb (Figure 1). Figure out how high the stand of ice is and the distance of the climbing route.

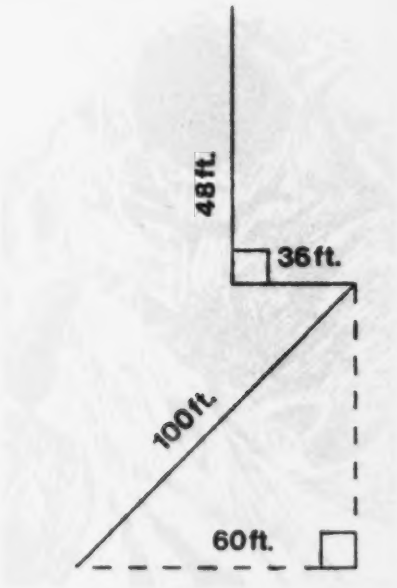


Fig. 1

- Ice climbers can fall twice the distance from their last point of protection (where the rope is anchored to the ice). With a little help from Galileo and some mathematics, calculate how fast and far ice climbers risk falling. Galileo showed that the acceleration due to gravity is 32 feet/second^2. Use this information to determine the final velocity of an ice climber who falls for 2 seconds. Translate the figure into mph. How far would the climber have fallen?

Anasazi Indians: Archaeologist Randy Morrison explains that unless artifacts are carefully cataloged, vital information about context is lost.

- Develop a system of cataloging artifacts in your classroom. Use graph paper and coordinate mapping to indicate where desks, tables, etc. are located. To make it even more challenging, try to develop a three-dimensional cataloging system for the classroom or for the items in a dresser or closet at home.

- On movie sets, continuity people must also use a system of cataloging to ensure that the people and objects that appear in one shot are in the identical place in later shots of the same scene. They use detailed drawings or markings on the set and take instant photos to catalog the scene. Look at the schoolyard or cafeteria as a movie set and catalog the objects found there. At a later time, return to the scene and recatalog the information. Then, using the two sets of catalogs, compare the information, noting how objects have been moved and how efficiently your cataloging systems have worked in preserving information.

"Depending on the way you have conceived of the problem space, you use various kinds of information drawn from memory, or given with the problem, to process the data so as to move toward your goal."

Allen Newell and Herbert A. Simon, *Human Problem Solving*

BEST COPY AVAILABLE

Speed Skiing: Engineer Paul Hamilton uses graphs to show the acceleration of a speed skier down a mountain slope.

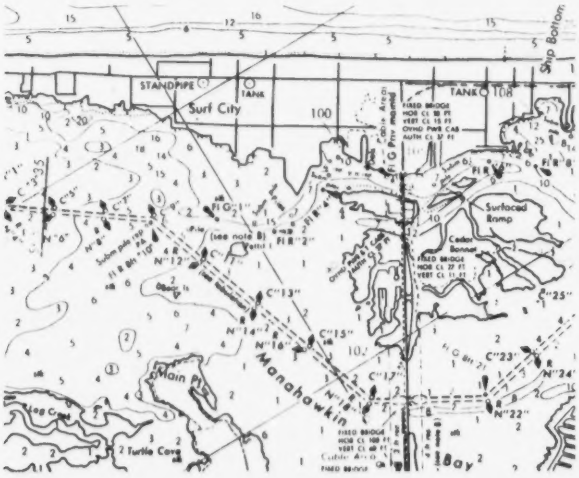
- Try your hand at graphing the results of some other sporting events. For example, sketch a graph comparing the speed of a sprinter in the 100-yard dash and the performance of a marathon runner. These graphs will be crude, but should display the difference in slope of speed and acceleration, a sometimes difficult concept to grasp. Then, try graphing the speed of a winning race car in the Indianapolis 500 and a winning horse in the Kentucky Derby.

- To give you practice in designing your own graphs and labeling the axes, try graphing the results of a high jump or long jump competition among classmates. Some students will graph speed against time; others will graph altitude against time. (An almanac or one of the many books on sporting events will supply more than enough information to expand this graphing exercise to include other sports.)

Fish Finding: The fishermen of Norway rely upon echo-locating technology to find fish in the vast North Sea. In earlier times they could navigate to the area of the fishing grounds, but once there, finding the fish was chancy. In some ways, this earlier method was very inefficient.

- Use a game like Battleship to illustrate the difficulty. In this game for two players, each player fires blind on the opponent. One player names a position on the game grid and the opponent responds either "hit" or "miss." The object is to find and destroy the other player's fleet. In this version, use graph paper and Cartesian coordinates to designate your location. One student takes the role of the fisherman; the other, the fish. Depending on your needs, determine a number of schools of fish and the size of the schools. Establish rules for how far the trawlers and fish can travel between moves. Then, challenge each other to devise search strategies.

- Mariners' charts show different kinds of information than land maps. For this exercise, look at a coastal or harbor chart and discuss what kinds of information are displayed (soundings or depths, hazards, land features, buoys or navigational aids, etc.). In sailing in or out of a crowded harbor, mariners often determine their position by triangulating (lining up the craft with two objects, such as a buoy and lighthouse). A compass bearing is taken along the line of sight between each object and the boat and then drawn on the map. The boat is located where the two lines intersect. As a class exercise, read out two bearings from two objects on the chart and have other students in the class try to fix the positions of the objects.



Mariner's map of Manahawkin Bay, off the coast of New Jersey, tells sailors what they need to know in order to navigate the waters. Included are depths, bridges, channel markers, buoys, hazards; dotted lines indicate the best route for safe passage.

Information With Class

Making Up Your Own Problems

This hypothetical dialogue between teacher and students provides examples of how to draw students into the process of devising and solving their own information problems.

"♦ ***Today we're going to make up our own information problems. And to get us started, I've collected some information for us to work with . . .***

Top 25 Registered Dog Breeds (1983)*

	Rank 1983	1983	Rank 1982	1982
Cocker spaniels	1	92,836	2	87,218
Poodles	2	90,250	1	88,650
Labrador retrievers	3	67,389	4	62,465
Doberman pinschers	4	66,184	3	73,180
German shepherds	5	65,073	5	60,445
Golden retrievers	6	52,525	6	51,045
Beagles	7	39,992	8	35,548
Miniature schnauzers	8	37,820	7	36,502
Dachshunds	9	33,514	9	32,835
Shetland sheepdogs	10	33,375	10	30,512
Yorkshire terriers	11	28,350	11	26,205
Chow chows	12	27,815	13	22,623
Lhasa apsos	13	27,087	12	25,945
Shih tzu	14	23,308	16	20,556
English springer spaniels	15	22,626	14	22,296
Siberian huskies	16	21,237	15	20,654
Collies	17	20,789	17	20,084
Pomeranians	18	19,691	18	18,456
Basset hounds	19	19,042	19	17,871
Brittanys	20	18,433	21	17,349
Pekingese	21	17,611	20	17,434
Boxers	22	17,365	22	16,301
Chihuahuas	23	16,946	23	15,867
Rottweilers	24	13,265	27	9,269
Boston terriers	25	11,855	25	11,361

*Source: American Kennel Club, Inc., New York, N.Y.

♦ ***Now we can make up some problems. My problem is deciding which breeds of dogs have gained most in rank between 1982 and 1983? Where would we look to find out this information? Let's see, the shih tzu went from 16 to 14. And rottweilers went from 27 to 24. That's three places, so that's the breed that had the biggest jump in ranking.***

That means that the number of rottweilers increased more than any other breed that year, right?

♦ ***Is that true or are you confusing "rank" with "actual numbers"? How could you test your theory?***

I guess I'd have to do a lot of subtracting to find out just how much difference there was in the numbers of each breed between 1982 and 1983. Sounds like a lot of work.

♦ ***Is it worth doing? When you're making up your own problems, you always want to ask yourself if the problem is worth the time and effort involved. As a teacher, I think this exercise is a good one for subtraction practice, but what else is it good for?***

Well, once we've done all that work of subtraction, we might have information to answer other questions.

♦ ***Like what?***

I'm not sure that I can answer that without first seeing the numbers.

"Real problems often involve an overwhelming amount of relevant information which cannot all be considered in (for example) making a decision. Or, some important information may be unavailable, yet an adequate decision must be made anyway."

Richard Lesh, *The Arithmetic Teacher*

♦ ***Then, let's get the numbers and see where they take us . . .***

Difference between 1982 and 1983:

Cocker spaniels	5,618
Poodles	1,600
Labrador retrievers	4,924
Doberman pinschers	−6,996
German shepherds	4,628
Golden retrievers	1,480
Beagles	4,444
Miniature schnauzers	1,318
Dachshunds	679
Shetland sheepdogs	2,863
Yorkshire terriers	2,145
Chow chows	5,192
Lhasa apsos	1,142
Shih tzu	2,752
English springer spaniels	330
Siberian huskies	583
Collies	705
Pomeranians	1,235
Basset hounds	1,171
Brittanys	1,084
Pekingese	177
Boxers	1,064
Chihuahuas	4,079
Rottweilers	3,996
Boston terriers	494

♦ ***With this new information, what are some other questions that we can now answer?***

I can see a lot more now than I could before. Rottweilers are *not* the breed that increased most in number. That's cocker spaniels. And you can easily see that doberman pinschers *decreased* most in number.

♦ ***You've just pointed out an important result of working with information: The more you work with it, the easier it is for you to see and solve all sorts of problems. Can you give me some more examples?***

How about some percentage problems, like finding out what percentage of the total of the "top 25 breeds" cocker spaniels represented in 1983?

♦ ***Good question. How would we do that?***

First we would need to find the total population for the top 25 breeds in 1983. I'll use a calculator. . . I get a total of 884,378. To get a percentage, I'll divide that into the total of cocker spaniels for 1983, which is 92,836. I get .10497. Rounding off, that gives me 10.5%. In other words, for the top 25 breeds, cocker spaniels represent more than 10% of the total population. That's a lot of cocker spaniels.

♦ ***As you can see from your own problems, they're getting more exacting as your information gets more exact. That's one of the interesting things about information problems: They can get as detailed and complex as you're willing to go with them. For instance, we could even go so far as to look at trends about population growth. Do we have enough information, for instance, to predict how the populations of different breeds might look in five or ten years?***

I don't think we can do that without knowing what's happened to the different breeds over a longer period of time. How can we predict the *next* five years if we don't know about the *last* five years.

♦ ***Good point. You're now talking about the limits of information and of the temptation to make too much out of too little information. But we've still got a long way to go with the information we've got. Let's try some more.***

A basketball team made 60 shots. 24% of the shots were from the free throw line (1pt. each) 75% were 2pt. 1% was 3pt. shots. How many pt. did the team make? What was the average for the 5 players in the game? If there are 15min per quarter, what was the average for each 5 min.? per quarter? The highest scorer was Froglegs Flipper with 80% of the shots, (all 2pt.) How many pt. did the other 4 average? Did they win?

Raul Reyes
Bullen Junior High School
Kerosha, WI.

On the Lighter Side

Teachers often remind students to read word problems carefully and to note important information. Many students persist, however, in quickly scanning the material and overlook important details when attempting to solve the problem.

That's "Bill the Hoofer's" story: He misreads the situation. He misses one key piece of information—the precinct sign. Then his assumptions about where he is blind him to other clues. It takes some inspired footwork to get him out of trouble.

Without ruining the joke, we hope students absorb Bill's lighthearted lesson.

A Comedy of Errors: The Lineup

"Here I am backstage at a Broadway theater. Guess they want us to line up here."

"Well, here we go. . . Broadway!"

Picture This

- Show your students the picture of Bill in the lineup. Have them study it for one minute. Then have them do the following exercises.
  - Which man is tallest? Which is shortest?
  - How many are wearing a hat?
  - How many are wearing a jacket?
  - How many have their arms at their sides?
  - Write a paragraph describing the scene in detail.
  - Make a sketch of the scene.

"Book him."
". . .Gosh, I'm excited! I've got to thank the producers and the director and the policeman. Policeman? Wait a minute. There's been a misunderstanding."

"You see, gentlemen, I'm an entertainer. I'll give you a command performance . . . . Proof positive of my true identity."

- Now have your students look at the picture again.
  - Which were more accurate, their paragraphs or their sketches?
- Divide the class up into groups of four or five. Put each group in a lineup at the blackboard. Mark each person's height on the board.
  - What is the difference between the height of the tallest person and the height of the shortest?
  - What is the average height of the class?
  - What is the average height of the boys in class? of the girls?

Information on Ice

"Ice climbing is psychologically more difficult than mountain climbing because the ice changes."

Mike Kennedy and Paul Sibley know how to get to the top. To climb Rigid Designator Falls, a 150-foot high frozen waterfall located near Vail, Colorado, they rely on acquiring the right information and using it wisely. It can make the difference between a safe, successful climb and one that leads to disaster and even death.

"In some ways," Kennedy says, "ice climbing is psychologically more difficult than mountain climbing because the ice changes." The ice climber must be constantly alert. If the ice is too soft, holds are not secure. Hard, brittle ice may collapse under the weight of the climber.

Direct sunlight can warm the rocks underneath, weakening the ice's bond to the stone. Hollow and honeycombed ice, with water running through, is also dangerous. Thin ice over rock makes it difficult for the climber to set "protection"—ice screws through which their safety lines are passed. Ice that "dinner-plates"—breaks off in large chunks—can result in a shower of debris, which can cause injury to those below or a climber to fall.

Because frozen waterfalls are often located at the bottom of natural avalanche paths, avalanches are another danger. To climb in safety, ice climbers conduct the same kind of survey that avalanche rangers do. They inspect the snowpack all around the route of a projected climb, often digging deep pits into the snow to get information about the stability of the snow layers. The objective is a successful climb with minimal risks, and this means relying on solid information, not guesswork. Sometimes, life-saving information is right before the climbers' eyes.

SCALING THE FANG: A climber attempts the Fang, a towering spire of ice near Vail, Colorado. Climbers must brave snow, avalanche and the dangerous warmth of the sun; their safety depends on the quality of their information.

Pitches and Belays

Only moments after the team had analyzed the ice at one site and judged it too wet and too soft to risk a climb, the 50-foot column of ice fractured and collapsed before their eyes.

Kennedy and Sibley found a safer and more secure site nearby at Rigid Designator Falls. In order to protect against long falls and because climbing ropes come in standard lengths of 150 or 165 feet, ice climbs are organized in *pitches* or stages. Before they attempt their assault on Rigid Designator Falls, Kennedy and Sibley must estimate how many pitches and how much time will be required for both climbers to reach the top. (Pitches don't always lead straight up. Instead, they go from one section of safe, negotiable ice to another.)

As the team makes its way up and across the ice face, one acts as leader and the other as *belayer*.* The leader goes first, tied to the belayer by a safety line. When he has gone as far as it is safe to go, he stops to set protection—an ice screw to which he clips his safety line. In the event of a fall, this ice screw, securely placed in stable ice, will keep the climber from falling to the bottom of the climb. He does risk falling twice the distance that he has climbed above his last piece of protection. After the lead climber has safely reached the top of one pitch, and set protection, he becomes the belayer for his partner.

*"Belay" is an old maritime term meaning stop. Thus, the belayer is one whose job is to stop his or her partner in the event of a fall.

Ice Axes and Crampons

Ice axes and crampons are basic equipment for the climb. Ice axes become extensions of climbers' arms: They use them to extend their reach, to chop holds in the ice and to haul themselves up bit by bit. Crampons are used to adapt a climber's feet to the ice in much the same way that a football player's spikes allow his feet to grip the playing field. The difference is that crampons are *real* spikes—steel teeth strapped to the sole of each boot for traction, with more teeth projecting straight out from the toe. The toe spikes allow a climber to kick toeholds into the ice.

As each pitch is completed, the climbers have to make decisions about the next. Remaining alert to changing conditions means constant information gathering. It's not safe simply to plot a route and then attack it head-on. Climbers must continually evaluate all the information available at any given moment, and they must be prepared to change course if the information warrants it. Their very lives may depend on it. ■

ON TO THE NEXT ONE: Belayer feeds out rope to lead climber Paul Sibley scaling Rigid Designator Falls, a 150-foot spire outside Vail. After one frozen waterfall collapsed at their feet, Sibley and fellow climber Mike Kennedy took extra care surveying the landscape before starting up this one nearby.

Ice, Water and Steam

Ice climbers face the same hazards as mountain climbers. There's a further danger, however: At any minute, the ice they climb may become unstable in reaction to changes—sometimes minute ones—in air temperature or pressure.

When climbers Mike Kennedy and Paul Sibley examine an ice column, they're looking for visual evidence of its stability. They're looking, in fact, at a complex relationship of water, temperature and pressure—the same relationship we take advantage of and take for granted every day.

Water commonly exists in three states, or phases: solid (ice), liquid (water) and gas (steam). The temperature at which water changes between solid and liquid states, called the *melting point* or *freezing point*, is 32°F at sea level. The temperature at which it changes between liquid and gas, called the *boiling point* or *condensation point*, is 212°F at sea level. But temperature is only half of this story, as you know if you've ever used a pressure cooker. The pressure in and around a substance is just as important as the temperature.

Pressure is force per unit area. If you weigh 100 lb and your shoeprint has an area of 25 sq in, then if you stand on one leg, you exert a pressure of 4 lb/sq in (psi) on the floor underneath your shoe.

Gases and liquids exert pressure in all directions on the objects they surround. The air exerts pressure on us and everything around us.

This provides a handy unit for measuring pressure. One atmosphere (1 atm) is the "standard" pressure of the air at sea level, 14.7 psi. This corresponds to a barometric reading of 29.92 inches of mercury.

Air pressure varies with altitude and depth. This is why your ears "pop" when you're on a plane or a fast elevator. If you're a swimmer, you encounter pressures greater than 1 atm whenever you swim underwater.

Scuba divers often dive to very great pressures. Pressure in water varies much more sharply with depth than pressure in air, due to water's greater density. At a depth of 30 ft, the pressure in sea water is about 2 atm. At the bottom of the Marianas trench in the South Pacific (depth 35,810 ft—the deepest ever measured), it's well over 1,000 atm. Pressures in excess of 1 million atm can be created in the laboratory.

From Fahrenheit to Celsius... and Back

The Celsius Temperature Scale, named for Anders Celsius (1701–1744), a Swedish astronomer and physicist, is also known as the centigrade scale because it takes the melting point of ice to be 0° and the boiling point of water to be 100° at 1 atm. The Celsius scale is generally used in scientific work.

The Fahrenheit Temperature Scale was devised by the German physicist Gabriel Daniel Fahrenheit (1686–1736). He designated 32° as the melting point of ice and 212° as the boiling point of water at 1 atm.

Use the following formulas to convert from one scale to the other:

$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{9}{5}C + 32$$

Substance	Melting point		Boiling point	
	°Celsius	°Fahrenheit	°C	°F
Nitrogen (N_2)	−209	−344	−195	−319
Oxygen (O_2)	−218			−296
Water	0	32	100	212
Salt (sodium chloride)	801	1474	1413	2575
Lead	327	621		2759
Gold		1945	2600	
Iron	1535	2795		5904

Melting and boiling points at 1 atm for some common compounds and elements

Table 1

Exercises:

1. Supply the missing values in Table 1 using the data given and the box on Celsius and Fahrenheit conversion.

2. At what temperature (degrees Celsius) does nitrogen melt? At what temperature does it boil? How many degrees are there in between?

3. Answer question #2 for each substance in the table.

4. For which substance is the difference between melting and boiling the greatest? For which is it the smallest?

Depth (ft)	**Pressure** (atm)
0	1.0
10	1.3
31	2.0
50	2.6
100	4.2
282	10.0
437	15.0
1000	33.0
3720	120
10,000	321
31,185	1000
35,280	1150

Pressures at various ocean depths.

Table 2

5. Using the information in Table 2, how deep are you when the pressure is 2.6 atm?

6. What is the pressure at 100 ft below sea level?

7. When is the pressure 1,000 atm?

8. About how deep do you have to be for the pressure to be about 500 atm?

9. Sperm whales may dive as deep as 10,000 ft. What is the pressure in psi at that depth?

10. There's a great deal of information in the two tables. Make up three additional questions to ask your classmates based on each table.

Further Readings

Chouinard, Yvon. *Climbing Ice*. San Francisco: Sierra Club Books, 1978.

While this is not a beginner's manual, it is easily the best book on the subject and is well worth looking at regardless of prior experience. This oversized volume has terrific pictures and clear text on the history and equipment of ice climbing and includes discussion of the formation and structure of snow, ice and avalanches.

Randall, Glenn. *Vertigo Games*. Sioux City, Iowa: W.R. Publications, 1983.

This book covers other sports but has a good chapter on ice climbing which includes photographs and a first-person account of a difficult climb on Rigid Designator Falls, where the ice climbing segment for INFORMATION was filmed.

***Rock and Ice Magazine* (monthly). Boulder, Colorado.**

Features a variety of articles on ice climbing that include pictures, interviews with climbers, information on equipment and book reviews.

Wren, Christopher S. "Ice Climbing on the Equator." *New York Times Magazine*, August 17, 1980.

Wonderful, first-person account of climbing the Diamond Couloir, an ice fall on Mt. Kenya in Africa. Describes use of different tools and the difficulties of climbing a sheer ice face at over 15,000 feet.

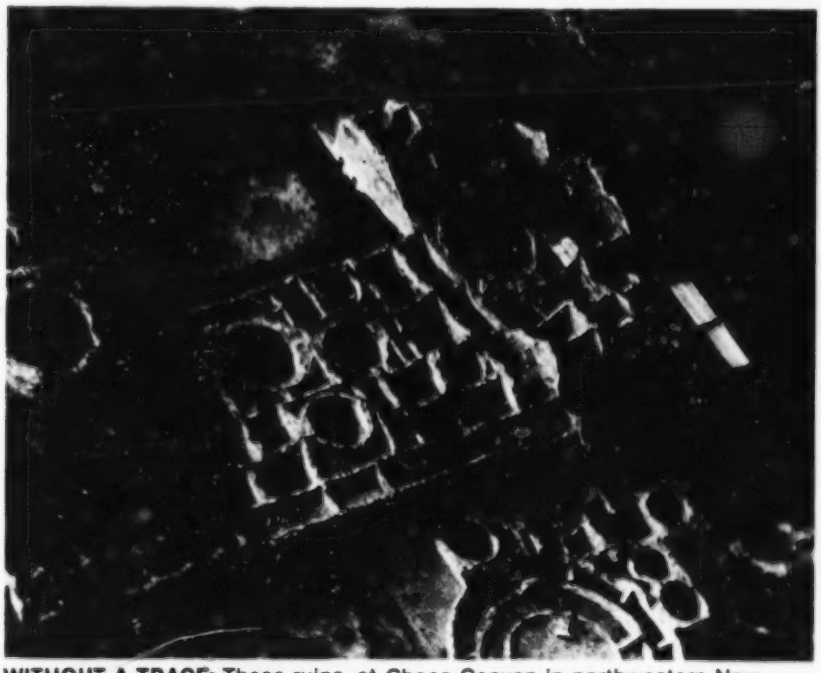
The Anasazi—What They Left Behind

Every shard of pottery, every stone or stick is a clue to the past.

Around a thousand years ago, a large part of the southwestern United States was populated by the Anasazi Indians. Their domain stretched from east of the Rio Grande through New Mexico and Arizona to southern Colorado and Utah. Their civilization is believed to have flourished for several hundred years, disappearing sometime about 1200 A.D. for reasons not fully understood. The Anasazi left no written records, no evidence of a spoken language—no obvious clues to their elaborate cities and buried artifacts. (For more information on the Anasazi Indians, see pg. 250.)

The body of archaeological evidence found at Anasazi sites like Chaco Canyon in northwestern New Mexico and Mesa Verde in southwestern Colorado strongly supports the idea that the Anasazi were a sophisticated people with unusual means of survival at their disposal. What, then, might have caused them to abandon their homes and eventually to disappear? Again, archaeologists cannot say for sure. They can only offer plausible *theories*—a natural disaster, perhaps, a famine or drought might have brought the Anasazi domain to a premature end.

Every shard of pottery, every stone or stick is a clue to the past. Modern archaeologists use precise techniques to plot where artifacts are found and meticulously document their findings. Every piece in an excavation is mapped and photographed before being removed. Digging in straight-faced trenches, archaeologists clearly record the contents of every layer, or stratum. They order the information with computers, so the data can be analyzed in hundreds of ways. Only by using the best techniques available to recover all possible information can archaeologists begin to decipher how this society might have functioned and why it might have disappeared.



WITHOUT A TRACE: These ruins, at Chaco Canyon in northwestern New Mexico, housed a thriving civilization of Anasazi Indians some 1,000 years ago. No one knows why they suddenly disappeared; in fact, very little is known about them at all.

Archaeologist Randy Morrison hopes to solve some of the mysteries by combing the ruins for the few clues not already removed by earlier excavations.

Information Forever Lost

Archaeology was only an infant science at the end of the nineteenth century when the first excavations began at Chaco Canyon and at other Anasazi sites. Early archaeologists did not dig carefully, layer by layer, making records as they went. While they did record which rooms certain artifacts came from, they kept no record of where objects were found in relation to one another. Vital information was forever lost.

New and better techniques yield information to answer some questions and pose new ones. Recently, an intricate network of roadways was discovered in and around Chaco Canyon. Although these roadways could only be observed by aerial surveys, Navajo elders told archaeologists of their possible existence. Following the roads from their source in Chaco Canyon to their end, archaeologists found more than 60 Chacoan ruins in 2,400 square miles. These roadways and "outlier" Chacoan ruins, with their ceremonial *great houses* and underground *kivas* not only tell a story of a technically advanced and sophisticated people, but of an organized society with numerous satellite communities as well. Artifacts also indicate that the Anasazi had trade ties with a larger world, from Mexico to the Pacific Coast.

Each new piece of information adds to understanding. As the field of reliable data about the Anasazi increases, questions are answered and more are posed. And the need grows for more and better information. ■

CHACO'S SILENT COMPANION: Ruins at Mesa Verde in southwestern Colorado (above) are as silent as Chaco Canyon. Among the unsolved mysteries is the significance of "T"-shaped doorways (left).

Cartesian Classroom

In INFORMATION, Randy Morrison notes that the location of an artifact itself is important information about the people who used it.

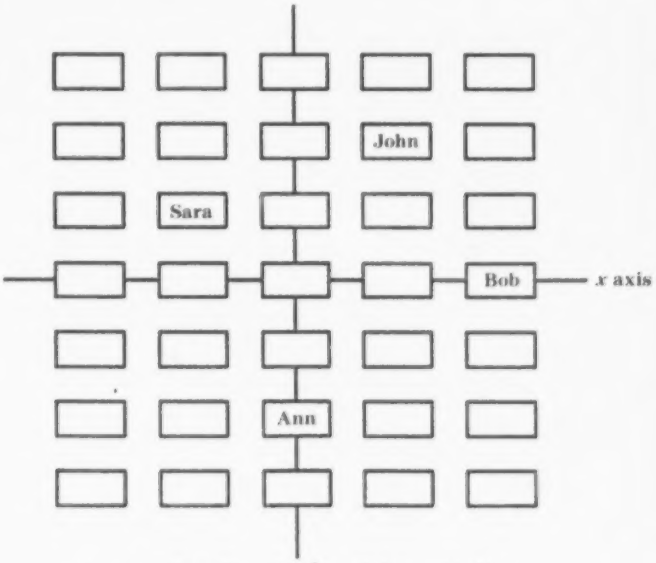
How do archaeologists record the locations of found objects? One way is to treat them as points on a grid. The area of exploration is divided into equal squares, like a piece of graph paper, and the location of the object is recorded using coordinates.

This system is related to one of the most important of all mathematical inventions, coordinate geometry, which describes the location of a point in a plane by giving its distance from a pair of perpendicular lines, called *axes*.

If your students haven't had any experience with Cartesian coordinates, here's a way to get them to think of coordinates as physical locations:

■ Imagine that each desk in the classroom is a square on a piece of graph paper. Now imagine that a pair of lines, the x and y axes, runs through the middle rows of the classroom, separating the room into quadrants.

Figure 1 shows an example of what a classroom diagrammed using Cartesian coordinates might look like. Now have your students do their own "classroom maps."



Ann: (0, −2) John: (1,2)
Bob: (2,0) Sara: (−1,1)

Roll Call at René Descartes Junior High School

Fig. 1

To introduce the idea of coordinates representing variables in a function, try the following exercise with your class:

■ Give the class a simple formula (for instance, $2y = 3x$, $x = 1/2y$ or $x = y - 1$) and show students how to find the coordinate pairs that can be substituted for the variables.

Have all students whose desk coordinates fit the formula identify themselves and prove that their coordinates are indeed on the graph of the function.

Exercises:

1. Using graph paper, "map" your classroom, identifying with coordinates the locations of such things as the teacher's desk, pencil sharpener, blackboard, etc.

2. Plot the following coordinates on graph paper:

(0,10)
(6, -1)
(20,0)
(-8, -2)
(12, -6)

3. Name the coordinates for each point on the graph in Figure 2.

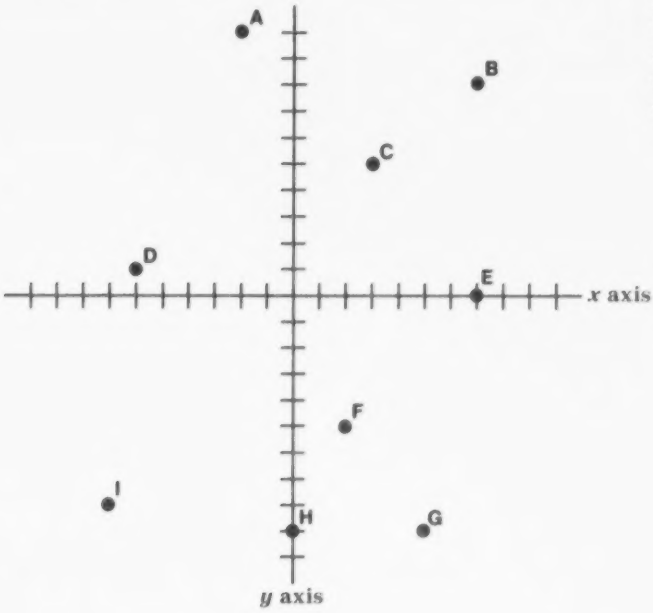


Fig. 2

Class Project:

4. Measure off an area of your schoolyard, a vacant lot or some other location in your town (you can use a pace as your unit of measurement) and develop a graph of the objects you find in that area. Let each square on the graph paper represent a unit of measurement. Use coordinates to locate each object on the graph.

CHALLENGE Problem:

5. Extend the coordinate graph you made of your classroom into three-dimensions. Use letters to indicate the third dimension to graph objects (A) on the floor, (B) in the middle of the room (such as the tops of desks), and (C) at ceiling level. Use a three-coordinate system (x,y,z) where x represents distance from the left wall, y represents distance from the back of the room and z represents space position a,b,c. Indicate the coordinates of items in your classroom. Can you think of a way actually to construct such a graph?

"When we form a problem representation, we not only add information and delete information, we also interpret information—that is, we use our knowledge of language and the world to understand problem information. . . We become acutely aware that we need knowledge to interpret problems. . .when we don't have that knowledge. . .we don't understand the initial state or the goal, and we have no idea what the operators might be."

John R. Hayes, *The Complete Problem Solver*

The Dating Game

Archaeologists record the location of information in three dimensions. As they excavate a site, layer by layer, they record at what depth each artifact is found, which gives them a clue to how old the object is.

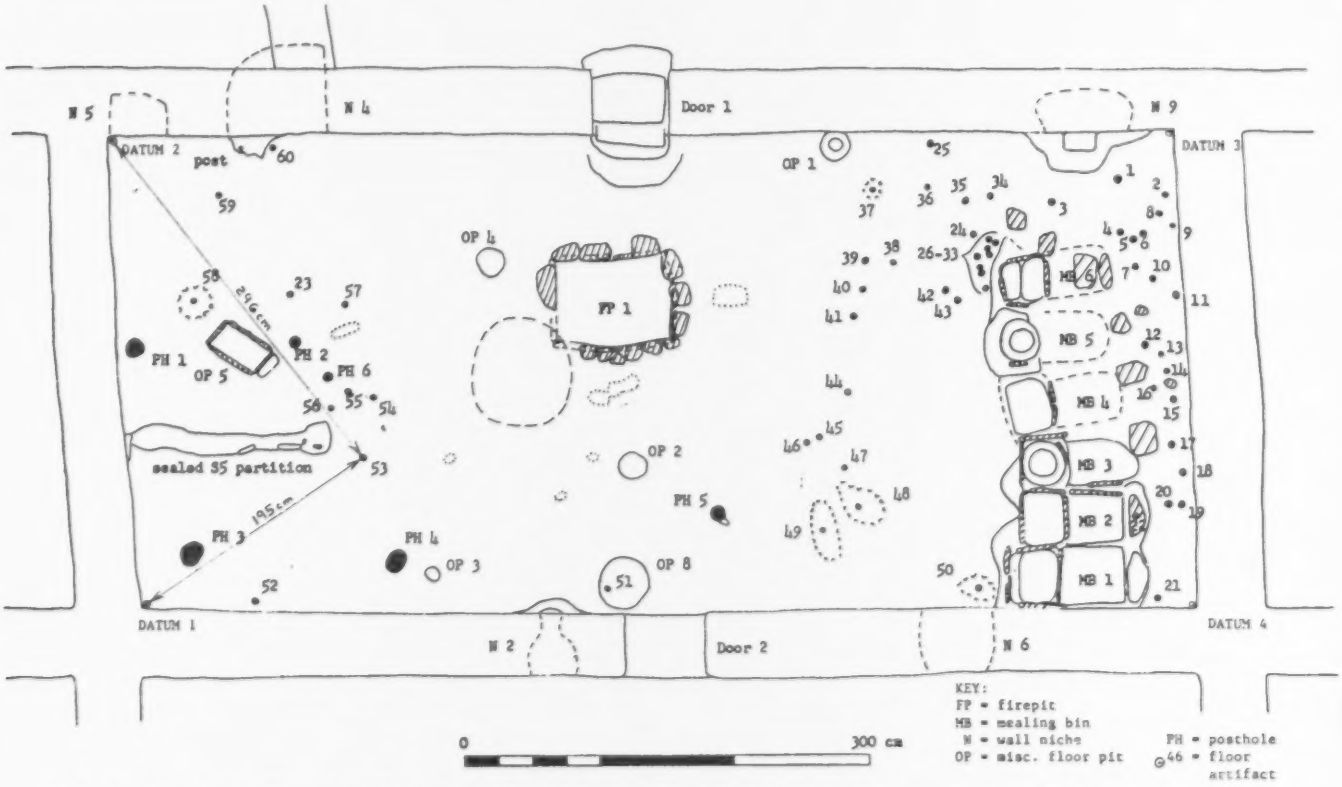
If you know the average rate of soil accumulation for an area, it's possible to estimate the age of an object by the depth at which it was found. (This method requires rather specific conditions of climate and soil composition and an intimate knowledge of the area. It's not the method of choice for definitive dating, but given the right conditions, it can be handy for a quick estimate.) The following exercises are hypothetical and assume a fairly constant rate of soil accumulation over a long period of time.

Exercises:

1. You're an archaeologist on a dig. The first object you've found is buried 12 feet below the surface. Local scientists tell you that the soil accumulates in this area at an average of 1 inch per year. How old is the object?

2. An object is found 200 feet below the surface. How old is it if an average of 2 inches of soil accumulates each year?

3. A 660-year-old object was found in an area where .4 inches of soil accumulates each year. How many feet below the surface was it found?



This drawing shows the top surface excavation of Room 110 of Pueblo Alto, at Chaco Canyon. Each artifact (mostly shards and stone debris) is numbered and located precisely. The concentration of artifacts around the mealing bins at right of drawing consist of manos and hammerstones used in grinding corn and other seeds. Archaeologists record, catalog and map every dig so thoroughly that, in theory at least, everything could be replaced exactly as it was before the first shovelful of dirt was removed.

4. How far down would you expect to find a 400-year-old object if soil accumulates at an average rate of 4 inches every 5 years?

5. How deep will a 2,000-year-old object be if .5 inches of soil has accumulated each year?

6. You've found an object that you can reasonably assume was used 200 years ago buried 20 feet below the surface. What is the average soil accumulation per year?

7. A 500-year-old object is found 25 feet below the surface. What's the average annual soil accumulation?

Answers:

1. 144 years: 12 in/ft × 12 ft × 1 year/1 in = 144 years
2. 1,200 years
3. 22 ft
4. 26.7 ft: 4 in/5 years × 400 years × 1 ft/12 in
5. 83.3 ft
6. 1.2 in: 200 years/20 ft = 10 years; it takes 10 years for one foot of soil to accumulate. So in 1 year, 1/10 ft or 1.2 in of soil is added.
7. 0.6

SCREEN TEST: Students use sifting screens to search for artifacts at Rowe Pueblo Excavation, Rowe, New Mexico.

Further Readings

Anderson, Douglas and Anderson, Barbara. *Chaco Canyon: Center of a Culture.* Globe, Arizona: Southwest Parks and Monuments Association, 1981.

A thorough history of the Anasazi, revolving around Chaco Canyon. Beautiful photographs.

Canby, Thomas Y. "The Anasazi: Riddles in the Ruins." *National Geographic*, November 1982.

An overview of the Anasazi culture—its achievements, its mysterious disappearance and the clues it left behind. Perhaps the most interesting yet information-dense article on these fascinating people.

Jones, David R. "Climbing to a Palace in the Desert," and Zollinger, Norman, "Chaco: Echoes of the Ancient Strangers." *New York Times* (Travel Section), June 10, 1984.

Two articles about traveling to the Southwest to get to know the Anasazi up close. Good information for anyone interested in visiting these locations.

Osborne, Douglas. "Solving the Riddles of Wetherill Mesa." *National Geographic*, February 1964.

An excellent description of five archaeological excavations of Mesa Verde, Colorado. Well-illustrated with photographs and paintings.

Simmons, James C. "In the Footsteps of the Anasazi." *American Way*, November 1983.

Firsthand account of trip to the Southwest to view archaeological sites; several good photos.

Speed Skiing: Going for the Limit

A computer model helps speed skier's determine which variables have the greatest impact on world-record speeds.

In speed skiing, there are no jumps, turns or gates to check the skier's speed. The object is to ski as fast as possible down the steepest and straightest course available.

At Velocity Peak near the old mining town of Silverton, Colorado, the skiing is among the fastest in the world. There, a *couloir*, or avalanche chute, drops down from an altitude of 12,500 feet at an average pitch of 31 degrees to a broad frozen lake 1,200 vertical feet below. The slope is extremely steep at the top, nearly 52 degrees. It is the possibility of attaining great speed on skis that brings young athletes like Joyce Loeffler to the spot. Technical engineer Paul Hamilton is at Velocity Peak to try to determine, through a computer model, how even higher speeds might be achieved.

New Input

Hamilton uses a computer to create a model of a speed skier on a course. He uses data gathered about world-class speed skier Paul Buschmann as a baseline model—a point of departure for measuring other conditions and other skiers.

The computer is also programmed with data such as the friction coefficient of skis on snow and atmospheric temperatures. The higher the air temperature, the less dense air is. It is therefore easier for skiers to reach greater speeds. But higher temperatures will melt the snow and make the going slower. Wind speed and wind direction are important factors as well.

FASTEST MAN ON SKIS: Austrian skier Franz Weber sets new world record of 129.302 mph at Velocity Peak. The 1983 record was set after the starting gate was moved from 12,500 feet to 12,560 feet; the extra altitude made possible greater speed.

There is a great deal of information still to be entered, including what Hamilton calls "the most mysterious figure of them all"—drag area. *Drag* is commonly thought of as "wind resistance." Hamilton is especially interested in the drag created by air that "sticks" to the skier as it rushes past, called the *adhesive boundary layer.*

A Big Drag

Skiers wear tight racing suits made of slick material to cut down on the friction caused by this adhesive boundary layer. Another kind of drag is created by the skier's wake—the turbulent flow of air that results in suction behind the skier. To minimize this, some skiers wear lightweight plastic fairings on the backs of their legs and aerodynamically designed helmets that help smooth out the airflow. All racers crouch way down on their skis in a drag-reducing position called the "tear drop" (from its shape) or the "tuck." With Buschmann in the tuck, Hamilton has measured the frontal area exposed to the air and made numerous computations to arrive at a value for drag area that he has also entered into his computer as part of his baseline model.

WINGS OF MERCURY: Fairings on skiers' helmets are aerodynamically designed to cut down on wind resistance and drag, thus increasing skier's speed.

SLOPED FOR SPEED: Velocity Peak's reputation for speed brings skiers from far and wide. Everything about the spot, from avalanche chute dropping 1,200 vertical feet at average pitch of 31 degrees, to steep 52-degree slope, makes Velocity Peak one of the steepest and fastest courses available.

While the tuck position is optimal for drag reduction, it is not beneficial to the skier's stability and safety. A full tuck tightly closes all the angles at the skier's joints; this leaves the skier with no body resilience with which to compensate for bumps on the course. Serious injury can result when a speed skier hits even a small bump while traveling at more than 100 mph. As a result, most racers use a modified tuck—standing up just a little on their skis with their upper bodies parallel to the snow so their legs can still function as shock absorbers.

Other essential information is the pitch and length of the slope from starting gate to the speed trap at the bottom. Air density computed at the top and bottom of the course is another important piece of information, as it greatly affects a skier's speed. At 12,560 feet, where the starting gate at Velocity Peak is located, the air is thin, thus allowing racers to build up greater speed for the lower altitudes where the air is denser.

Trapping Speed

With information on all the variables entered, the computer calculates the skier's progress every half-second, from a dead start at the gate to the moment the skier clears the speed trap at the bottom. According to Hamilton, his computer program is especially useful in citing which variables have the greatest impact on a skier's speed. For example, the computer showed that moving the starting gate from 12,500 feet to 12,600 feet would result in speeds of up to 130 mph. In 1983, the start was moved up to 12,560 feet. From that altitude, Franz Weber of Austria set a new world record: 129.302 mph.

Joyce Loeffler didn't set a world record, but she wasn't aiming for one. Her goal was to hit 100 mph, and she did it. On her second run down the course, she hit the trap at 112.465 mph. With more experience and the help of the information generated by Hamilton's program it is likely that she will be able to improve her time. ■

Thinking Graphically

Practice in reading as well as making graphs, like Paul Hamilton's speed skiing graphs, is important to prepare students to deal with the wealth of information presented to them every day.

The actual calculations that Paul Hamilton's computer program performs are very advanced, involving variables such as air temperature, atmospheric density, course slope, skier's body weight and drag. But the graphs that result from those calculations provide a good opportunity for class discussion and exercises on how to interpret graphs.

What Do They Say?

Paul Hamilton's graphs, (Figs. 1-3), show how changes in several variables can effect a speed skier's performance. The graphs give Hamilton an overview of the relationships between the many separate data points.

Figure 1 shows the curve resulting from the plotting of Franz Weber's 1982 world record run, during which he reached a speed of 126.238 mph. This graph shows speed in mph recorded at 1/2-second intervals (x axis) versus the altitude of the skier (y axis).

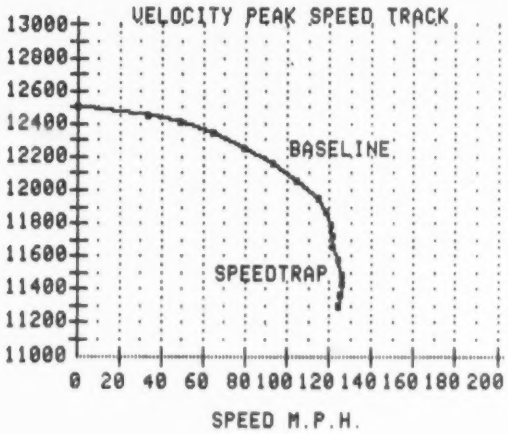


Fig. 1

Figure 2 shows the effect of altitude on speed. Note that an increase of a little over 3 percent in altitude boosts speed by a dramatic 15 percent. The graph conveys the significance of this increase with more immediate impact than data plotted in tabular form.

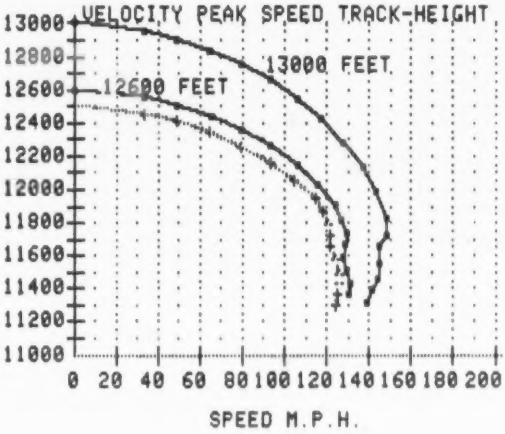


Fig. 2

Figure 3 shows the effect of body position and drag on speed. The projected improvement in performance if a skier were able to take and maintain an idealized tuck (or teardrop) position is astonishing. (The graph shows it is close to an idealization of an object falling down an inclined plane.)

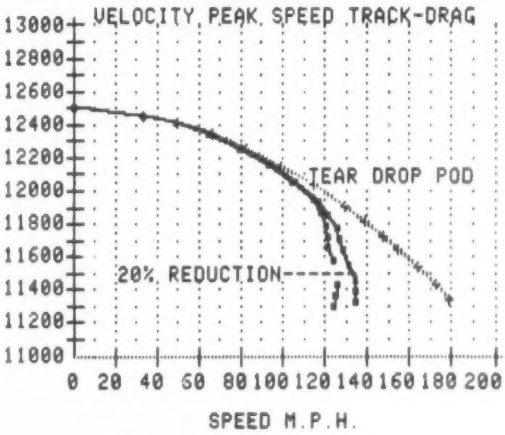


Fig. 3

These relationships, which make themselves known only over a large number of data points, are immediately visible on Hamilton's graphs.

What Axes Does He Use?

Labeling what the x and y axes will represent is perhaps the most crucial step in graphing, especially when you're presented with a variety of possibilities. A good graph is as clear and illuminating as possible, and the variables must not be picked casually.

Paul Hamilton had a choice from among three possible variables: *altitude, speed* and *time* down the slope.

One of the things he wanted to measure was the effect of altitude on speed, so it wasn't hard to choose altitude and speed as the variables for Figure 2. Using the same two variables in the other two graphs as well gave him consistency in all three graphs.

Hamilton chose altitude and speed because that combination told him what he wanted to know—how various factors affect the performance of a speed skier—in a form that allowed an at-a-glance comparison of the three graphs.

His graphs are unusual in the choice of variables, but that's in part because speed skiing is unusual. With many other sports, where the object is to finish first or fastest—to gain the highest *average* speed—altitude and time would have been a more logical combination of variables. With speed skiing, however, time is not an important variable: The skier is clocked only in a "speed trap," instead of being judged by overall time.

The lesson here: A good graph is designed to make the information tell the story you want to convey. In some cases, that may require a "different" approach. Consider all the possibilities.

Engineer Paul Hamilton's research in computer modeling is aimed at finding the limit of human performance in speed skiing.

Exercises:

1. Using Figure 1, what was the altitude of the skier when he was traveling at 20 mph? At 80 mph?

2. How fast was he going when he reached a height of 12,400 ft? 11,800 ft?

3. At what altitude was he traveling fastest?

4. Why is each graph a curve and not a straight line?

5. What variables combine to influence the speed of a skier?

6. Bring examples of graphs from newspapers and magazines, both editorial and advertising, to class. Which graphs are meant to inform, and which to persuade? How informative are the former, and how persuasive the latter? Discuss how the variables are plotted and why they were chosen.

7. In small groups of about 5 students each, collect data on the different groups for graphing. Choose an activity you can perform in class (such as jumping rope, doing push-ups or sit-ups, etc.). Then have each member of the group perform the activity for a specific period of time, e.g., two minutes. Record your data—how many times each member can perform the activity over the time period—and then graph individual results and the group average in several ways.

Next, record the same activity over several different time periods. Start with 10 seconds; then 20; then 30. Make a graph of the results. Next, see if the groups can predict accurately how many repetitions they'll be able to do in 40 seconds, based on the information already in the graph. Here are some questions for discussion:

a. Is the graph a straight line? If so, what does that show? If not, then why not?
b. What variables affect the number of repetitions one can do in a given amount of time?

77

8. In each of the following examples, decide what the variables are and then do a sketch graph (a rough graph which shows approximate relationships without actual quantities) to describe the relationship between them. Explain your graph in words.

How does. . .

a. the time taken to fly to New York from Chicago depend on the speed of the aircraft?
b. the length of time you spend eating depend on the size of the meal?
c. the rate at which you eat vary during a large meal?
d. the value of a car depend on its age?
e. your height vary with your age?
f. the water level in your bathtub vary before, during and after a bath?

CHALLENGE Exercise:

9. Paul Hamilton's graphs show his data in the form of a parabola, showing that the distance traveled is proportional to the square of the velocity.

Most professional statisticians shy away from this kind of graph, preferring the clarity of a straight-line graph whenever possible.

If you want to fit Hamilton's graph into a straight line, try using the square root of the altitude on the y axis. If you have a computer in the classroom, it should be easy to change the variable and plot a new graph.

Downhill Computer

The following program, written in Applesoft BASIC for the Apple II series computer,* plots a graph like the ideal graph for a speed skier (teardrop shape, no friction—see page 64). The variables VI and VF represent the skier's velocity at the top and bottom, respectively, of each 1 meter of descent.

```
10    HGR
20    REM    ********************
30    REM    INITIALIZE VARIABLES
40    REM    ********************
50       VI = 0
60       XX = 4
70       YY = 1
80    REM    *************************
90    REM    DRAW COORDINATE SYSTEM
100   REM    *************************
110   FOR J = 1 TO 31
120      HPLOT J * 9 - 6,154 TO J * 9 - 6,150
130   NEXT J
140   FOR J = 0 TO 15
150      HPLOT 0,J * 10 TO 3,J * 10
160   NEXT J
170   REM    *************
180   REM    DRAW GRAPH
190   REM    *************
200   REM    ALTITUDE IS DIVIDED INTO
202   REM    149 INTERVALS
210   REM    VI IS THE VELOCITY AT THE TOP
212   REM    OF EACH INTERVAL
220   REM    VF IS THE VELOCITY AT THE
222   REM    BOTTOM OF EACH INTERVAL
230   FOR I = 1 TO 149
240      Y = I
250      VF = SQR (VI^2 + 2 * 9.8)
260      X = VF * 5 + 4
270      HPLOT X,Y TO XX,YY
280      VI = VF
290      LET XX = X:YY = Y
300   NEXT I
310   END
```

*Line changes for adapting this program to the IBM PC BASIC. (Note: IBM PC's must be equipped with a color graphics monitor adaptor to run this program.)

```
10   Screen 1
120  LINE (J * 9 - 6, 154) - (J * 9 - 6, 150)
150  LINE (0, J * 10) - (3, J * 10)
270  LINE (X,Y) - (XX,YY)
```

'Chuting a Picture

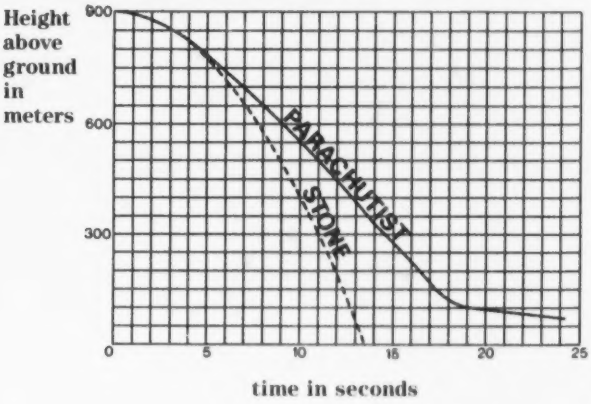


Fig. 1

This graph shows how the height of a parachutist varies with time as she jumps from an airplane. Study it and try to discover what it means.

Exercises:

1. Write a paragraph describing in as much detail as possible exactly what happens to the parachutist, when it happens, and how you can tell. Include in your description answers to the following questions:

a. At what height is the aircraft when the parachutist jumps?
b. When was she falling most rapidly?
c. When did the parachute begin to open?

2. During the period between 8 seconds and 12 seconds after jumping, the parachutist fell from 650 meters to 450 meters. Find these readings on your graph. Calculate her average speed in meters per second for this interval.

Now, complete the table below:

Time interval	Distance fallen	Average speed
From 0 to 4 seconds		
From 4 to 8 seconds		
From 8 to 12 seconds		
From 12 to 16 seconds		
From 16 to 20 seconds		
From 20 to 24 seconds		

Table 1

3. At the same moment that the parachutist jumps from the aircraft, a stone is dropped. A graph of the stone's progress is shown on the graph (Fig. 1) by the dotted line.
Find out as many facts as you can about what happens as the stone falls. For example:

a. When did it hit the ground?
b. When was it falling most rapidly?
c. What was its average speed during the last seconds of its fall?

Compare the graph of the stone with that of the parachutist. Explain the differences.

Answers:
1a. 900 m
b. Immediately before 16 seconds
c. 16 seconds
2. 50 m/sec
3a. 13.5 sec
b. Immediately before hitting the ground
c. 125 m/sec

Further Readings

Dorworth, Dick. "A Roman Circus and the Records Fall." *Ski*, November 1982.

An entertaining first person description of the first championship speed skiing event held at Velocity Peak, Colorado.

***Fast Tracks* Quarterly Newsletter of International Speed Skiing (One East Liberty Street, Suite 412, Reno NV 89501).**

Up-to-date information on the annual scheduled speed skiing events around the world.

Hamilton, P. and Tracy, Dr. Richard. "Testing the Speed Limit by Computer." *International Speed Skiing*, 1983 (One East Liberty, Suite 412, Reno NV 89501).

This article elucidates Paul Hamilton's speed skiing computer program, explaining how the program models a skier's run and what variables it considers. Accessible to high school students.

International Speed Skiing Magazine, published yearly, provides in-depth articles by and about world-class speed skiers, statistics on recent events, race schedules; has good photographs.

Miller, Peter. "Speed Skiing." *The New York Times Magazine*, February 20, 1983.

An article describing what speed skiing is all about from a number of different skiers' points of view. Skiers talk about how it feels to go straight downhill at over 100 mph — and live to tell about it.

Most, Bruno W. "Speed Demons on Snow." *American Way*, November 1983.

A short article containing interviews with and photographs of speed skiing champions, with descriptions, in their own words, of what is required to be a "speed demon."

Perlman, Eric. "The Ballistics of Speed Skiing." *Science 80*, January/February 1980.

A well-informed description of how a speed skier ensures maximum aerodynamic efficiency. This article discusses the effects of drag and ski friction on a speed skier's performance.

Fishing with Sonar

The same technology that is used to locate and catch fish is now being used to protect against too many fish being caught.

During World War II, sonar, which was intended for use in locating enemy submarines, sometimes found fish instead of U-boats. Large fish or schools of fish would interfere with the sonar signals and cause alarms to sound and escort ships to swing into action.

After the war, engineers sought to capitalize on sonar's capacity to locate fish. The result was the development of sophisticated echo-sounding devices that can not only pinpoint the location of the fish but can also differentiate among kinds of fish, their numbers and even their size. The latest echo-sounders can even distinguish between different schools of fish swimming in layers, one above the other.

With this technology, fishermen no longer rely solely on their years of experience to find schools of fish. Sonar helps them decide where and how deep to drop their trawls. It also gives them a rough idea of how many fish they are likely to find at a given location.

But where the new technology has solved one set of problems, it has created another: The danger now is that fishermen might misuse this new information and fish a stock too low—that is, harvest so many fish that the species itself is threatened. It almost happened once. After World War II, the increased efficiency of the North Atlantic fishing fleet depleted the herring stock until it was dangerously low. The fishermen from all the North Atlantic countries agreed to stop fishing for herring until the stock increased.

NET RESULT: Modern fish-finding techniques, using sonar-type devices that track fish by echo location, have taken a lot of the uncertainty out of catches like this one. The devices have proven so successful for Norwegian fishermen that they have had to set quotas, to avoid depleting the stock of fish on which their livelihoods depend.

80

ECHO, ECHO. . .: New echo locators, such as this color video sounder, are now used not only to find where the fish are in the ocean's depths, but how dense the schools are, and even what varieties of fish are to be found.

Setting Limits on Success

To prevent this type of catastrophe from occurring in the future, experts such as Gudmund Vestnes of the Marine Research Institute of Bergen, Norway, have developed an information-gathering network for the fishing industry. They use the same equipment that helps *catch* the fish to collect information that protects against too many fish *being caught*. "We collaborate with research vessels from all the North Sea countries," he says, "as well as those from Great Britain, the Soviet Union and others." That information enables the Institute to calculate how much of each species should be harvested each season. "The tonnage of fish is calculated," Vestnes says, "and this is given to the political authority, which divides the fish stocks among the countries. The amount we advise the fishing authority to take out is divided among the fishermen," so each vessel gets its quota.

Control over the individual boats is precise. "They can't go out before they are given a quota for the kind of fish they are permitted to catch," Vestnes states.

Year after year, season after season, the fishing boats go out as they have for centuries. The work is still hard and dangerous; the winter gales are of blood-chilling ferocity. The information that is available now means a better chance of success on every voyage; and when information is wisely used to manage the harvest, the fish get an even break, too. ■

Further Readings

Dunlap, G.D. *Navigating and Finding Fish with Electronics*. Camden, Maine: International Maine Publishing Company, 1974.

Straightforward and sophisticated description of how electronic devices are used to find fish. Lots of photos and illustrations; only 80 pages. This book is recommended to the student or teacher who is specifically interested in how electronic fish finding works.

Time-Life Books, Eds. *Navigation*. New York: Time-Life Books, 1975.

A clearly written and well-illustrated introduction to coastal and inland water navigation. Chart and map reading, navigational aids such as lighthouses and buoys, radio navigation and sonar fish finding are all covered here. Simple description of the use of sonar for navigation and fish finding.

von Brandt, Andres. *Fish Catching Methods of the World*. London: Fishing News (Books), 1972.

Detailed but very interesting survey of fishing methods around the world. Lots of photos and drawings; covers fishing from Canada to China, Italy to Thailand, Africa to California. Thorough and fascinating for any student (or teacher) who is interested in all the different ways fish are tracked and caught around the world. (110 Fleet Street, London, EC4, England).

Warner, William W. *Distant Water*. Boston MA: Little, Brown and Company, 1983.

Fascinating and well-written account of the North Atlantic fishing industry, told from the point of view of the author, who spent months on the North Atlantic fishing vessels of five different nations. Warner discusses the effect of increased technological efficiency on fishing stocks, and, in the appendix, illustrates the different types of trawl nets and sonar fish finders currently in use.

How SONAR Works

SONAR is an acronym for *so*und *na*vigation *r*anging. It is used to locate objects underwater or to aid communication between submarines or between a submarine and a surface vessel. The sonar or echo-sounding device consists primarily of a *transducer*, which converts electric energy to acoustic energy and back again to electric energy, in much the same way as a telephone receiver works. The transducer is in direct contact with the water, usually suspended from the hull of a surface vessel.

Once sound waves have been "projected" into the water through the transducer, the transducer then functions as a hydrophone and "listens" for the return signal or echo of the sound waves as they bounce back from the ocean floor or from any other object that might lie in their path.

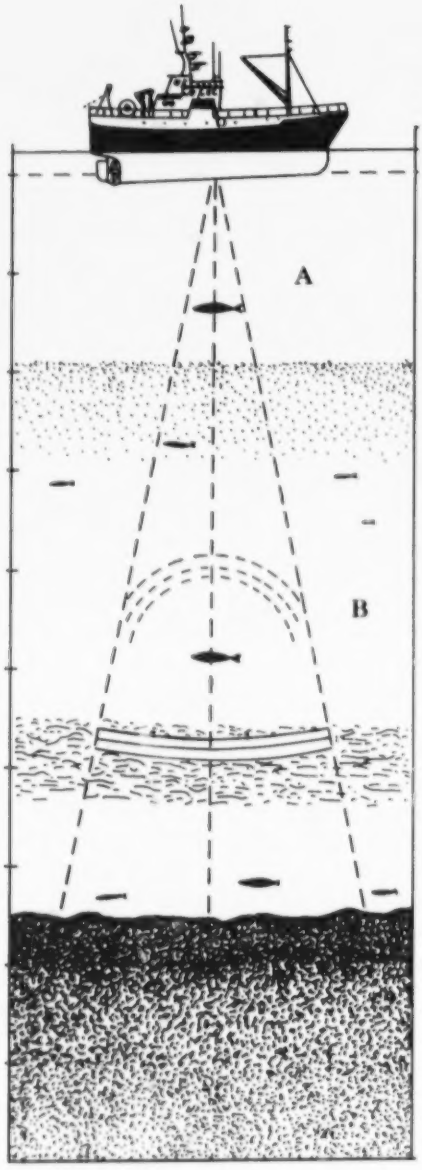
The transducer then converts this returning echo back to electric energy which may be analyzed to determine the range, bearing and nature of the object intercepted. The great advances in echo-locating technology in the last four decades have been in reading these signals to get even more and better information.

The Comings and Goings of Sound

While fishermen think about fishing, the sonar equipment they use makes calculations. Based on the speed of sound in water, the equipment can calculate how far away the fish are according to the length of time it takes for the reflected sound to return to the ship.

Sound travels at different speeds in different materials, depending on how well the material conducts it. In fact, sound travels faster through most other substances than through air. The speed of sound in water, for instance, is roughly five times the speed of sound in air.

The speed of sound in a material will, in general, depend on the temperature and the pressure and on impurities in the material (salt in water or humidity in air, for example), as well as on the material itself.



In the drawing, fish (A) and fish (B) have the same reflective properties. Between them are a thin layer of plankton and several fish of varying sizes. As the sound pulse from the echo locator travels down through the water, the wavefront area increases in proportion to the square of the distance from the transducer, and the intensity of the sound is correspondingly reduced. Although fish (A) and fish (B) are at different depths, the receiver compensates for the loss of signal strength and makes both echoes appear to be equal—alerting the fishermen that the two fish are the same size.

Use this table as data for the exercises that follow.

Medium	Speed of sound (m/sec)
Air (dry, sea level, 0°C)	331
Air (dry, sea level, 20°C)	334
Water (pure, 15°C)	1450
Water (pure, 25°C)	1500
Water (sea, 6°C)	1470
Water (sea, 25°C)	1530
Copper (20°C)	3560
Aluminum (20°C)	5100
Iron (20°C)	5130
Mercury (liquid, 25°C)	1450
Beryllium	12900

Experimentally determined speeds of sound for various materials.

Table 1

Exercises:

1. How long does it take sound to travel (in dry air) 1 km at sea level at 0°C? Express the speed in mph.

2. Devise a rule of thumb to determine the distance of a thunderstorm based on the time difference between the lightning and thunder. (Note: Light travels approximately 186,000 miles per second, so for the purposes of your rule of thumb you can think of it as instantaneous.) Would your rule of thumb be different if you used the actual speed of light in your calculations?

3. Express the speed of sound in sea water at a temperature of 6°C in kph.

4. If the ocean depth is 800 m, how long does it take for a sonar pulse to travel to the bottom, reflect and return to the ship? Assume the temperature of the sea water is 6°C.

5. Using the information in Table 1, write three questions about how sound travels through different materials.

CHALLENGE Exercises:

6. A standard pulse rate for sonar devices is 48 pulses per minute. What is the farthest away an object in the sea (6°C) can be in order that its reflected pulse reaches the ship before the next pulse is emitted? Assume that the duration of the pulse itself is negligibly short. How would your answer change if the temperature were 25°C?

7. Gunnar Nielsen is the captain of the fishing vessel *Christiana*. The vessel is in a region of the North Sea where the bottom is flat and level over a fairly large area. He has just heard over his radio a warning that an old mine has been found floating at the surface and is soon to be intentionally detonated. His sonar hydrophone picks up a large disturbance 5 minutes and 32 seconds later. One half second later a fainter disturbance is detected by the hydrophone. After another 6.5 seconds, Captain Nielsen hears a distant explosion. Ignore the humidity in the air and assume the sea temperature is 6°C and the air temperature is 0°C.

a. How far away was the mine?
b. How do you explain (reasonably) the second underwater sound?

Answers to CHALLENGE Exercise 7:

7a. Let d denote the distance (meters) from the *Christiana* to the mine. Let t_{air} and t_{sea} denote the travel times of the sound in air and sea, respectively. According to the problem, $t_{air} - t_{sea} = 7$ seconds.

Also, $d = t_{sea} \times 1470 = t_{air} \times 331$

Hence, $t_{sea} = d/1470$ and $t_{air} = d/331$

$7 = t_{air} - t_{sea} = d(1/331 - 1/1470)$

So, $d = 7(1/331 - 1/1470) = 2900$ m

b. The second sonar detection is the reflection off the sea floor of the sound of the explosion.

Get Set

1. Answers will vary here. As an extension of this exercise, have students justify their organization.

2. Sometimes it's fairly obvious how things are organized; sometimes there are many possibilities. For instance, the answer for (a) could as easily be "the set of all cities on the Great Lakes" as "the set of all American cities not bordering on an ocean" or "the set of cities whose names begin with 'B,' 'C' or 'D,' among other possibilities. The answer for (b) could be "the set of all fictional characters who solve mysteries" or, more specifically, "the set of all fictional mystery solvers who have been featured in motion pictures or television shows." And so on.

Additional Exercise:
Have your students take one individual item from each of the sets in #2 and make up new set definitions for which the other items would *not* fit.

For example: The set of Presidents of the United States would include "Cleveland," but would not include "Chicago," "Detroit," "Duluth" or "Buffalo."

3. The straight sides of *polygons* are what distinguishes them from the members of the other set presented.

4. Two characteristics determine *gliffiahumps:* They must have two eyes and three humps, but they can have straight or curved sides.

Creating a Catalog

A catalog is a means of *organizing information*. When modern archaeologists uncover a ruin, they catalog each and every artifact so thoroughly that everything could be replaced nearly as it was before they began digging.

Stage managers in theater productions and continuity people on movie sets must also use a cataloging system to make certain that people and objects are properly placed on the stage or set. Librarians have a standardized system for locating books and putting them back on the shelves.

Such systems are quite elaborate. Some require detailed drawings; others are numerical.

Have your students develop their own cataloging system for the objects and people in their classroom at a given moment in time. They should make their catalog as detailed as possible, so that by looking at the information they might recreate how the classroom looked at that precise moment.

One way to check the accuracy of their system is to take a photograph of the classroom after they have completed their catalog. Then, at some future date, have the class put all the objects and people back into the positions they were in at the time of cataloging. Check the results by looking at the photograph and take another for comparison purposes. Then have your students refine their system based on the results.

Get Set

Here are some questions and a classroom game about one of the ways we organize information: the use of *sets*.

1. How are the following items usually organized? What are some other ways they might be organized?

a. groceries in a supermarket
b. toys in a toy store
c. screws, nuts, bolts and nails in a hardware store
d. records or cassette tapes

2. What are some of the sets from which the following subsets might have been taken?

a. Cleveland
Chicago
Detroit
Duluth
Buffalo

b. Nancy Drew
Sherlock Holmes
James Bond
Hardy Boys

c. camels
elephants
canoes
trains
planes

d. 2
4
12
26
90

3. Distinguish between the two sets below. What sets a *polygon* apart?

These are POLYGONS:

These are *not* POLYGONS:

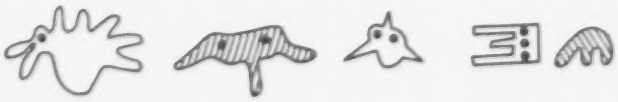
Which of these are POLYGONS?

4. Now, a harder one. Figure out which characteristics distinguish the set of *gliffahumps*.

These are GLIFFAHUMPS:

These are *not* GLIFFAHUMPS:

Which of these are GLIFFAHUMPS?



5. Make up some of your own "set" problems to try out on your classmates. Remember that one or more distinguishing characteristics will define a set.

Taking Up Space

This problem will help students distinguish between relevant and irrelevant information. The answer is that it doesn't matter whether the ships are 5,000 or 50,000 km apart. What is relevant is the speed at which they're traveling. At 1 minute before collision, they will be 8 km + 12 km or 20 km apart.

Missing Numbers Bureau

In the tradition of the great detective stories, you can deduce "the whole story" armed with only partial information, *if* that information fits a logical pattern.

If this exercise can be done at all (it can), the student must find the necessary relationships in the first three columns, which are the only ones to be completely filled in. These relationships must fit *all* three columns.

There are two important relationships apparent in the first three columns:

1. The value in the top row of each column is the sum of the values in the second and third rows of the same column.

2. The value in the bottom row of each column is the product of the values in the second and third rows in the same column.

Although the instruction is "fill in the missing numbers," the problem is actually "find the relationships inherent in the data given." The problem is solved when those relationships are perceived and verified; after that, it's a matter of calculating and filling in the blanks.

On the Right Track

Students accustomed to word problems that serve up, step by step, all the information necessary for their solutions are only flexing a fraction of their problem-solving muscles. In addition to computational drills, students should spend time on exercises that develop their analytical skills. Take, for example, the venerable word problem "On the Right Track."

At first glance, the problem contains a lot of information: the speeds of the trains; the distance between them; the speed of the fly. It has all the makings of a plug-in-the-numbers rote exercise. It may be tempting, at this first glance, to use that information to try to figure out how many back-and-forth trips the fly takes. Students who try this will quickly find it a waste of time.

Solving this problem is a two-step process, and the first step is to solve for the crucial piece of information that isn't given: the amount of time the fly spends in the air. This is the same as the time it takes for the two trains to meet. The formula is:

$$r_1t + r_2t = d$$

or,

$$25t + 15t = 120$$
$$40t = 120$$
$$t = 3 \text{ hours.}$$

The fly travels for 3 hours at 100 mph, covering a total of 300 miles in back-and-forth trips.

Taking Up Space

Two spaceships are moving straight toward each other on a collision course. One ship is going 8 kpm, the other 12 kpm. Assume they are exactly 5,000 km apart. What will be the distance between them 1 minute before they collide?

Missing Numbers Bureau

5	8	7				10		14	15
2	4	5	3	7		6	5		
3	4	2	4	6	9		8		
6	16	10			36			49	56

Table 1

Fill in the missing numbers using the information that is already there.

In One Form or Another

Finding the information you need can be a two-step process: finding the data, then, performing some mathematics to get it in the form you want. What do you need to know in order to find out:

1. The average speed of an airplane flying from Los Angeles to New York?

2. The cost of a 5-lb bag of potatoes?

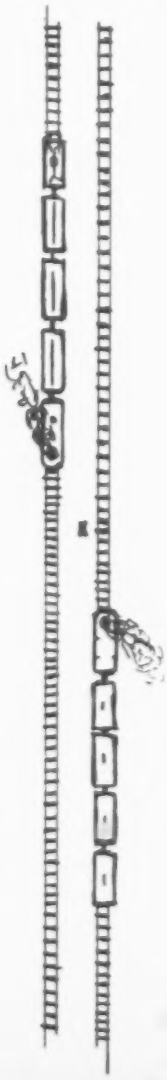
3. The average cost of a long-distance phone call in your household?

4. The rate of increase of the temperature of a pot of water when heated to boiling?

On the Right Track

It is 120 rail miles from Penn Station in New York to Union Station in Hartford. As one train leaves New York for Hartford at 25 mph, another train leaves Hartford for New York at 15 mph. At the same time, a fly takes off from the front of the Hartford-bound train flying straight toward the other train at 100 mph.

On meeting the second train, the fly immediately turns around and heads straight back toward the first train. If the fly keeps going back and forth from one train to the other, with undiminished speed, how far will the fly have flown by the time the two trains meet?



Teacher's Notes

(see opposite page)

Race Tracks

This is an exercise in analyzing information, which is given not in numbers or words, but in pictures. To solve, students must make several deductions:

- The left side of each dip in the graph represents a drop in speed, and the right side represents an acceleration. So each dip on the graph probably corresponds to a bend in the track. The deeper the dip, the greater the deceleration and thus the sharper the bend.
- The track has three bends. This rules out B and G.
- The first bend is sharper than the third. This rules out C and E.
- The first bend is not as sharp as the second. This rules out F.
- The straightaway after the second bend is longer than the first. This rules out A.

Based on a careful analysis of the information in the graph, D is the only track which meets all the criteria.

Running to Conclusions

In question #3, it takes some thought to decide on the scale of the vertical axis. The runners' times are so close together that too great a distance between intervals on the time scale would make it difficult to plot the graph. In Figure 1, the time scale (on the vertical axis) extends from 3 minutes to 4 minutes, 20 seconds at 10-second intervals.

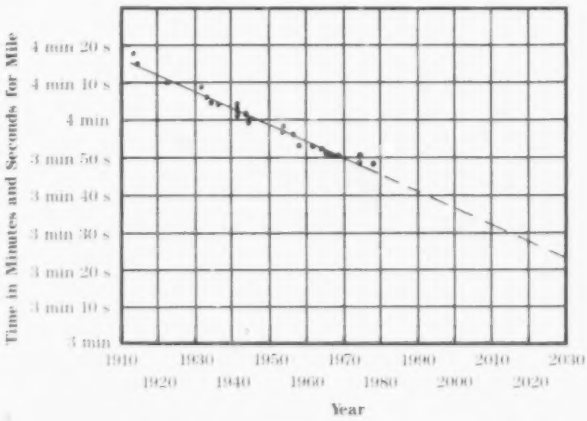


Fig. 1

Without getting into some of the more complex methods of projection, the graph does suggest one intuitive way to project the world's record for the year 3000 (question #4)—by extrapolation from a straight line connecting the first and last data point. But extrapolation is only possible if a function continues to behave in the same way.

At an average change of about −0.4 seconds per year, runners would decrease the world's record for the mile by 407.6 seconds by the year 3000, thus achieving an impossible time of −3:00:03—finishing the race more than 3 minutes before they started it.

Though the graph appears to be a straight line, it will ultimately become nonlinear as the limit of human performance is reached. Without more information, it is impossible to estimate the mile record for the year 3000.

Class Project:
Have your students find out their average growth rate using their current height and birth height. What happens when they project their height for the age of 50?

Race Tracks

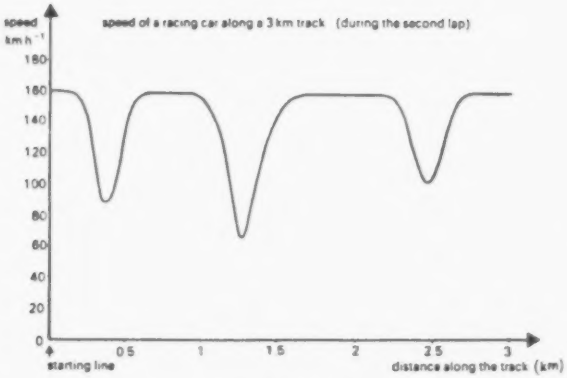


Fig. 1.

Figure 1 shows how the speed of a motorcycle varied as it completed the second lap of a race along a flat 3-km track. Study the information in it carefully. Which of the tracks in Figure 2 matches the graph?

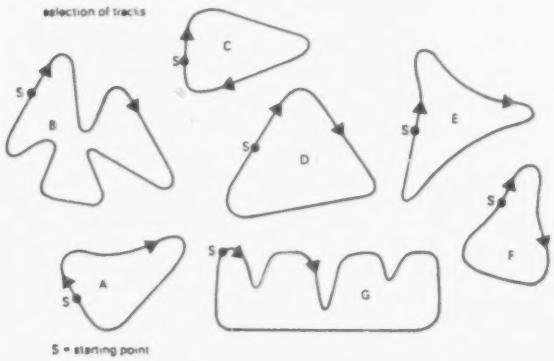


Fig. 2.

Running to Conclusions

4:14.4	John Paul Jones	USA	1913
4:12.6	Norman Taber	USA	1915
4:10.4	Paavo Nurmi	FIN	1923
4:09.2	Jules Ladoumegue	FRA	1931
4:07.6	Jack Lovelock	NZL	1933
4:06.8	Glen Cunningham	USA	1934
4:06.4	Sydney Wooderson	GBR	1937
4:06.2	Gunder Haag	SWE	1942
4:06.2	Arne Andersson	SWE	1942
4:04.6	Gunder Haag	SWE	1942
4:02.6	Arne Andersson	SWE	1943
4:01.6	Arne Andersson	SWE	1944
4:01.4	Gunder Haag	SWE	1945
3:59.4	Roger Bannister	GBE	1954
3:58.0	John Landy	AUS	1954
3:57.2	Derek Ibbotson	GBR	1957
3:54.5	Herb Elliott	AUS	1958
3:54.4	Peter Snell	NZL	1962
3:54.1	Peter Snell	NZL	1964
3:53.6	Michel Jazy	FRA	1965
3:51.3	Jim Ryun	USA	1966
3:51.1	Jim Ryun	USA	1967
3:51.0	Filbert Bayi	TAN	1975
3:49.4	John Walker	NZL	1975
3:49.0	Sebastian Coe	GBR	1979
3:48.8	Steve Ovett	GBR	1980
3:47.3	Sebastian Coe	GBR	1981

World records for the mile run from 1913 to 1981.

Table 1

1. How much faster did Sebastian Coe run in 1981 than John Paul Jones in 1913?

2. Add another column to the table showing running speeds in mph.

3. Plot the runners' times on a graph. Do the data points suggest that a straight-line graph would be appropriate?

4. Can you use the graph to project the world's record for the year 3000? Backward to the year 1500? Do your results make sense?

RESTATEMENT

How Do I See It?

RESTATEMENT: *How Do I See It?*

Running Time: 19:14 minutes

Documentary Segments (4):

1. ***DINOSAURS:*** Paleontologist James Farlow challenges the assumption that dinosaurs were slow, lumbering creatures. **With no live dinosaurs to study, can he find a suitable replacement to fill the great beasts' shoes?** ..92
2. ***LOCKS:*** An industrial design class, given an assignment to redesign a 50-year-old door lock, ponders a key question in lock design: **How do you make it easy to lock others out, without locking yourself in?** ..98
3. ***SWAMP:*** The Okefenokee Swamp presents a face of ageless serenity, but a closer look reveals a teeming center of flora and fauna that make up its complex life cycle. **Can a new kind of geometry predict the swamp's future and help prevent ecological disaster?** ..102
4. ***SKI BUS:*** A pair of Olympic ski jumpers and an engineer team up to solve the ski jumper's perennial problem: practice time away from the slopes. **Can a specially equipped school bus substitute for an expensive wind tunnel?** ..110

Comedy Segment:

5. ***WALLPAPERING:*** "Mr. Manley the Paperhanger" mistakenly hangs diagonal stripes on Mrs. Hopkins' walls. **If he can't change the way the stripes are, can he at least change the way people see them?** ..90

Math Concepts:

DINOSAURS: Distance rate time, exponents, negative numbers, formulas, modeling, prediction linear equations, graphing

LOCKS: Three-dimensional modeling, ratio proportion, scale drawing

SWAMP: Geometrical modeling, fractal geometry, exponents, progression, fractions, topology, prediction, formulas, dimensions, curves, graphing

SKI BUS: Distance rate time, ratio proportion, angles, vectors, metric system, formulas, exponents

Problem-Solving Concepts:
Solving by analogy; prediction by modeling; problem comprehension; problem translation.

HOW TO SPOT A RESTATEMENT PROBLEM

A problem calls for restatement if. . .

- ***It comes to you in a "foreign language."*** *Translating the problem into your own terms lets you understand it, or makes it easier to solve.*
- ***It doesn't suit your own problem-solving style.*** *Most people have a particular way of solving problems. Visually oriented people might draw diagrams; verbally oriented people might talk it out with a friend. Restating the problem puts it in a form you can work with.*
- ***It is not instantly understandable.*** *Restatement problems need some degree of interpretation for the sake of clarity and conciseness.*

Throughout the chapter, the $X =$ sign indicates pages containing exercises and problems for classroom use and discussion.

SHIMMERING CITY: Reflection of Amsterdam buildings in shimmering water creates an abstract restatement of a city.

RESTATEMENT

How Do I See It?

"In other words..." How many times a day do we hear or use this phrase? It's a signal: It indicates that an idea is about to be explained in a different, and hopefully, more accessible, way. Often these *restatements* are made specifically in terms of personal experiences, relating something new and perhaps confusing to something else already understood.

This phrase—and its extensions—should be part of every problem solver's tool kit. Restatement clarifies, simplifies and opens doors. *Restating a problem in one's "own language" personalizes a problem, makes it more familiar and adds interest.*

Restatement is the way to understand and explain situations which, because of their magnitude, distance or the commitment of time or resources, are beyond the realm of demonstration. Architects show what a building will look like through blueprints and scale models; designers make sketches; sculptors make cartoons; project managers track their progress on charts; astronauts train in chambers that simulate weightlessness; engineers and inventors communicate their ideas through sketches, models and prototypes.

Restatement is also a convenient timesaver. Businesses often show numerical trends and relationships in the form of graphs. The graphic restatement facilitates the understanding and interpreting of data and allows busy executives to forgo detailed analyses in favor of more productive uses of their time.

Why **Restatement?**

As a problem-solving skill, restatement holds a pivotal position. It is important as a *communicative* tool, to clarify and define obscure problems. It is a *modeling* tool, to simplify large and complex problems. Not least, it is a *motivational* tool, defining problems in personal—and thus more compelling—terms.

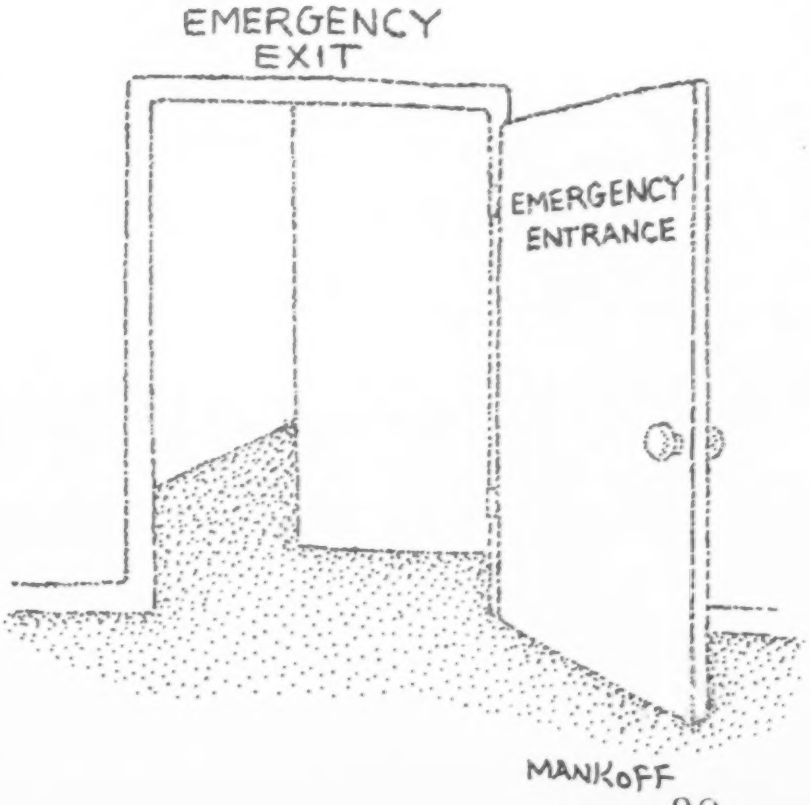
Every student, when faced with a problem, should start by asking a question: "What *is* the problem?" This is often very different from "what does the problem say?"

If the answer is "I don't know," the student must find a way to restate the problem in whatever form it takes to understand it better. Otherwise, there's no hope for a solution.

The restatement process is individual. It will vary with the student and with the problem. It might involve rephrasing the question, drawing a picture or diagram, even acting out the problem situation or talking it through with a friend or with oneself.

For example, a student might be given the following problem:

> *There are 125 players in a tennis tournament. Each player who loses a match is eliminated from the tournament. The winner must win every match he or she plays. How many matches must be played before there is a winner?*



If the problem isn't clearly understood, the student might try to find an algebraic formula or draw a tree diagram to solve it—wasting a lot of time, and perhaps giving up in frustration. A simple restatement of the question serves to focus the problem and make the solution obvious: "How many people must be eliminated before there is a winner?" Since there are 125 players in the tournament, 124 must each lose one match. So a winner will emerge after 124 matches.

One of the most frequent and apt criticisms about word problems is that they don't pertain to students' experiences. The charge might just as well be made that most word problems are presented in a standard form, regardless of whether the form suits the student's ability to understand them. The ability to restate a problem gives students the means to define problems in *their* terms, not their textbooks'.

Teaching **Restatement**

Restatement is a problem-solving tool that would get a lot more use if students understood its power. Besides, it's often fun—its "let's pretend" aspects have a strong sense of play. Teachers can build on that association to make students more motivated and capable problem solvers.

Restatement simplifies complex problems. Some problems are too large and unwieldy to be solved by conventional methods. How do NASA scientists determine lift-off times and flight paths to the moon? Modeling the situation, whether by drawing, graphing, acting out, using mathematical formulas or other means, reduces a problem's complexity by eliminating non-essential details.

Restatement alerts the student to missing data. In addition to weeding out unnecessary information, making a model alerts the problem solver to what information is missing. This is not always obvious in the wording of the problem, but becomes apparent in the process of stripping the problem down to make the model.

Restatement ensures understanding. A problem-solving axiom: If you can't restate a problem in your own words, you don't understand it. A good test of whether a student understands any problem, no matter the size, is to have him or her try to explain it to someone else.

Restatement promotes creativity. Encouraging students to look for new approaches and analogies with other problems they've solved breaks them out of the "one tool/one job" mind set. Restatement draws on the full store of past experience for solutions, often making far-reaching connections.

Restatement gets students involved in "other people's problems." Putting problems in their "own language"—expressing problems in terms of their own experience—puts students *into* the problems instead of *against* them. With increased understanding comes increased involvement . . . and greater problem-solving skills. ■

PROBLEM SOLVING AT PLAY: Restatement can make complex problems interesting by letting the imagination take over, emphasizing the "play" aspects of problem solving.

Restatement in the Classroom

The most basic way to restate a word problem is to change the words. This can help, but it's not always enough. Students have different communicative strengths and weaknesses; some react more strongly to words, some to symbols or pictures. There are techniques you can develop in your students that allow them to see their strengths and can help them see problems in the ways that best suit them as individuals.

Drawing: Graphic representation is an important problem-solving technique with mathematical applications. Restatement of a problem situation in pictorial terms can cut complex, information-dense word problems down to size. For students who have difficulty in decoding even simple word problems, a drawing can tell the story where words only confuse.

BRINGING THE CITY DOWN TO SIZE: A city is much too big to work with on its own terms. Urban planners build scale models to get the big picture in a smaller package.

Graphing: Indispensible in business and science, graphs allow quantities of information to be absorbed at a glance. Graphing encourages students to think about numbers in terms of relationships and patterns, not merely isolated quantities. Students should develop graphing skills early and practice them often.

Formulas: Formulas restate a problem in terms of its bare essence: the data to be operated on and the operations to be performed. In a broader sense, a student skilled at matching a problem with the appropriate formula is simply restating the problem in terms of previously solved problems, which might have been worded very differently. Students who have not yet learned about formulas can compare and analyze related problems. What makes them similar? What techniques or operations do they have in common?

Role Playing: Students who have special trouble with word problems can benefit from acting out the problem situation, translating from the "textbook language" of the word problem to the students' everyday language. For instance, problems involving money can be staged using play money or even pieces of paper. This gives an added dimension of life to the textbook; it makes the problems more relevant and more interesting to students because *they are creating them* in the form of skits. The solution becomes part of the play. ■

"In many cases, an external representation is very helpful for solving problems. Drawing a sketch, jotting down lists, writing out equations, and making diagrams can help us to remember information and to notice new relations in the problem."

John R. Hayes, *The Complete Problem Solver*

Behind the Scenes: RESTATEMENT On Film

In months of combing files, staring at microfilms, and conducting telephone interviews, our researchers developed some rules of thumb for what would make a good film story. If the initial reports made us ask ourselves, "*What* is that?" or "How did they do that?" we knew we had a hot lead. And if we asked ourselves, "What made them do it *that* way?" we knew we not only had a prospect for the series, but a likely RESTATEMENT story.

The documentary segments in RESTATEMENT show how some very inventive people have tackled their problems in surprising ways.

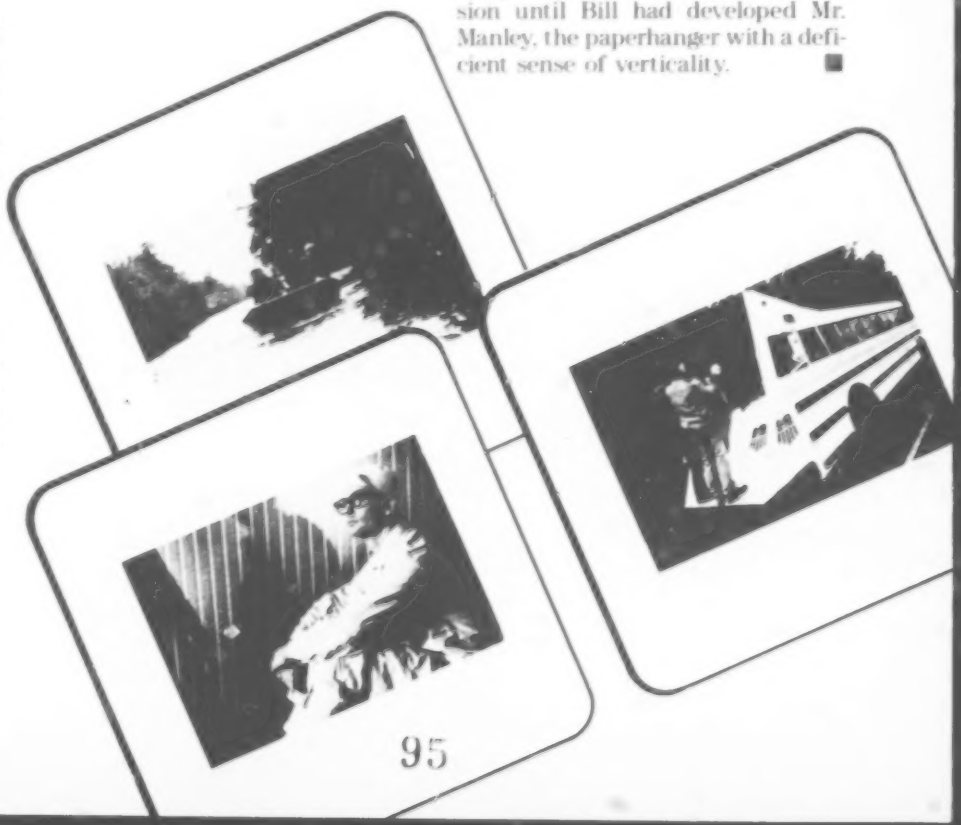
Dinosaurs: Dinosaurs are perennial favorites with young students, and therefore a logical topic for us to include in CHALLENGE. There's only one problem with filming dinosaurs: there are no dinosaurs left to film and it's difficult to get the feel of the real thing from skeletons and models. Paleontologist Jim Farlow had a similar problem studying dinosaurs, trying to model how they moved and how fast they might have run. He solved his problem (and ours) by finding a living model he could study—an ostrich. This segment illustrates the use of models as a means of restating a problem; it's also a good jumping-off point for discussing the derivation and use of formulas and distance/rate/time.

Locks: The objects that surround us everyday—our desks, chairs, lamps, cars, even eyeglasses—are someone's solution to a common problem. In CHALLENGE, we wanted to look at one of these solutions that we take for granted and see if it couldn't be solved in another way. We found our story in Noel Mayo's industrial design class at the Philadelphia College of Art, where students were redesigning a common door lock. The class takes the lock apart and asks why it is the way it is and what else it might be. Their sketches and models are a lesson in scale and three-dimensional thinking.

Swamp: We were just plain intrigued by *fractals*. This esoteric branch of geometry produces such startling and wonderful patterns. But how were we to incorporate it into our film series? We found our answer in, of all places, the Okefenokee Swamp, where ecologist Bernard Patten and mathematician Harold Hastings both investigate the complex ecosystem using mathematical models. Hastings' fractal model may provide an early warning system for the fate of the swamp's magnificent cypresses.

Ski Bus: Serious ski jumpers now use wind tunnels in training to improve their aerodynamic profile. We thought this would be an excellent topic for CHALLENGE. But when we talked to ski jumpers Jeff and Jon Denney, we found an even better story. The Denney family doesn't train in wind tunnels. With the help of neighbor/engineer Bob Mittelstadt, they built their own—out of a *school bus*! In the RESTATEMENT film, you'll see a wind tunnel cruising down the highway at 55 miles per hour, a memorable image and the basis of some interesting material about lift and drag as well as modeling.

Wallpapering: This comedy segment started with a play on words, an inspired pun by executive producer Denny Crimmins. After defining "restatement" as an individual *slant* on solving a problem, he said what about having Bill do something on a slant, all wrong, of course. It was just a hop, skip and an all night script session until Bill had developed Mr. Manley, the paperhanger with a deficient sense of verticality. ■



Following Through: Problem Ideas from the Film

Students will find plenty of fodder in RESTATEMENT for making up their own problems. Here are some examples to get them started.

Dinosaurs: The formula James Farlow uses to calculate the running speed of a dinosaur (see page 95) was derived by R. McNeill Alexander from his observations of different animals.

- How would you clock the running speeds of various class pets—dogs, cats, hamsters, rabbits, gerbils, etc.? Can you use the data to come up with a formula that would link their speed with physical characteristics such as weight or height? Would you expect such a formula to work all the time?

Locks: Designers, inventors, engineers and architects make good use of scale drawings and models to work out the kinks in their plans before committing time and money to actual production.

- Choose a common object—for instance, a school desk—and redesign it. Could it be made more useful? More comfortable? More attractive? Make scale drawings and models of your new design.

- Make a scale drawing on graph paper of an object in your classroom. It could be a book, an eraser, a chair or anything you want. The drawing needn't be scaled smaller—it could be a large drawing of a small object, or it could be scaled 1:1 (life-sized). Explain why you chose the scale you used. Did your drawing give you any ideas as to how the object might be improved?

TENNIS, ANYONE? Stunt fliers Ivan Unger and Gladys Roy turn the wings of an airplane into a flying tennis court for a quick set before landing (1925).

Swamp: Both Harold Hastings and Bernard Patten use models to help them understand the complexity of ecological cycles in the Okefenokee Swamp.

- A graph is a kind of model. Graph the cycle of sunsets over the seasons, using the times for sunset listed in a newspaper as your data points. Make it a big graph and keep it on the wall of the classroom, adding a data point to it as often as possible.

- Try to draft an economic model of your finances, including how much money you take in as income (allowance, babysitting money, money for chores, etc.) and how much is taken up by expenditures. Show the income and expenditures in boxes of proportional size.

Ski Bus: Olympic skiers Jeff and Jon Denney use a "wind tunnel on wheels" to give them more practice time in the air when conditions do not lend themselves to being able to practice on an actual hill.

- During a jump, a ski jumper is actually in the air for about 4 seconds. If a skier practices 6 jumps a day, 5 days a week, over a period of 3 months, how many jumps does he make? How much time does he spend in the air? How many jumps could he make if he were able to practice 12 months of the year? What percent increase would this represent?

- The history of sports is full of stories of how different sporting events came into being through the process of restatement. Have you ever thought of how changing one small rule of a common game might change how the game would be played? For instance, what would happen if baseball officials suddenly decided to do away with second base? Or if tennis officials introduced a larger ball with much more bounce into the game? Think of some other "sports restatements" and how they would change the game being played.

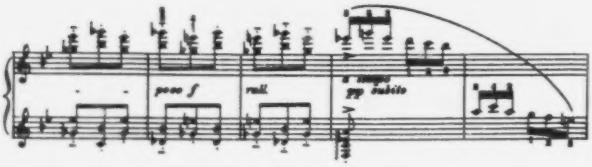
GETTING STARTED: **Thinking RESTATEMENT**

Before viewing the RESTATEMENT film, try the following exercises with your class to get them thinking about the topic.

1.

- maps
- formulas
- sheet music
- graphs
- scale models
- Morse code
- shorthand
- blueprints
- equations

These are restatements of various ideas, relationships or objects. For example, maps are restatements of geographical relationships; formulas are restatements of mathematical relationships; sheet music is a restatement of the actual sound of music.



What about the others? Of what are they restatements? What other forms of restatement can you think of? Why do we have them? What problems do they help us solve?

2. Graphs are effective and versatile ways of restating relationships. **What do the following graphs represent? How would you put the relationships expressed in the graphs into words?**

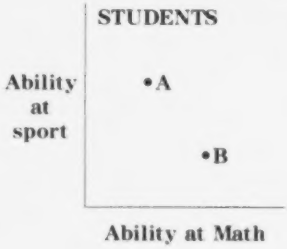


Fig. 1

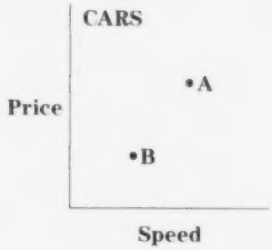


Fig. 2

3. Restate the following math problem *without* using numbers.

Mary is a flying veterinarian whose territory covers a sizable portion of Alaska. She leaves her office and flies due south for 20 minutes at 140 mph to vaccinate Sam's cows. Then she flies due east for half an hour at 130 mph to take the stitches out of Mrs. Grant's horse's leg. By the time she leaves Mrs. Grant's, she has only 45 minutes to get back to the office to take care of Mr. Taylor's cat. How fast must she fly to make it back in time?

There are many possible representations of this problem. Here's one version:

Mary flies south at one rate, and then east at another rate. If she flies straight back to where she started, at what rate must she travel to get back to work at a given time?

Making a sketch is another way to restate the problem so that the essential relationships may be seen more clearly. Then, a formula can be developed and the numbers to solve the problem plugged in.

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Mary's path—first due south, then due east, then home—traces the sides of a right triangle, as Figure 3 shows. The trip home is the hypotenuse.

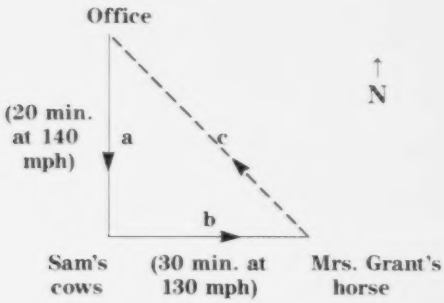


Fig. 3

To solve: First, find the distance back to the office ($c^2 = a^2 + b^2$). Mary has 45 minutes, or 3/4 of an hour to travel that distance. So if d = the distance home, and r = rate of travel in mph, then, since $\boldsymbol{d = r \times t}$,

$$d = r \times 3/4$$

or,

$$4/3d = r$$

Mary must travel at (4/3 × distance home) mph to keep her date with Mr. Taylor's cat.

For Openers

Here are some questions to ask your students *after* they've seen the RESTATEMENT film.

- What is "restatement"? What are some of the ways you can restate a problem?
- Describe a problem you might have had that was easier to solve when you restated it.
- How do the stars of the film segments restate the problems they are faced with?
  1. In what way were James Farlow's ostriches restatements of a dinosaur? What did Farlow learn from them?
  2. How did the students at the Philadelphia College of Art restate the ordinary door lock? How was restatement an important part of the design process?
  3. What restatements helped Bernard Patten and Harold Hastings learn about the ecological cycles of the Okefenokee Swamp?
  4. How is a ski bus a restatement for a wind tunnel? What did the skiers hope to learn?
  5. How does "Mr. Manley the Paperhanger" use restatement to his advantage and to the satisfaction of his client?

"Verbalize, use language to simplify the statement of the problem, write it down. Use mathematical or graphical pictorial models. Use abstract models such as symbols and equations, or use concrete models in the form of objects. . . . A model is a simpler representation of the real world problem; it is supposed to help you."

Moshe F. Rubinstein, *Patterns of Problem Solving*

Stalking a **Restatement** Problem

Making Up Your Own Problems

The following is a hypothetical conversation between teacher and students. It is intended to suggest ways to encourage students to make connections between problem-solving concepts in the film, their own experience and their knowledge of mathematics.

"

- ***Okay. Who's going to be first with a restatement problem? Diane, how about you?***

  I don't have any problems.

- ***No problems at all? It doesn't have to be a real problem. You can make one up. Didn't you and Cindy work together mowing lawns last summer?***

  Yes. We're going to do it again this summer, too.

- ***Suppose the principal called you and asked you to take care of the school grounds. How much time do you think it would take you?***

  I don't know. . .a lot, I guess.

- ***Let's try to find out. Let's do even more than that: We'll work out a way that would let us find out not just for the school, but for any property.***

  How do we do that?

- ***We'll build a model. We'll work out a formula that you can use in any situation. What do you think we'll need to build our model?***

  I don't know. . .what do you mean by "model"? Are we going to make something, like a model airplane?

- ***Not exactly. By "building a model," I mean we're going to make a miniature version of an actual lawn-mowing job. But instead of using glue and plastic, we're going to use numbers. See what I mean?***

  Yes. But what numbers do we use?

- ***What numbers do you think are important? What makes one lawn-mowing job different from another?***

  The size of the lawn.

- ***Good. That's an important part of our model. What else?***

  The number of people working.

- ***Okay. That's part of our model, too. Anything else?***

  That's all.

- ***Alright. Now we'll figure out how to make the model. First we need something to base it on: a piece of lawn small enough to mow fairly quickly, but large enough to base some deductions on.***

  My backyard isn't too big, but it's not too small either.

- ***Let's use that. Now, we need a way to measure it.***

  A ruler or yardstick? That sounds too hard.

- ***I think you're right. We need a way that we can apply even to very large lawns. We don't have to be exact, but we should be pretty close.***

  What if we paced it off?

- ***That would work just fine. What's the area of a backyard?***

  The length times the width.

- ***Right. So we have part of our model. We'll call it AREA. Now, how long does it take you to mow your backyard?***

  I don't know. My brother always does it.

- ***Well, we don't need to know for our model anyway. We'll just call it TIME TO MOW. Now are you beginning to get the picture?***

  Yes. If I know how long it takes to mow my backyard, and I know how big the lawn is, then I can measure another lawn and figure out how long it would take to mow it, too.

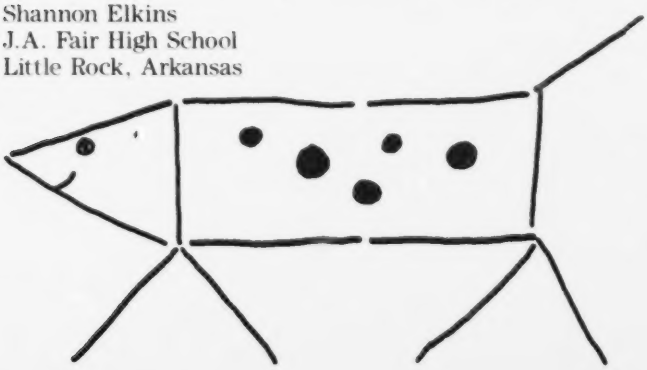
- ***Right. Let's put it into a formula. We'll use subscript 1 on everything that has to do with your lawn, and subscript 2 on everything that has to do with the lawn you're comparing it to. So the formula will look something like this:***

  $\text{TIME TO MOW}_2 = \text{TIME TO MOW}_1 \times (\text{AREA}_2 \times \text{AREA}_1)$

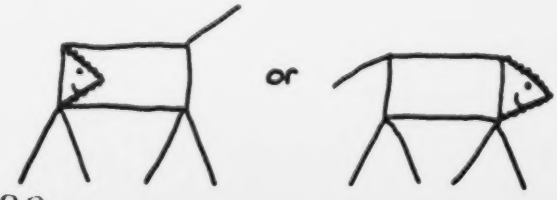
With 13 toothpicks, make a toothpick creature like the one shown

The toothpick creature is looking to your left. Can you make it look to the right, by moving only 2 toothpicks?

Shannon Elkins
J.A. Fair High School
Little Rock, Arkansas



Solution



♦ ***Do you see why it goes that way?***

I'm not sure...

♦ ***Alright. First you want to know how much bigger (or smaller) the new lawn is than your lawn. So we divide the new lawn's area by the area of your lawn. Let's say the answer is two: that means it's twice as big. Then it should take twice as long to mow the new lawn—so TIME TO MOW$_2$ = TIME TO MOW$_1$ × 2. See?***

I see. But what if the new lawn has lots of rocks? Then it will take longer.

♦ ***A model of your lawn will tell you how long it should take to mow a lawn like your lawn. For a lawn that's very different, you'd have to have another model, or estimate the extra time it would take.***

What if Cindy and I work together? Then it won't take as long. Do we need another model for that?

♦ ***All you have to do is mow your lawn together and you'll find out how much time it takes together. You use that number in your model instead of the time it takes you alone. Now we'll finish with an example. Let's say your lawn is 100 by 100 paces, and it takes you 15 minutes to mow it. Let's say the school lawn is 450 by 600 paces.***

My lawn's area is 1000 square paces, and the school's is 450 × 600...130,000 square paces. Wow, that's a lot!

♦ ***How long would it take you?***

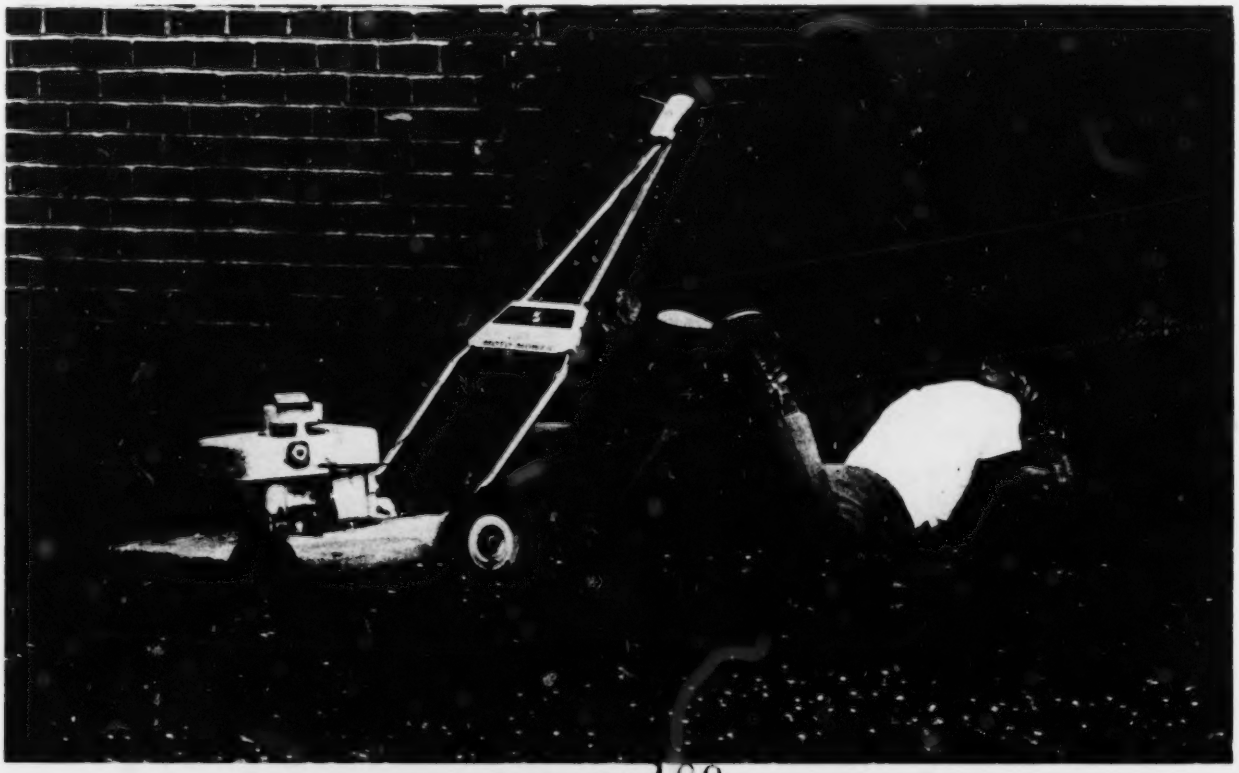
It would take 15 minutes times 130,000/1,000...that's 15 minutes times 130...1,950 minutes, divided by 60...32½ hours. Nearly a whole working week!

♦ ***I think you're going to need some help. What if Cindy works with you, and together it takes you only 10 minutes to do your lawn?***

That would be 10 minutes times 130, or 1,300 minutes, or 21⅔ hours. That's much better!

♦ ***Let's review. In order to let you figure out how long it should take to mow an unfamiliar lawn, we used your lawn as a model. We made a formula using the time it takes you to mow your lawn. Now you should be able to measure a new lawn and figure out how much time it should take to mow it, alone or with Cindy.***

I think I'll try it this summer. But I'll have to do it when my brother's not home. I don't want him to see me mowing our lawn....It might give him ideas! ”



On the Lighter Side

Mr. Manley hopes to earn his stripes on Mrs. Hopkins' wall. Instead, he misreads his problem situation. Because of a lopsided view of the world, he hangs the paper with the stripes running diagonally instead of vertically. An unconventional solution saves the day. Sawing off portions of the legs of his employer's furniture, Mr. Manley gives the chairs and tables the same slant as the wallpaper.

His restatement of the living room is a hit! Mrs. Hopkins and her new living room are the toast of the neighborhood, and Mr. Manley's professional reputation gets a happy boost.

Is this solution a good one or a lucky one? What do your students think? Is it a brilliant idea, or is Mr. Manley just angling his way out of a jam?

A Comedy of Errors: Wallpapering

Mrs. Gerard Hopkins had hired the one wallpaper hanger in Weehawken with a broken leg.

Mr. Manley—perhaps as a result of his accident, and the prescribed plaster cast about his ankle—had come to see things with his own particular bias.

A New Slant on Life

- Mrs. Hopkins was so impressed with Mr. Manley's wallpapering that she hired him to tile the bathroom floor. The floor to be tiled was 6 feet by 4 feet. Each tile is 8 inches square. How many tiles does Mr. Manley need to buy to finish the job.
- It takes Mr. Manley 3 hours to paper Mrs. Hopkins' living room when she is out. It takes him 4 hours when she's at home. How long does it take him if Mrs. Hopkins goes out for 1 hour and then spends the rest of the afternoon at home?

But it wasn't until much later, thinking his first professional wallpapering job complete, that Mr. Manley paused to consider his work. It was then that the nature and cause of his dilemma became apparent.

Mrs. Hopkins took a seat and her great affinity for stripes carried the day. This would be a cruel ending but for the fact that Mrs. Hopkins became known among her friends as a trendsetter for her daring new decor, and Mr. Manley much sought after as an interior decorator.

■ With his success with Mrs. Hopkins' living room, Mr. Manley has been hired to paper three walls in the neighbor's home. Two walls are 8 feet by 14 feet, and the third wall is 5 feet by 10 feet. Each roll of wallpaper is 2 feet wide by 40 feet long. How many rolls of wallpaper should Mr. Manley purchase in order to complete the job for Mrs. Hopkins' neighbor?

A Case for a Dinosaur Detective

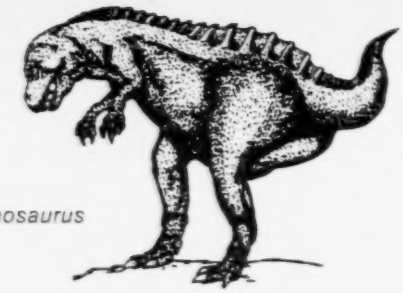
"If Alexander's formula is correct, the dinosaurs were either running—some of them at about 26 miles per hour, faster than a human athlete—or remarkably long-legged. I'm convinced that at least one of them was probably not an unusually long-legged dinosaur."

James Farlow is a peculiar sort of detective. Instead of dusting for fingerprints at the scene of an unsolved crime, he studies footprints. But these are not human footprints. The footprints Farlow studies were formed millions of years ago by dinosaurs who wandered onto muddy tidal flats. Over the centuries, the mud turned into stone, leaving a permanent record.

Farlow is a "dinosaur detective," called an *ichnologist*—an expert in the tracks and footprints of prehistoric creatures. He tries to read the clues in the footprints to understand more about the creatures that might have made them. How big were they? How fast were they moving? The latter question, in particular, has led Farlow to some fascinating theories.

Dinosaur Valley State Park in Texas is a trove of footprints and trackways; to Farlow it is a gold mine of useful information. He focuses his work on tracks he believes were left by *Acrocanthosaurus**, a two-legged, carnivorous dinosaur most likely related to the better-known *Tyrannosaurus rex*.

SEEING ACROSS THE TIME BARRIER: To paleontologist James Farlow (left), dinosaur tracks represent more than fossilized footprints: "In a sense you're seeing a living animal in the process of hunting for food, fleeing from an enemy, doing the various kinds of things that animals do in the course of their lives." Farlow theorizes that dinosaurs could do those things a lot faster than experts once believed. His studies of Texas tracks, including the *acrocanthosaurus* footprint above, have convinced him that dinosaurs could reach speeds of at least 26 miles per hour, faster than the fastest human athletes.



Acrocanthosaurus

Making Tracks

In the field, Farlow studies the footprints closely, but the task is not a simple one. "It's hard to tell the tracks of a blacktail or a mule deer from those of a whitetail deer," Farlow says, "and sometimes moose tracks from elk tracks. The problem is even worse for extinct animals such as dinosaurs."

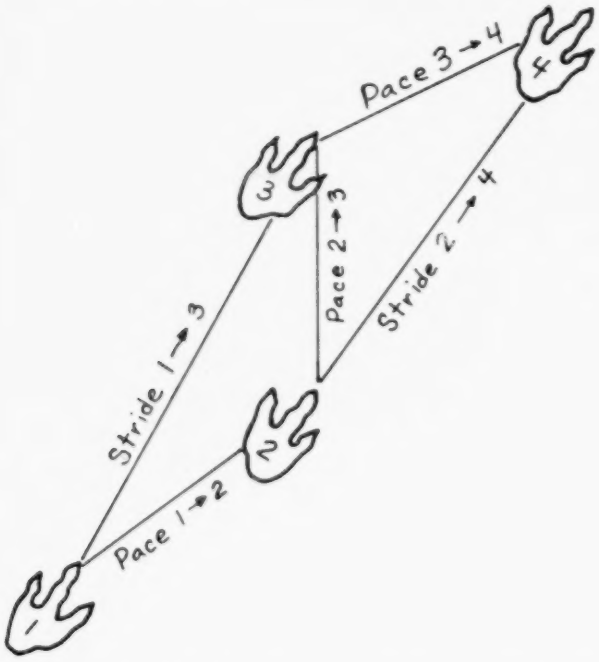
Dinosaur *trackways*—or series of footprints—provide some clues for Farlow. *Pace length* (the distance between a left and a right footprint) and *stride length* (the distance covered between two successive left or right steps) can indicate how efficiently a dinosaur moved, and can suggest things about its age and appearance. The *gauge* (breadth) of the track tells whether the legs were close together or set far apart.

In the 1970s, British biologist R. McNeill Alexander gathered data on all kinds of living animals (from gerbils to elephants, ostriches to humans) and created a formula for calculating an animal's velocity based on the relationship between its hip height and the length of its stride. (See page 95 for more on the Alexander formula.) He concluded that, among other things, a running animal produces a stride length about twice the length of its walking stride. Using data derived from dinosaur tracks, he calculated that dinosaurs were slow walkers.

In Texas, however, Farlow has measured some tracks with stride lengths of 21 feet. "If Alexander's formula is correct," Farlow says, "the dinosaurs were either running—some of them at about 26 miles per hour, faster than a human athlete—or remarkably long-legged. I'm convinced that at least one of them was probably not an unusually long-legged dinosaur."

**Acrocanthosaurus* means "high-spined lizard," a name derived from the tall ridge of spines running down the animal's back. The beast is believed to have been about 10 to 12 feet tall, with powerful hind legs.

AN UNLIKELY UNDERSTUDY? Not at all. The ostrich is more like the *acrocanthosaurus* in the way it moves than most other living creatures. Farlow observed how ostriches' tracks grew deeper in the toe as the big birds accelerated, and deeper in the heel as they slowed down. The same, he reasoned, must be true of dinosaur tracks.



READING THE TRACKS: A *trackway* is a series of footprints made by the same animal. The distance between successive footprints is the animal's *pace*, while the sum of two successive paces is its *stride*. When an animal is running, its stride length is about twice as great as its walking stride. Farlow calculates the dinosaur's approximate hip height from the length of its footprint; the Alexander Formula uses the hip height and stride length to yield the animal's velocity.

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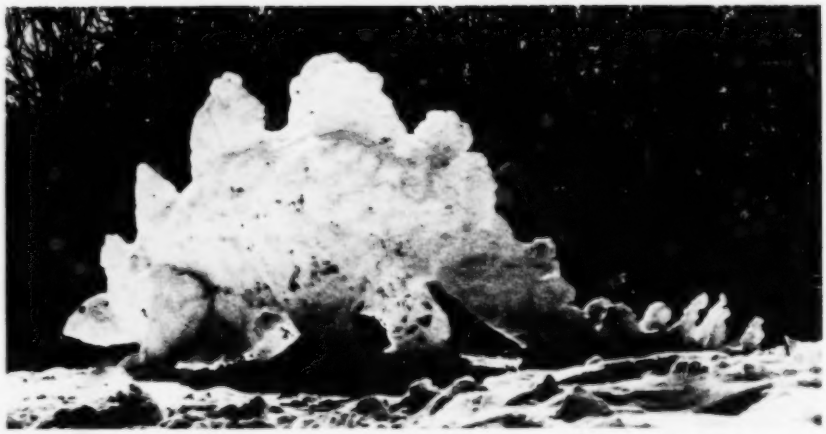
A Model Bird

In order to read the footprints better, Farlow wanted to recreate the movement that produced particular kinds of tracks. As a model, he needed a living animal that shared important characteristics with acrocanthosaurus. He chose the ostrich. The ostrich, Farlow says, is "about the closest thing to a bipedal (two-footed) dinosaur that we have today in terms of overall structure and size."

And so Farlow found himself at the zoo in Fort Wayne, Indiana, coaxing several jittery male ostriches into making tracks—walking, trotting and running—to see how this change in speed changed the shape of the footprints they left behind.

It was no easy task. The birds were not very cooperative. In their confined quarters they were never able to get close to their top speed of 50 to 70 mph.

But Farlow was able to observe the way in which the birds' toes dug in as they moved faster and left deeper heel marks as they slowed down. Using ostriches as stand-ins for dinosaurs has given Farlow a clearer mental picture of the beasts that walked—and probably ran—across a muddy tidal flat 100 million years ago. ■



FROZEN IN TIME: Was this unlucky *stegosaurus* caught in the ice age? No, it's a modern snow sculpture—an artistic restatement of a long-dead dinosaur.

Further Readings

Colbert, Edwin H. *Dinosaurs: An Illustrated History*. Maplewood, N.J.: Red Dembner Enterprises, 1983.

Perhaps the best introduction to dinosaurs—their history, areas where fossils have been discovered, methods of unearthing and preserving their fossils. Well-illustrated with photos and drawings.

Gilbert, John. *Dinosaurs Discovered*. New York: Larousse, 1981.

A good overview of dinosaur discoveries around the world, this book includes information on the scientific methods used to unearth and clean dinosaur fossils. Well-illustrated; accessible to junior and senior high school students.

Horner, John R. "The Nesting Behavior of Dinosaurs." *Scientific American*, April 1984.

A fascinating description of the discovery of large numbers of dinosaur eggs and nests in Montana and how scientists have used this discovery to form new theories of the social relationships among these extinct reptiles.

Langston, Wann, Jr. "Pterosaurs." *Scientific American*, February 1981.

This article is a detailed study of pterosaurs, flying reptiles that endured for 135 million years. Some with wingspans of up to 12 meters, these enormous reptiles are thought to represent the link between dinosaurs and birds.

Mossman, David J., and Sarjeant, William A.S. "The Footprints of Extinct Animals." *Scientific American*, January 1983.

Probably the best article dealing with the type of research Jim Farlow is doing with dinosaur footprints. The authors describe dinosaur footprint sites around the United States, types of information that can and cannot be gleaned from the tracks and how plaster casts of individual footprints are made. Well-illustrated with line drawings, charts and photographs.

Ostrom, John. "A New Look At Dinosaurs." *National Geographic*, August 1978.

Interesting, well-illustrated article outlining recent theories about and explorations of dinosaurs. Included are the study of footprints and fossils and a summary of dinosaur extinction theories.

Sattler, Helen Rodney. *Dinosaurs of North America*. New York: Lothrop, Lee and Shepard Books, 1981.

This copiously illustrated book provides an excellent introduction to the dinosaurs that roamed the North American continent 200 million years ago. It includes a description of acrocanthosaurus, the dinosaur believed to have made the footprints James Farlow is studying in the film. For junior and senior high-school students.

The Mathematics of Modeling

How do you make sense of something you'll never be able to see? One technique is to use a mathematical model. The model is designed to behave in the same way as the original. Therefore, you can make predictions based on how the model performs. Models are used to explore a wide variety of areas in which firsthand experience is difficult or impossible to achieve. City planners, for example, might need to determine how many hospitals a city with a population of 100,000 should have. They might use a similar city and its hospital system as a model, with the assumption that their city would have similar needs.

Although details of the calculations vary substantially with different models, the procedures used for working them have a lot in common:

- Gathering data and information about the objects you're studying—*or about things like them*, if the objects themselves aren't available to be studied.
- Summarizing that information in mathematical form—possibly using a graph or a mathematical formula.
- Testing to see whether the graph or the formula makes sense. Does it capture the data you have? Is it reasonable on its own terms?
- Making predictions about the objects of interest, based on the graph or formula.
- Checking to see whether the predictions are plausible.

How do you model the motion of a dinosaur? You can start with other animals and see if you can find a general formula describing their speed. In the 1970s, R. McNeill Alexander took data from running tracks of different animals—among them humans, horses and elephants—and analyzed it until a pattern emerged.

Alexander's formula summarizes the results. In simplest form, the velocity of the animal is approximated by

$$v = .25g^{0.5}s^{1.67}h^{-1.17} \text{ meters/second,}$$

where g is the acceleration due to gravity (9.8 meters/second^2), s is the stride length (the distance between successive left or right footprints) and h is the hip height of the animal; s and h are measured in meters.

Checking the Model

After building the model, you need to test it. First, is it reasonable on its own terms? Does the formula's prediction agree with what will happen in real life?

Second, do predictions made on the basis of the formula seem plausible? This was what James Farlow was checking when he was studying the running patterns of ostriches. With a rather small ostrich (hip height 1.3 meters), Farlow got a running stride length of 7 meters. Using the Alexander formula, you get a velocity of about 15 meters per second, or 53 kph—about 33 mph (a reasonable value).

CHALLENGE Exercises:

1. This exercise explores the variable s (stride length) in the Alexander formula.

a. First, examine the formula:

$$v = .25g^{0.5}s^{1.67}h^{-1.17} \text{ meters/second,}$$

$$\text{or } v = \frac{.25g^{.05}s^{1.67}}{h^{1.17}} \text{ meters/second.}$$

Since 1.67 is between 1 and 2, if $s > 1$, $s^{1.67}$ represents a number larger than s^1 but smaller than s^2.

Suppose s gets larger.

Is $s^{1.67}$ smaller or larger? Since the formula calls for *multiplying* by $s^{1.67}$, does v get smaller or larger when s gets larger?

b. Check your answers against your own experience. If you run faster, are your strides likely to be longer or shorter? That is, when v gets larger, do we expect s to be smaller or larger?

c. Compare the answers to parts **a.** and **b.** Does part **a.** suggest that a larger s should be associated with larger or smaller v? Does part **b.** suggest that a larger v should be associated with a larger or smaller s? Do parts **a.** and **b.** agree or disagree? As a result, do you have more or less confidence in the formula?

2. In a similar fashion, explore the role of h (hip height) in the Alexander formula:

a. $$v = .78s^{1.67}h^{-1.17*} \text{ meters/second,}$$

$$\text{or } v = \frac{.78s^{1.67}}{h^{1.17}} \text{ meters/second.}$$

Suppose h gets larger. Is $h^{1.17}$ smaller or larger? Since the formula calls for *dividing* by $h^{1.17}$, does v get smaller or larger when h gets larger?

b. Consider the implications of the model. Think about the footprints left as a very short person (with a short hip height), h, walks. Would a tall person have to walk very slowly, or run very fast, to land in the same footprints? In other words, if s (the stride length) is fixed, would a larger value of h result in a smaller or a larger value of v?

c. Compare the answers to parts **a.** and **b.** Does part **a.** suggest that a larger s should be associated with a smaller or larger v? Does part **b.** suggest that a larger v should be associated with a smaller or larger s. Do parts **a.** and **b.** agree or disagree? As a result, do you have more or less confidence in the formula?

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*$.25g^{.05} = .25(9.8)^{.05} = .78$

"Use an analogy whenever you can think of one. An analogy provides a model which serves as a guide to identify the elements of a problem as parts of a more complete structure. It also helps recognize phases as elements of a complete process. . . . In many cases, metaphors help convey understanding."

Moshe F. Rubinstein, *Patterns of Problem Solving*

A Class Project: Keeping Pace with Alexander

You can test the Alexander formula by trying it out as a class. This is a class project for which you'll need a calculator, a stopwatch, a meter stick and a place to do some running. Give each student a head start to get up full speed, and clock how long it takes to run 100 meters. While one student is running, another should count the number of steps, n, it takes to run 100 meters while a third clocks the runner's time. For each student, you can then do the following calculations.

1. The reading on the stopwatch gives the time T it took to run 100 meters. If you divide 100 by T, you get the real velocity V, in meters/second.

2. If you divide 100 meters by the number of steps, n, that the student took, you get the length of an average pace. (A stride is two paces.) This says that $S = 2 \times 100/n$.

3. Measure the hip height, h (from hip bone to floor), for each person, in meters.

4. Using the Alexander formula, $v = \dfrac{.78s^{1.67}}{h^{1.17}}$, and a calculator, you can calculate the prediction that the formula makes for the person's speed.

5. A comparison of the values of V and v will indicate how well the formula works for your class. You can graph the set of all pairs (V, v). If the formula is perfect, all the points will lie on the line $V = v$. How close is the set of points for your class? See the graphs in Figures 1 and 2, and compare your class' graphs with them.

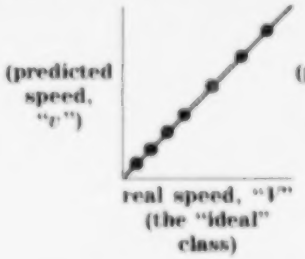


Fig. 1

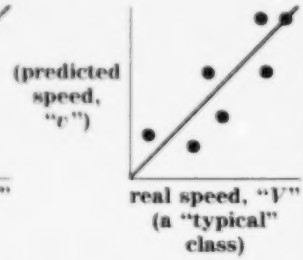


Fig. 2

New Keys to Old Locks

"Before improving anything, one needs to know what one is dealing with. What is the existing product like? What does it do, and why? Then you can begin to ask, what could it be? How far could we take it?"

In Noel Mayo's classroom, real-life problems—and solutions—are part of the curriculum. As part of their studies in industrial design at Philadelphia College of Art, Mayo's students are sometimes challenged with assignments brought to the class by local businesses.

John Taylor, president of Taylor Lock Company, asked Mayo's students for a new corporate logo to use on his letterhead. They provided it.

Characteristically, however, their teacher saw the new logo as only the first step. The Taylor product itself, an ordinary door lock, had not changed in 50 years. Why not consider a new design that might also improve its performance?

By asking the question, "What's a lock?" Mayo's students demonstrated to themselves and to Taylor executives that an ordinary door lock could be anything but ordinary.

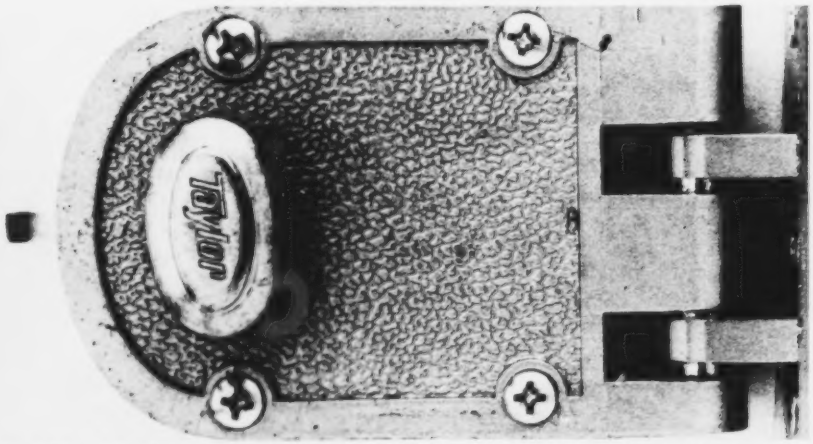
The First Step

Each student tackled the problem of coming up with a new lock design individually. As Mayo directed his students, the first step was to gather information. Before improving anything, you need to know what you're dealing with. What is the existing product like? What does it do, and why?

"Look at all of the competitors' products, whatever they are, so that you're intimate with the idea of what this thing *is*," Mayo says. "Then you can begin to ask, 'what could it be? What could it really be? How far could we take it?' The work is in trying to gather in the information by defining the problem clearly and continuing to define it, while throwing out clichés and bringing in preposterous ideas. All of a sudden, it clicks."

It was clear to the students that the primary purpose of a lock is security. In fact, the manufacturer's research showed that many people buy locks just *after* they have been robbed.

But what makes a lock secure? And secure for, and against, whom or what? Mayo's students found that for some it is a matter of how a lock looks, how it sounds, how it functions, how it is put together or any combination thereof.



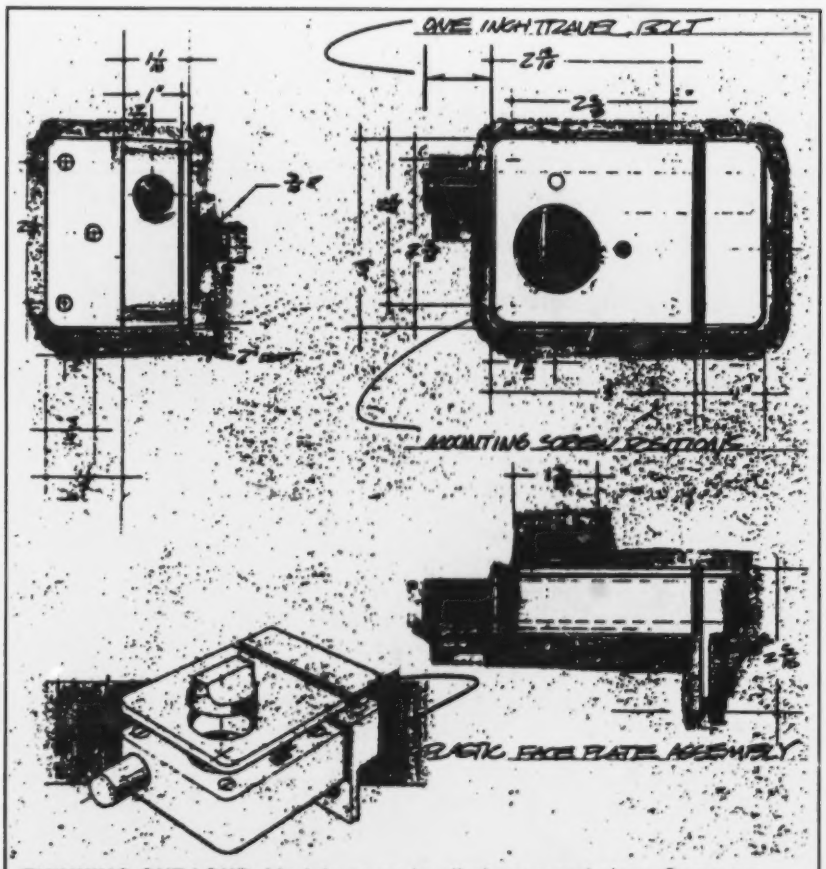
Noel Mayo (left) gave his students at the Philadelphia College of Art an unusual design project: to come up with a new design for the Taylor Lock Company's venerable door lock (above). In the process of thinking through their designs, students explored the relationship between form and function.

Looking into the Lock

Adam Sherman, one of Mayo's students, concentrated his efforts on the lock's appearance. He took the lock apart to see how it was cast and finished; he tried to put himself in the place of the buyer. What were potential buyers looking for in a lock that he might be able to show them in his design?

His final design features a large red bolt visible at the back whenever the lock is open. At a glance, without walking up to it, a person can tell whether the door is locked. Besides being reassuring, Mayo's students realized that this could be an important feature for the visually handicapped.

Philip Chant posed the problem differently: "I began to think that trying to open and close the lock, which was its function, was very annoying," he remembers. "You have to twist a tiny little knob. Why is it so tiny?" His design features a large overhanging piece in place of the troublesome knob. The lock can be opened or closed with an elbow as well as a hand—a design feature convenient for anyone who goes to the door with arms full of groceries, and important for some of the handicapped.



THINKING OUT LOUD: Models come in all shapes and sizes. Some are sketches, some are elaborate mathematical simulations requiring the world's largest computers. All are ways of "thinking out loud" or "thinking with your hands." This rendering by art student Philip Chant is a model of his design for a new door lock. It allows him to see the lock in three dimensions, to examine it from several angles, to clarify and correct his thinking before sinking additional time, effort and money into the design.

Model Locks

Like all Mayo's students, Chant worked with models—another kind of restatement. Some models are three-dimensional: the designer can hold them up and turn them around. Others are sketch models. Students continually shift their perspective between two and three dimensions, so that they can alternate the processes of creating and analyzing their problem.

Sketch models were very important, says Chant. By holding the models against a door, he realized that they looked so big and bulky that they were actually more frightening than reassuring.

One of the features of good design, Mayo points out, is that it restates the point of view for both the manufacturer and the user. The way to succeed, he maintains, is not simply by making a product easier to manufacture, but by satisfying the customer as well.

"We have a whole different ball game [in design today]," says Mayo, "and the need for sensitive ideas related to human beings becomes more critical and much more challenging. It really deals with trying to solve problems for people, to make their lives more comfortable and useful, to raise the quality of life." ■

Mind Sets

When Noel Mayo's students undertake redesigning the Taylor Company's 50-year-old door lock, the first thing they must do is overcome their mind sets*, their preconceived ways of thinking about what a lock *should* look like. The lock is so familiar that only by a complete reexamination of what a lock is and does can students become free to think of something new.

As a general concept, mind set is not necessarily a bad thing. In fact, in some situations it can be downright helpful. For example, when you see a red traffic light, you know automatically that the message is "stop." On the other hand, mind sets can definitely hinder the creative process. Ingrained preconceptions can inhibit a problem solver from finding an effective solution.

It's easy to think of tools, including mathematical ones, as essentially custom-made, and to use one tool for one task and *only* one task. In the mind of the problem solver, each tool has a *fixed function*.

But what happens when this problem solver is confronted with an unfamiliar task? Here the ability to restate a problem—to make an analogy with a task one already knows how to perform—can point to a solution and lead to new uses for old tools.

*This discussion of *mind set* and some of the accompanying exercises are adapted from Glenn D. Allinger's article, "Is Your Mind in a Rut," *Mathematics Teacher*, May 1982.

Exercises:

1. How many triangles are there in Figure 1 (ΔADG, $CE \parallel BF \parallel AG$)? How many trapezoids?

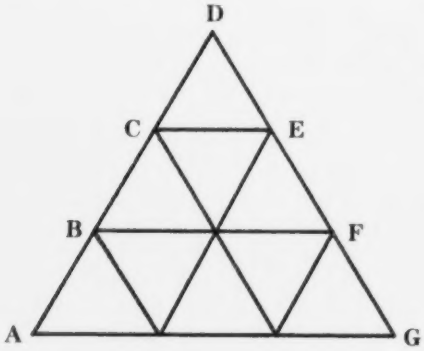


Fig. 1

2. How many squares are there in Figure 2? How many rectangles?

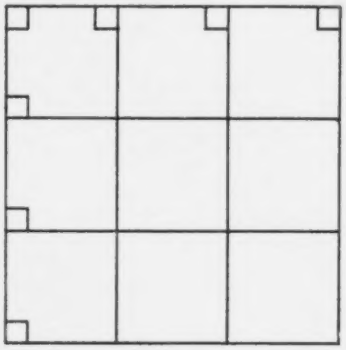


Fig. 2

3. Given $\angle WXY$, $\overline{WX}$, $\overline{XY}$ and $\overline{YZ}$ in Figure 3, construct a trapezoid WXYZ.

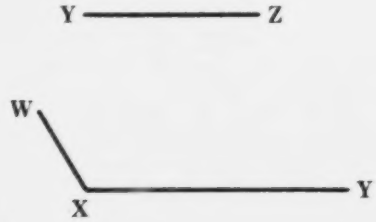


Fig. 3

4. Given four 4*s* and any operations (+, −, ×, ÷), represent all whole numbers 0–20.

Examples: $(4 + 4) - (4 - 4) = 7$
$4 + 4 - 4 - 4 = 1$

5. List 50 uses for a pencil.

6. Draw 5 different rectangles that have area = 100. Label the dimensions of each of your rectangles.

7. Draw 5 different parallelograms with base = 10 and with area = 100.

8. Draw 5 different parallelograms with base = 20 and with area = 100.

"One purpose of acting out or representing a problem is to help students understand the problem's applicability to real life."

Edward J. Davis and William D. McKillip, *Problem Solving in School Mathematics*

Picking Locks

Solving this puzzle should be an open-and-shut case. Each of the ten keys (**A–J**) pictured below fits one (and only one) of the ten locks (**1–10**). The puzzle is to match key to lock by examining their distinctive shapes and sizes.

Answers:

1. D	6. A
2. B	7. H
3. J	8. G
4. E	9. I
5. F	10. C

Okefenokee Swamp: Unraveling the Mysteries

"When I started coming down here to the swamp it was a very confusing place to me. There is a plethora of biological and physical and chemical processes going on. It's very hard to make any sense out of all that complexity, diversity and large size."

The Okefenokee Swamp is as beautiful as it is mysterious. Stretching from southeastern Georgia to northern Florida, the swamp supports an astonishing range of wildlife—from microorganisms to insects and birds, snakes and alligators, bears and deer. The majestic cypress trees, standing out from the pines and magnolias, create an atmosphere of eerie primordial stillness.

In an attempt to unravel some of its mysteries, Bernard Patten, an ecologist at the University of Georgia, is using mathematical models to study life in the Okefenokee. He is attempting to understand the whole system (or *ecosystem*) by breaking it into smaller, more analyzable parts.

"When I started coming down here to the swamp, about eight or ten years ago, it was a very confusing place to me," Patten says."There is a plethora of biological and physical and chemical processes going on. It's very hard to make any sense out of all that complexity, diversity and large size."

QUIETLY TEEMING: It looks calm, serene, even suspended. But look closer. The Okefenokee Swamp is fairly bursting with activity of a tremendous variety of plant and animal life. How do all these life forms fit together in the swamp's ecosystem? To answer that question, it's necessary to restate the swamp in terms of a less complex, more accessible model.

Random Happenings?

Patten believes that underlying the swamp's seeming complexity and randomness is an organizational plan that is accessible to analysis. If something happens at one level of the swamp system, its effect will be felt at other levels.

The model can take a bird's-eye view of how individual changes might affect the swamp's ecosystem as a whole. Extensive lumbering, for example, could raise the water level; attempts to reclaim land for development by draining parts of the swamp could reduce the water level. The vegetation cycle could be affected if all spontaneous forest fires were extinguished before running their course.

The cypress trees are a good case in point. After a drought in the 1950s, huge areas of the swamp were left vulnerable to devastating fires. While it might appear to have been a disaster of major proportions, ecologists have come to understand that such fires are necessary to the swamp's survival. Cypress seeds need dry conditions in which to germinate. Fires burn off older trees to make way for new growth, and the cycle of cypress trees continues.

Unfortunately for ecologists, this kind of cycle may take 1,000 years or more, too long for study. So Patten and his team attempt to model the swamp. With only 25 years' records of rain and water levels in the swamp as starting data, they meticulously collect current data on the water systems (input and output) of the swamp and feed it into a mathematical model of the water cycle of the swamp. As new data is collected, it is also evaluated, and the mathematical models are revised as required. Although mathematics can simplify the problem and help make Patten's conclusions clearer, it can't make them perfect.

Clues from Geometry

Harold Hastings, a mathematician from Hofstra University, was visiting Patten when he saw another possible mathematical clue to the mystery of the Okefenokee. He noticed that the irregular patches of vegetation shown in infrared aerial photos of the swamp bore a distinct resemblance to the geometric curiosities known as *fractals*. He thought fractals could be a key to mathematically predicting change in the vegetative cycle of the swamp, offering a different type of mathematical model and a new means for more detailed analysis for yet another element of the swamp's ecosystem.

Computers can generate pictures of fractals using a parameter called the *fractal exponent*, which, according to Hastings, "governs the pattern of the area and the relative numbers of small or large patches." Matching the computer pictures to aerial photographs, Hastings is able to assign a fractal exponent to the irregular distribution of patches of vegetation in the swamp.

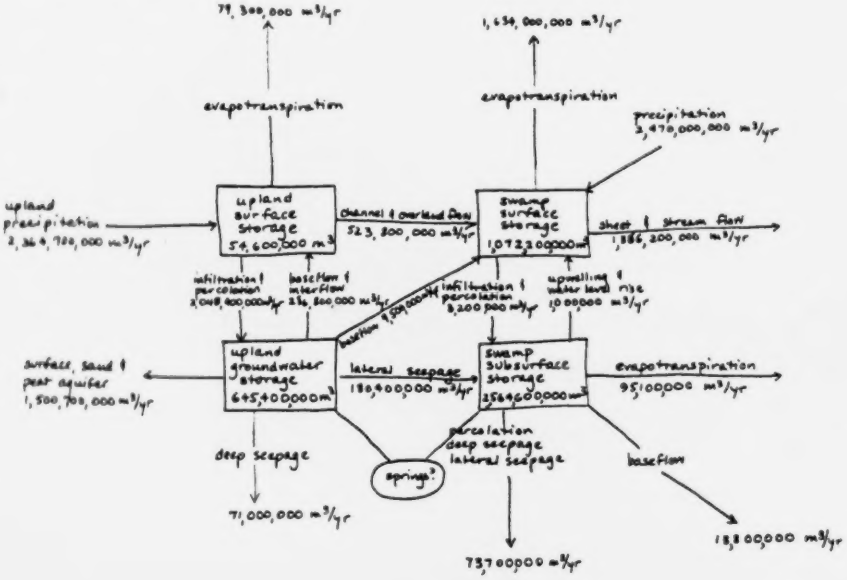
Rough but Reasonable

By using fractals in conjunction with a model of plant succession, Hastings has hypothesized that fractal exponents could be used to predict the fate of individual plant species in areas of the swamp.

The fractal forecast is not absolute or precise; it does not say precisely where the cypress will be by the year 2000. Like a general weather forecast for the month ahead, it is a rough but reasonable description of what will probably happen. "We can tell you about the characteristics of those regions," Hastings says, "and a little bit about the long-range evolution of the swamp."

The mathematics is extremely complicated, but according to Hastings' model, the cypress trees—the very symbol of the Okefenokee—will flourish in some areas while dying out in others.

Mathematical models constructed by Patten and Hastings provide information about the workings of the swamp. Scientists now know a little more about how the course of nature —or the incursions of humans—will affect individual plant and animal species and the swamp itself. ■



Patten's systems analysis model of Okefenokee Swamp hydrology: A 1982 version.

THE OKEFENOKEE, AS SEEN AT WATER LEVEL: What does a swamp look like? Bernard Patten and his team at the University of Georgia think the Okefenokee Swamp looks something like this, at least in terms of its water budget. The numbers are estimates, based on real data, common sense and experience. This conceptual model—like all good restatements—allows Patten and his colleagues to focus their research efforts. The data they collect is used to test and refine the model, which in turn leads them to better research results.

Further Readings

"Fractals for Modeling Ecosystems." *Science News*, 11 June 1983.

A short and concise description of Harold Hastings' work exploring the ecosystem of the Okefenokee Swamp.

Gibson, Dot Rees. *The Okefenokee, Land of the Trembling Earth*. Waycross, Georgia: Dot Gibson, 1974.

A well-written introduction to the Okefenokee Swamp, including lots of photographs, a history of the swamp and information on the flora and fauna of the region.

McDermott, Jeanne. "Geometrical Forms Known as Fractals Find Sense in Chaos." *Smithsonian*, December 1983.

This is the clearest and most concise description of fractals we have seen to date. It describes what a fractal is, how fractals were discovered, and their uses, including computer animation for movies. Excellent drawings and photos; accessible to high school students.

Mandelbrot, Benoit B. *The Fractal Geometry of Nature*. San Francisco: W.H. Freeman, 1982.

The definitive book on fractal geometry, this well-illustrated volume describes the discovery of this "new mathematics"—the geometry of irregular (and hence, most natural) shapes. Much of this is very complicated, but there are sections that are accessible to high school students, and the drawings and illustrations are magnificent.

Sorensen, Peter R. "Fractals." *Byte*, September 1984.

A description of how fractals can model natural landscapes on a computer. This article surveys recent research into computer graphics and fractals, and explains how mathematics and the "real world" can visually intersect on a computer screen. Good for high school students who are interested in computers.

Thomsen, Dietrick E. "Making Music—Fractally." *Science News*, 22 March 1980.

The author explains research that hypothesizes that the structure of music is based on fractals. Short, but gratifying to a student or teacher interested in music.

FRACTALS: A Picture Worth a Thousand Words

Mathematics is the study of patterns—in science and art, in reality and imagination. Often visual patterns, or geometry, are the easiest to recognize and the most powerful to use. In mathematics as in other places, a picture can often be worth a thousand words.

Fractals are a new class of patterns that seem to describe many situations that otherwise appear chaotic or inexplicable. The morass of detail in the Okefenokee Swamp is a perfect example: Layers of vegetation, interlocking ecosystems, constantly changing water and drainage patterns provide a complex structure that seems to defy explanation. Those who know fractals can "see" in this complexity the clue to a fractal: replication of complexity from the aerial view to the laboratory microscope.

A VARIABLE YARDSTICK: The wiggling of fractal shapes produces another distinctive but distracting feature: The answer you get when measuring the length or area of a fractal depends on the length of the measuring rod you use. If you walked along the California coast, in and out of every bay, it might be 10,000 miles. If you became the size of a crab and walked around every pebble, the distance could increase to 100,000 miles.

Ordered Chaos

Fractal patterns arise commonly when some natural structure—such as a cloud, a swamp or lung tissue—is created by repeated application of small changes. These changes are usually governed by certain general rules of nature, but within the general rules much random variety occurs. The resulting object—a bumpy cloud, a serpentine swamp, a maze-like lung—reflects both order and chaos, both rule and randomness. Fractal models help separate the order from the chaos, thus providing scientists with good visual images of shapes to look for.

Fractal shapes are fascinating and bizarre, being not quite as orderly as floor tiles yet not quite as random as dice. They appear to be, and they are, a form of *ordered randomness*, in which random variation can occur within a generally growing pattern.

A New Dimension to Geometry

The enormous variety allowed within fractal patterns, and the regularity of that variety, gives rise to the first important characteristic of fractal patterns: Their pattern repeats itself at all levels of magnification. The night sky is a good example: Whether viewed with naked eyes, or through binoculars, or through a telescope, the general pattern of stars are the same. So are some coastlines: From an airplane, from a cliff or on foot, the bends and curves of jagged, irregular coastlines exhibit the same type of meandering features.

The name "fractals" comes from yet another feature, itself quite startling: Fractals represent fractional dimensions. The inherent randomness of a fractal causes so much wiggling in the shapes that a line may begin to fill up an area, or a surface may begin to fill up a solid region.

The surface of a smooth sphere—the ideal sphere of Euclidean geometry—is precisely two-dimensional. But the surface of the earth, with mountains and arroyos and plowed fields and wave-capped oceans, is more like a fractal of dimension 2.2. Computer-generated fractals, programmed to imitate dimension 2.2, look surprisingly like aerial photographs of a twin planet earth, with mountains and ridges and lakes that never were, but might have been.

Exercises:
Construct Your Own Fractals

1. The Snowflake Curve. Begin by drawing an equilateral triangle. Then add another equilateral triangle, producing a six-pointed star. Repeat this construction on each "point" of the star. Then repeat again. . . if you can. (See Fig. 1.)

Now make a table for each stage of the construction, with the number of segments, the total perimeter and the total area. (See Table 1.)

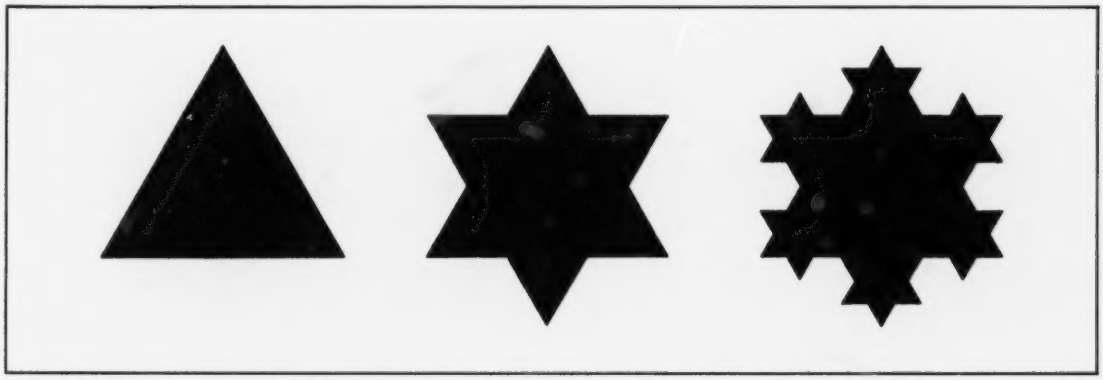


Fig. 1.

Snowflake Curve

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage n?
Number of segments	3	$3\times4=12$	$3\times4^2=48$	$3\times4^3=192$	$3\times4^4=768$	$3\times4^{(n-1)}$
Length of each segment	1	$1/3$	$1/3^2=1/9$	$1/3^3=1/27$	$1/3^4=1/81$	$1/3^{(n-1)}$
Perimeter	3	$\frac{3\times4}{3}=4=$ 4.000	$3\times4^2=$ $48/9=$ 5.333	$3\times4^3=$ $192/27=$ 7.111	$3\times4^4=$ $768/81=$ 9.481	$3\times(4/3)^{(n-1)}$
Number of *new* triangles	1	3	$3\times4=12$	$3\times4^2=48$	$3\times4^3=192$	$3\times4^{(n-1)}$
Area of each new triangle	$\frac{\sqrt{3}}{4}$	$\frac{1}{3^2}\frac{\sqrt{3}}{4}$	$\frac{1}{9^2}\frac{\sqrt{3}}{4}$	$\frac{1}{27^2}\frac{\sqrt{3}}{4}$	$\frac{1}{81^2}\frac{\sqrt{3}}{4}$	$\frac{1}{3^{2(n-1)}}\frac{\sqrt{3}}{4}$
Total area of new triangles	$\frac{\sqrt{3}}{4}$ $=.4330$	$\frac{3}{3^2}\frac{\sqrt{3}}{4}$ $=.1443$	$\frac{3\times4}{9^2}\frac{\sqrt{3}}{4}$ $=.0642$	$\frac{3\times4^2}{27^2}\frac{\sqrt{3}}{4}$ $=.0285$	$\frac{3\times4^3}{81^2}\frac{\sqrt{3}}{4}$ $=.0127$	
Total area	.4330	.5773	.6415	.6700	.6827	

Snowflake Curve

Table 1

X=

2. The Rings of Cantor. Begin by drawing a line or bar segment about 9 inches long. Underneath it, draw a copy of this segment without the middle third: The copy will have two parts, each 3 inches long, with a 3-inch gap in between. Underneath this draw a new pattern, this time omitting the middle third of each of the remaining pieces. Repeat as often as possible, until what remains is a thin pattern of points. (See Fig. 2.)

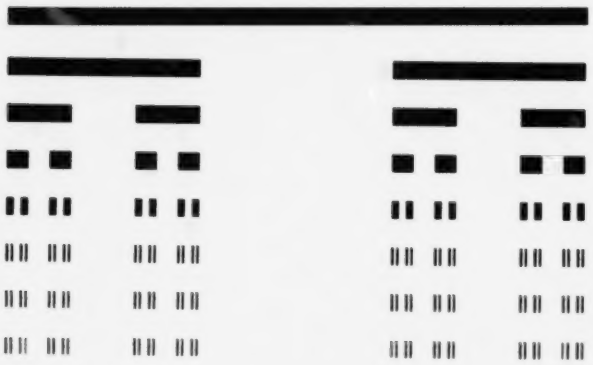


Fig. 2

After finishing, connect the midpoint of each segment to the midpoint of the segment above it from which it came. This will produce a structure called a "binary tree," useful for decision processes in business and in computing.

Add up the length of all the segments removed in constructing the Rings of Cantor. If the original *line* were of length 1, then the first step removes a segment of length 1/3. The second step removes two pieces of length 1/9; thus in total 5/9 are removed. The next stage removes 4 pieces of length 1/27. Make a chart of this process for the first 10 stages, and see what happens to the length of the pieces that are removed, and of those that remain.

3. Apollonian Circles. Carefully draw three large circles that just touch each other, bounding a triangular region with curved sides. Then put inside this triangle another circle that touches the other three. This yields three curvilinear triangles. Now repeat the process in the three new triangles: In each, put the largest circle that you can, just touching the sides. Repeat this process if you can. (See Fig. 3.)

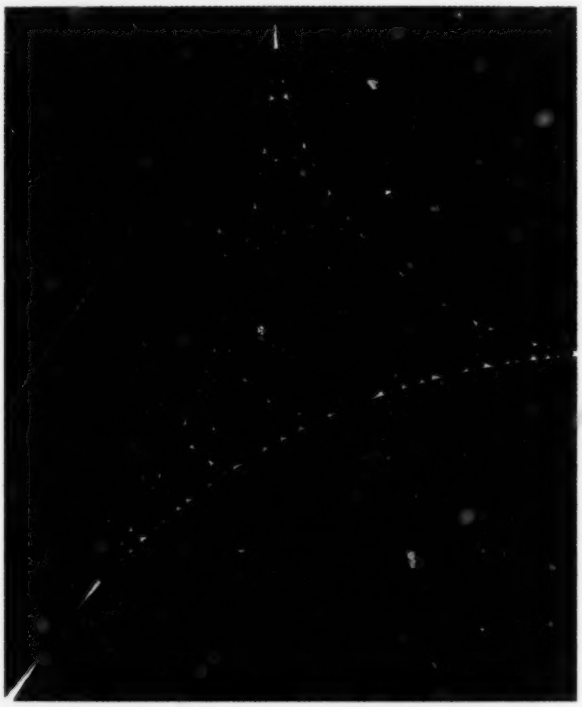


Fig. 3

". . .a problem solver, faced with a new problem, must first of all translate this problem into a form which he or she can readily understand."

Frederick Reif, *Problem Solving and Education*

4. Pascal's Patterns. Use a piece of graph paper and a calculator to make a large triangle of numbers in which each new number is the sum of the two that are above it. The resulting figure is called Pascal's Triangle and is the basis for many interesting mathematical calculations—including fractal design. (See Fig. 4.)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Fig. 4

Now go through the triangle erasing every even number, and changing every odd number into a black square. (See Fig. 5.)

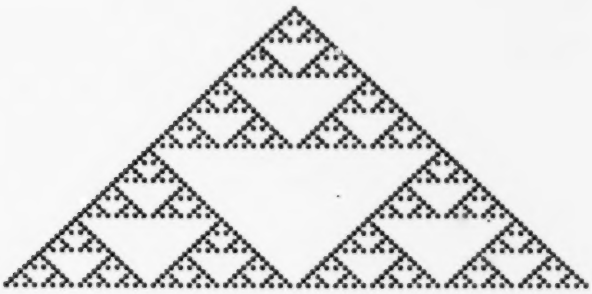


Fig. 5

The first 64 lines of Pascal's Triangle *modulo two* (black squares indicate ones; white squares indicate zeroes).*

*Copyright 1984, by permission of the Mathematical Association of America.

To get a different pattern, erase the even numbers; color red those numbers that are divisible by three; and color black all the rest.

For a multicolored, mosaic effect, try dividing each number in the triangle by a larger value and assigning colors to the remainders. For instance, if you use modulo 6, possible remainders are 0, 1, 2, 3, 4, 5, so you might have:

0 = white
1 = black
2 = red
3 = blue
4 = green
5 = yellow

"Frequently, problem solvers will make an external representation of some parts of the problem. They do this by drawing sketches and diagrams or by writing down symbols or equations which correspond to parts of the internal representation. Such external representations can be enormously helpful in solving problems."

John R. Hayes, *The Complete Problem Solver*

X=

Make Your Own Fractals . . . With a Computer

If you have an Apple II series computer, you can make your own fractals with the following program, written in Applesoft BASIC.*

The program generates fractal curves by moving a point from one position to another in the plane repeatedly through the same function.

When it starts, the program asks for the value of the complex number *lambda*, a constant. Different values of *lambda* will yield different fractal curves. Based on *lambda* and an initial value of the constant *z*, the program draws the position of *z* each time it is put through the function.

The program also asks for the scale; that is, the size of the "window" that the screen represents. A value of 4 is good for most pictures; you can experiment.

You'll get very different pictures from different values of *lambda*. Some are shown below (Figs. 1–3).

Each picture takes about 15 minutes to compute on the Apple II.

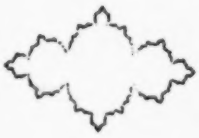


Fig. 1

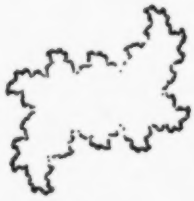


Fig. 2

Fig. 3

Lambda = (3,0) **Lambda = (0,1)** **Lambda = (1,0)**

Listing 1: Greg Turk's Fractal Program Written in AppleSoft BASIC*

```
10      CX = 140:CY = 96
20      X = .50001:Y = 0
100     GOSUB 5000
110     HGR2 : HCOLOR = 3
120     FOR I = 1 TO 10: GOSUB 4000: NEXT
140     GOSUB 6000
160     GOSUB 4000
180     GOTO 140
999     END
1000    REM SQUARE ROOT OF X,Y
1005  T = Y
1010  S = SQR (X • X + Y • Y)
1020  Y = SQR (( - X + S) / 2)
1030  X = SQR ((X + S) / 2)
1040    IF T < 0 THEN X = -X
1050    RETURN
2000    REM FOUR OVER L
2010  S = LX • LX + LY • LY
2020  LX = 4 • LX / S
2030  LY = - 4 • LY/S
2040    RETURN
3000    REM X,Y TIMES L
3010  TX = X:TY = Y
3020  X = TX • LX - TY • LY
3030  Y = TX • LY + TY • LX
3040    RETURN
4000    REM FUNCTION OF X,Y
4010    GOSUB 3000
4020  X = 1 - X
4030    GOSUB 1000
4040    IF RND (1) < .5 THEN X = -X:Y = -Y
4050  X = 1 - X
4060  X = X / 2:Y = Y / 2
4070    RETURN
5000    REM GET VALUES
5010    TEXT : HOME
5020    INPUT "WHAT IS LAMBDA? (X,Y) ";LX, LY
5030    GOSUB 2000
5040    INPUT "WHAT IS SCALE? ";SC
5050  SC = 2 • CX / SC
5060    RETURN
6000    REM PLOT X,Y
6010    HPLOT SC • (X - .5) + CX,CY - SC • Y
6020    RETURN
```

*Line changes for adapting this program to the IBM PC BASIC. (Note: IBM PC's must be equipped with a color/graphics monitor to run this program.)

```
100     SCREEN 2:KEY OFF:CLS
5010    SCREEN 0
6010    PSET (SC * (X - .5) + CX + 150, CY - SC * Y)
```

Best Way to Fly? Take the Bus

Ski jumping in a school bus/wind tunnel isn't easy. . . but two minutes in a harness with air streaming around the ski jumper is 30 times longer than the ride he can expect to get going off a 90-meter jump.

The problem with ski jumping is that there's little time to practice. Even on the big 90-meter hills, a skier's flight will last only about four seconds from take-off to landing. Because the U.S. ski-jumping season is short (about three months) and there are few world-class jumping hills in this country, even serious competitors like Jon and Jeff Denney of the U.S. Ski Jumping Team may spend a total of only five to ten minutes *per year* actually in the air.

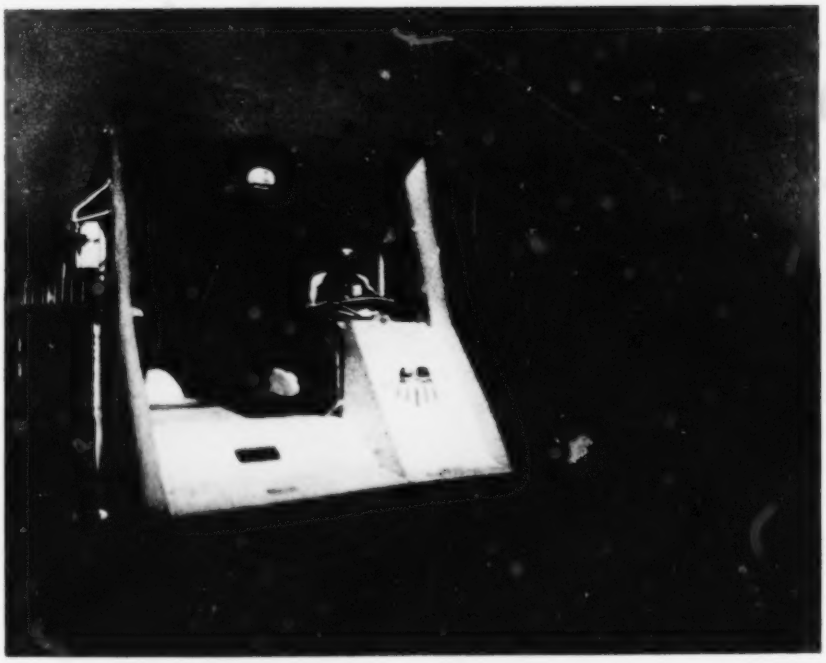
But practice time in the air is crucial to a ski jumper's performance. The sport is largely a matter of aerodynamics. Once a skier leaves the ramp, he must set himself in a "flying" position, cupping his body and angling his skis to approximate an airfoil, thus generating lift like an airplane wing.

The Denneys were looking for a way to get practice time in the air in the off season. They also wanted an accurate way to monitor which body positions would give the best results during flight.

"We wanted to learn something that we could learn in a wind tunnel," says Jon Denney, "except it would have cost us $5,000 an hour to learn it."

Wind Tunnel on Wheels

Together with Robert Mittelstadt, an engineer and former ski jumper, they set out to create a "cheap wind tunnel." What they came up with was an ingenious restatement, all the more so for its improbability: a school bus sawed open at each end. When



SKI JUMPING SANS SNOW: Ski-jump champion Jon Denney takes a ride on the wind in converted school bus "wind tunnel."

driven at 55 miles per hour—a ski jumper's average "flying" speed—down the highways around their home near Duluth, Minnesota, the bus's forward movement creates an airflow like that around a ski jumper in flight.

Instead of seats, the bus contains a large, counterbalanced beam with a harness at one end. The harness holds the skier in the air. On board are electronic instruments to measure *lift* (L) and *drag* (D). (For an explanation of lift and drag and how they affect a skier's performance, see page 114.) The bus enables the Denneys to concentrate on measuring and improving their L/D ratio, the key to improved performance in ski jumping.

"All other parameters being equal," Mittelstadt explains, "the skier with the best L/D ratio for a given airspeed will win events. To get maximum distance, a skier must train himself to fly at the best angle of attack for his particular weight, body area and airspeed. . . . *Any* improvement will show up in longer jumps."

Measuring Lift and Drag

Ski jumping in a school bus isn't easy. About two minutes is as long as a jumper can hold his position in the harness. But that two minutes, with air streaming around him, is *30 times* longer than the ride he can expect going off a 90-meter jump.

The on-board electronic instruments figure the skier's L/D ratio by measuring the force of his body against the beam. He can shift his aerodynamic position to see how different positions affect the L/D ratio. The position that gives the best readout—the best ratio of lift to drag—will be the one he'll try to achieve in actual competition.

The first ski-jumping competition in the United States, in 1887, was won with a jump of 37 feet. Today, with modern equipment and streamlined clothing, a good jump from a 90-meter Olympic hill can exceed 400 feet.

For the truly courageous, there are about a half-dozen 120-meter hills in the world, only one of which is in the Western Hemisphere (at Michigan's Copper Peak). Skiing off one of these "super hills" is like going off the top of a 24-story building. World-class ski jumpers soaring off 120-meter jumps can achieve distances greater than 600 feet.*

Where is the limit? Wherever it is, innovators like the Denneys and Mittelstadt will look for ways to extend it. ■

*The world record ski jump off a 120-meter hill was set by Natti Nykaenen of Finland with a jump of 185 meters (606 ft. 11 in.) in 1984, in Oberstdorf, West Germany.

ENGINEERING SPEED: Engineer Robert Mittelstadt straps skier Denney into a harness for a ride on the wind. The harness allows the skier to try out a variety of body positions; when the onboard instruments indicate that the best ratio of lift to drag has been reached, the skier knows what the position "feels" like and can try to replicate it in competition.

Further Readings

Epstein, Sue Hoover. "Skiing on Air." *Science 84*, January/February 1984.

A step-by-step description of what happens during a ski jump, from takeoff through landing. The article contains illustrations showing the angle of the skier in relation to the skis and in relation to the hill, and describes the technological and scientific know-how needed to compete successfully in today's world-class competitions.

Gelband, Myra, ed. "Fly the Friendly Van." From the Scorecard section of *Sports Illustrated*, 7 December 1981.

This article describes an early version of the "ski bus," designed by Steve Gaskill, head of the U.S. Nordic combined team. Gaskill devised an apparatus that harnessed the ski jumper to a 6-foot scaffold atop a van. The van was then driven down the highway at 55 mph, like the ski bus designed by Bob Mittlestadt.

Swift, W.M. "A Lift Out Of Life." *Sports Illustrated*, 4 February 1980.

This article profiles the oldest of the Denney brothers, Jim, who was the best ski jumper in the United States in 1980. It describes his training, and the American challenge to Europe's 20-year domination of the sport.

Flying Time

When a ski jumper stretches himself out during a jump, with his head nearly touching the tips of his skis, he is taking advantage of the same principles that keep an airplane aloft.

The shape a skier approximates during a jump is called an *airfoil*. This elongated teardrop shape is the basic shape of a cross-section of an airplane wing, designed to use the physics of airflow to keep the plane in the air. (See Fig. 1A.)

During flight, the airfoil exerts a force on the air that is, by virtue of the airfoil's shape and tilt, downward and slightly forward (see Fig. 1B).

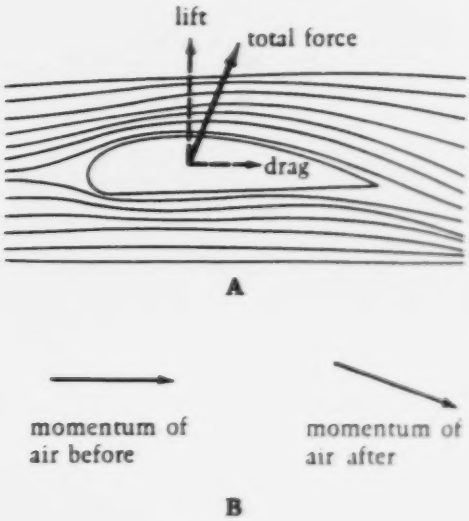


Fig. 1 (A, B)

Newton's third law says that if object A exerts a force on object B, then object B will exert a force of the same strength, *but in the opposite direction*, on object A. Thus the air, as "object B" (the airfoil being "object A"), exerts a force on the airfoil that is upward and slightly backward. The upward part is called *lift*, and the backward part is called *drag*. They are always found together; it's the ratio between them that is crucial to both the pilot and the ski jumper.

Here is how an airplane wing works:

- During flight, the air that is coming toward the wing separates. Some passes over the wing and some under; then the two streams come back together.
- Because of the design of the airfoil, the air going over the top has a greater distance to travel than the air passing underneath. So the air *over* the top of the wing goes faster relative to the surface of the wing.
- This higher speed corresponds to lower air pressure. In other words, the air under the wing exerts a greater pressure than the air over it.
- The greater air pressure underneath the wing results in lift.

When a plane flies at a constant speed at a constant height, the engines work just hard enough to keep the drag from slowing the plane down.

But the skier has only his own body with which to work. Without compensating engines, it's inevitable that drag will take its toll, putting a limit on how far he can fly.

X=

123

Angle of Attack

To stave off the inevitable loss of speed and altitude, at least for a while, the skier uses his body as an airfoil. The skier aims to keep drag down as much as possible while maintaining maximum lift, striving for a balance that will allow him to remain in the air for as long as possible and simultaneously go as far as possible. (The optimum ratio of lift to drag, according to engineer Robert Mittelstadt, is 2:3; practically speaking, a ratio of 1:3 is probably the best a skier can achieve. Lift and drag are both measured in pounds of thrust.)

The instruments on the ski bus measure the effects of lift and drag on the skier suspended in the bus's harness. When the instruments show an optimum lift-to-drag ratio has been achieved, the skier can learn what it feels like to be in that particular body position. Then, when he's actually in flight off a jumping hill, he'll try to maneuver his body until he assumes that same position.

TAKING OFF: Thorger Hemmestvedt's jump of 103 feet (drawing, left), which was respectable by 1893 standards, wouldn't even put him in the running today. Modern jumpers like Germany's Klaus Ostwald (above) try to approximate the perfect airfoil shape in achieving jumps in excess of 600 feet off of 120-meter hills.

"When good mathematics students use mathematical ideas to solve everyday problems, they are less likely to 'look for a similar problem' than to 'interpret the given problem from several perspectives.'"

Richard Lesh, *The Arithmetic Teacher*

The Skier's Dilemma

Maintaining altitude means maintaining lift. But if there's too much lift, the accompanying drag will keep the ski jumper from going out too far.

If drag is reduced too much, lift will also be reduced. The ski jumper will go very fast, but he will nosedive down the hill.

Since the object is to get the most forward speed while spending more time in the air, the ski jumper must find the optimum combination of the least possible drag with the greatest possible lift.

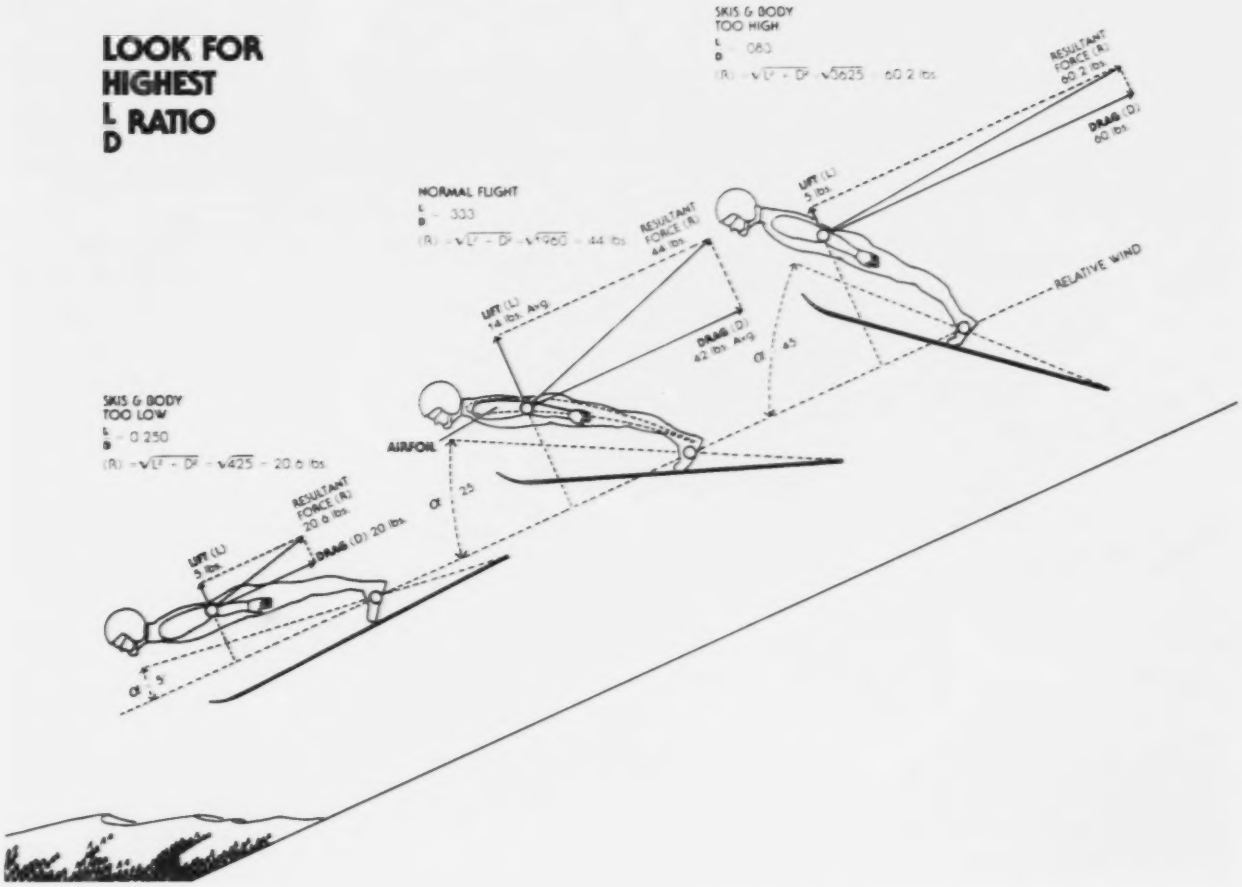


Fig. 1

The ski jumper nosedives down the hill. His drag is low, allowing him to keep more of his speed, but his lift is low as well.

The result: **His flight and jump distances are short.**

Fig. 2

The ski jumper's body and skis achieve optimum lift relative to drag.

The result: **Speed and air time are increased to combine for a longer jump.**

Fig. 3

The ski jumper is too high relative to the flight path. His drag is high and his lift is low.

The result: **His forward speed is significantly reduced—he doesn't go very far.**

X=

Vectors

A *vector* is a quantity that has direction as well as size. Lift is an example of a vector having a force (measured in pounds) *and* an upward direction.

Forces like lift and drag can be restated as vectors and represented as directed line segments. The arrowhead and angle of the segment indicate the direction of the vector; the length of the segment indicates the size of the vector. A vector X is represented as $\vec{X}$. Two vectors are equal if their length is the same.

Exercises:

1. Vectors are added to one another by putting the tail of one vector to the head of a second vector. The sum is the vector drawn from the tail of the second to the head of the first.

Add the following vectors:

Fig. 1

2. Do you get the same result above whether you put the tail of the first vector to the head of the second or the tail of the second to the head of the first?

3. Add the following sets of vectors:

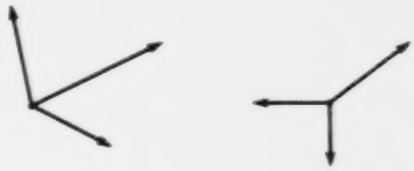


Fig. 2

Do you get the same result above whether you add the first vector to the sum of the second and third or add the sum of the first and second vector to the third?

4. What does it mean if the sum of vectors is a point?

5. Since the lift and drag forces of a ski jumper are perpendicular, the vectors that represent them make a 90° angle. The Pythagorean theorem can be used to compute the *resultant force*. Find the resultant force for each of the following. Determine the lift/drag (L/D) ratio for each.

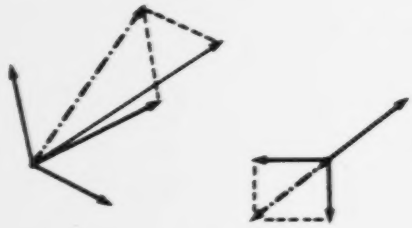
a. lift = 9 lb; drag = 40 lb
b. lift = 12 lb; drag = 35 lb
c. lift = 11 lb; drag = 60 lb

Answers:

1.

2. Yes, vector addition is commutative.

3.



Yes, vector addition is associative.

4. The sum is zero; the vectors "balance each other out."

5. **a.** resultant force = 41; L/D = .225
b. resultant force = 37; L/D = .343
c. resultant force = 61; L/D = .183

Teacher's Notes

(see opposite page)

Close Relations

Analogies, like ratios and graphs, are statements of relation and powerful problem-solving tools. In order to use analogies, students must develop skills in understanding the precise class of relation they represent.

Many exercises in analogy are multiple-choice drills in which the students can guess the answer without ever defining the relationship.

The object of this exercise is to focus on relationships. Your students may even be able to improve on the list below.

Answers:

	Column A	Column B	Relationship
1.	PIT / PEACH	SUN / SOLAR SYSTEM	Part to whole
2.	RACE / TRACK	SWIM / POOL	Action to object
3.	BLAST / GUST	BLARE / ROAR	Synonyms
4.	WALK / RUN	ADD / MULTIPLY	Degree
5.	HAND / ELBOW	FOOT / KNEE	Part to part
6.	ENGLAND / LONDON	CANADA / MONTREAL	Place
7.	SILK / RAYON	BUTTER / MARGARINE	Association
8.	BIRTH / DEATH	INTRODUCTION / CONCLUSION	Antonyms
9.	SURGEON / SCALPEL	BUTCHER / CLEAVER	Purpose
10.	RAIN / FLOODING	ICE / SKIDDING	Cause and effect
11.	SINKS / ROCK	FLOATS / FEATHER	Characteristic
12.	MAY / FEBRUARY	NOVEMBER / AUGUST	Sequence
13.	DISHES / BREAK	CLOTHING / RIP	Object to action

For Further Discussion:
Using the same list of classes of relationships, have your students make up their own analogies for each category.

Then try making up other categories of relationships. One-half of the class can make up the categories and the other half can make up analogies to go with them.

A Clash of Symbols

Answers:

a. woman
b. good or love (woman and child)
c. man
d. day (man with outstretched arms, head touching heaven)
e. field
f. feather (two of them)
g. bamboo (two stalks)
h. thicket (three trees)

For Further Discussion:
Some of the characters express abstract ideas with concrete objects. For instance, "day" is symbolized by a "man with outstretched arms, head touching heaven."

In what way are these restatements of concepts which could not otherwise be represented pictorially? Are they restated in a way that virtually any culture could understand? Have your students develop pictographs for the following:

- late
- night
- love
- hungry
- sleepy
- happy
- over

What other words might readily lend themselves to pictorial restatements?

Close Relations

Match the analogies in Column A with a similar analogy from Column B. Then discuss how they are related.

	Column A	Column B
1.	PIT	FLOATS
	PEACH	FEATHER
2.	RACE	SWIM
	TRACK	POOL
3.	BLAST	NOVEMBER
	GUST	AUGUST
4.	WALK	SUN
	RUN	SOLAR SYSTEM
5.	HAND	BLARE
	ELBOW	ROAR
6.	ENGLAND	FOOT
	LONDON	KNEE
7.	SILK	INTRODUCTION
	RAYON	CONCLUSION
8.	BIRTH	BUTCHER
	DEATH	CLEAVER
9.	SURGEON	ICE
	SCALPEL	SKIDDING
10.	RAIN	CANADA
	FLOODING	MONTREAL
11.	SINKS	BUTTER
	ROCK	MARGARINE
12.	MAY	ADD
	FEBRUARY	MULTIPLY
13.	DISHES	CLOTHING
	BREAK	RIP

A Clash of Symbols

Chinese is a language of symbolic restatement. Its characters are based largely on *pictographs* (pictures of things) and *ideographs* (pictures representing the ideas of things). Try to match the Chinese symbols in Figure 1 with their English equivalents.

Fig. 1

_____ thicket (three trees)
_____ woman
_____ good or love (woman and child)
_____ man
_____ day (man with outstretched arms, head touching heaven)
_____ field
_____ two feathers
_____ two stalks of bamboo

Teacher's Notes

(see opposite page)

Roller Coasters

A graph is a restatement of a relationship. Students often need to be reminded of the difference between *pictures* of a physical event and *abstract representations* of it in graph form.

Writing paragraphs will focus their attention on what is happening as the roller coaster moves down the track. The paragraphs need not be too specific, but should outline the car's journey from A to O. For instance, a paragraph might begin:

> The roller coaster begins to accelerate after B, picking up speed rapidly as it nears E, then slowing to H . . .

The curves on the speed/distance graphs of the roller coasters go in the opposite direction of the drawings. This makes sense, because *down* on the drawing (that is, the roller coaster's acceleration) means *up* on the graph (that is, higher numbers on the graph's speed axis).

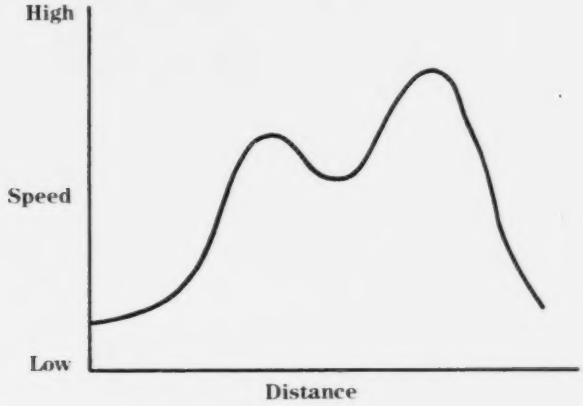


Fig. 1

Getting the Points

Possible verbal restatements for graphs **1–6.**

1. A is heavier than B.
B is taller than A.

2. A is faster than B.
B is older than A.

3. B is more expensive than A.
B is bigger than A.

4. A and B are the same temperature.
A is heavier than B.

5. A is hotter than B.
B is later than A.

6. B has larger feet than A.
A is more intelligent than B.

Note: It might be helpful to students in working with graphic restatements to imagine *what* is being graphed in each case.

For Further Practice:
Have your students develop additional graphic representations of relationships.

"At any level. . .a major difficulty in teaching problem solving lies in convincing students to record the details of a problem on paper. . .in ways they *find useful."*

Joel Schneider and Kevin W. Saunders, *Problem Solving in School Mathematics*

Roller Coasters

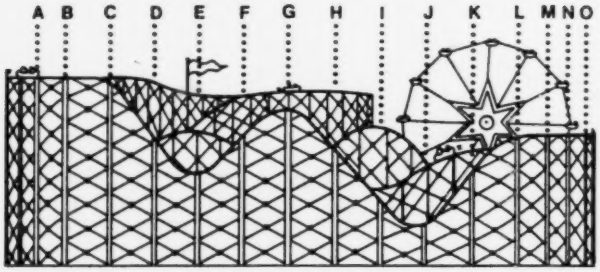


Fig. 1

Figure 1 shows the track of a roller coaster. Between point A and point B, the roller coaster travels at walking speed.

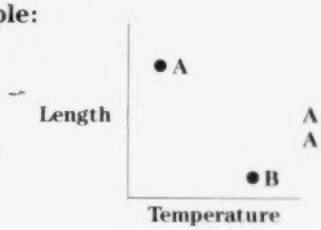
Write a paragraph describing how the speed of the roller coaster varies as it travels along the track from point A to point O. Then, sketch a graph that shows how the speed varies as it travels along the track.

Getting the Points

Graphs are pictorial representations of relationships, which are often more easily understood when put into words. These examples don't need any numbers for the relationships between points A and B to be clear.

Restate each of the graphs in Figure 2 (**1–6**), writing short sentences that compare A with B.

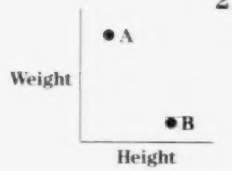
Example:



A is longer than B
A is colder than B

Fig. 1

1.



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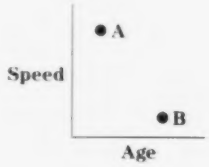
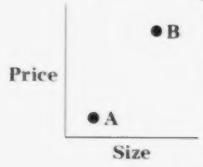


3.



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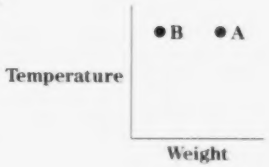
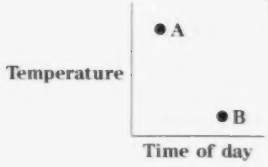
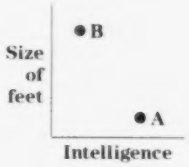


5.



6.



Teacher's Notes

(see opposite page)

Waiting for the Bus

Answer:

In a Pig's Ear

Answers:

1. 242 litters can be accounted for in a season with this system. The five sections of base 3 represented on a pig's left ear correspond to 1, 3, 9, 27 and 81. Two notches in each of those sections would add up to 242.

2. Litter no. 140; pig no. 11.

TRIPPING THE LIGHT TRULY FANTASTIC: Imaginative restatements of familiar ideas are a Hollywood staple. Here Fred Astaire takes to the dance "floor" in *Royal Wedding*.

Waiting for the Bus

Each person shown in Figure 1 is represented by a point on the graph in Figure 2. Label each point on the graph with the name of the person it represents.

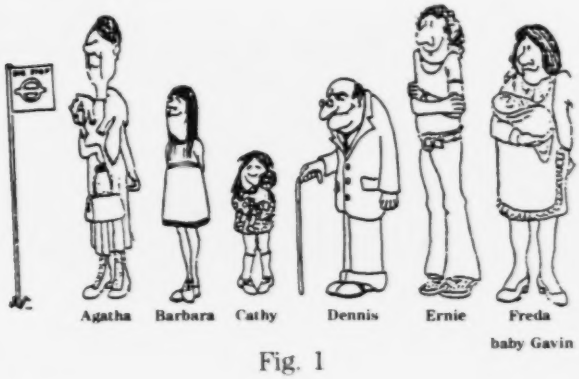


Fig. 1

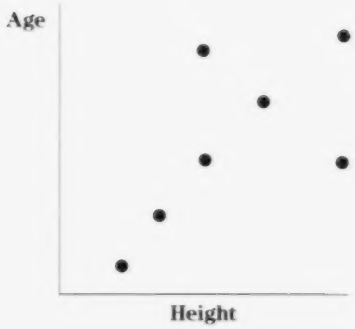


Fig. 2

In a Pig's Ear

Pig farmers use the *base-3* system to identify their pigs. Marks on the pig's right ear represent which litter it came from; marks on the left ear represent the birth order in that litter.

The right ear is divided into five sections (see Fig. 1). The left ear uses only three sections since the number of pigs in a litter rarely exceeds 26, the highest number that three digits in base 3 can express (two 1s, two 3s and two 9s).

1. How many different litters can a pig farmer identify in a single season with this system?

2. Identify the pig below (Fig. 2), according to its markings.

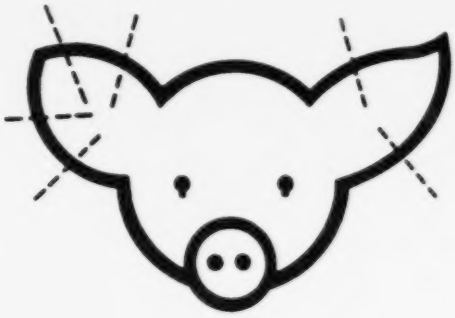


Fig. 1

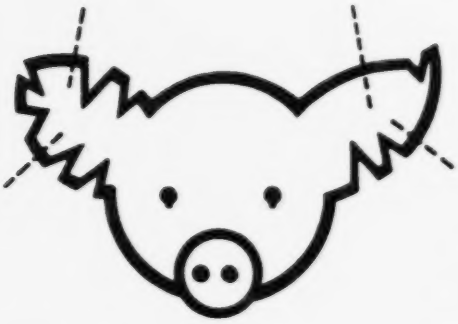
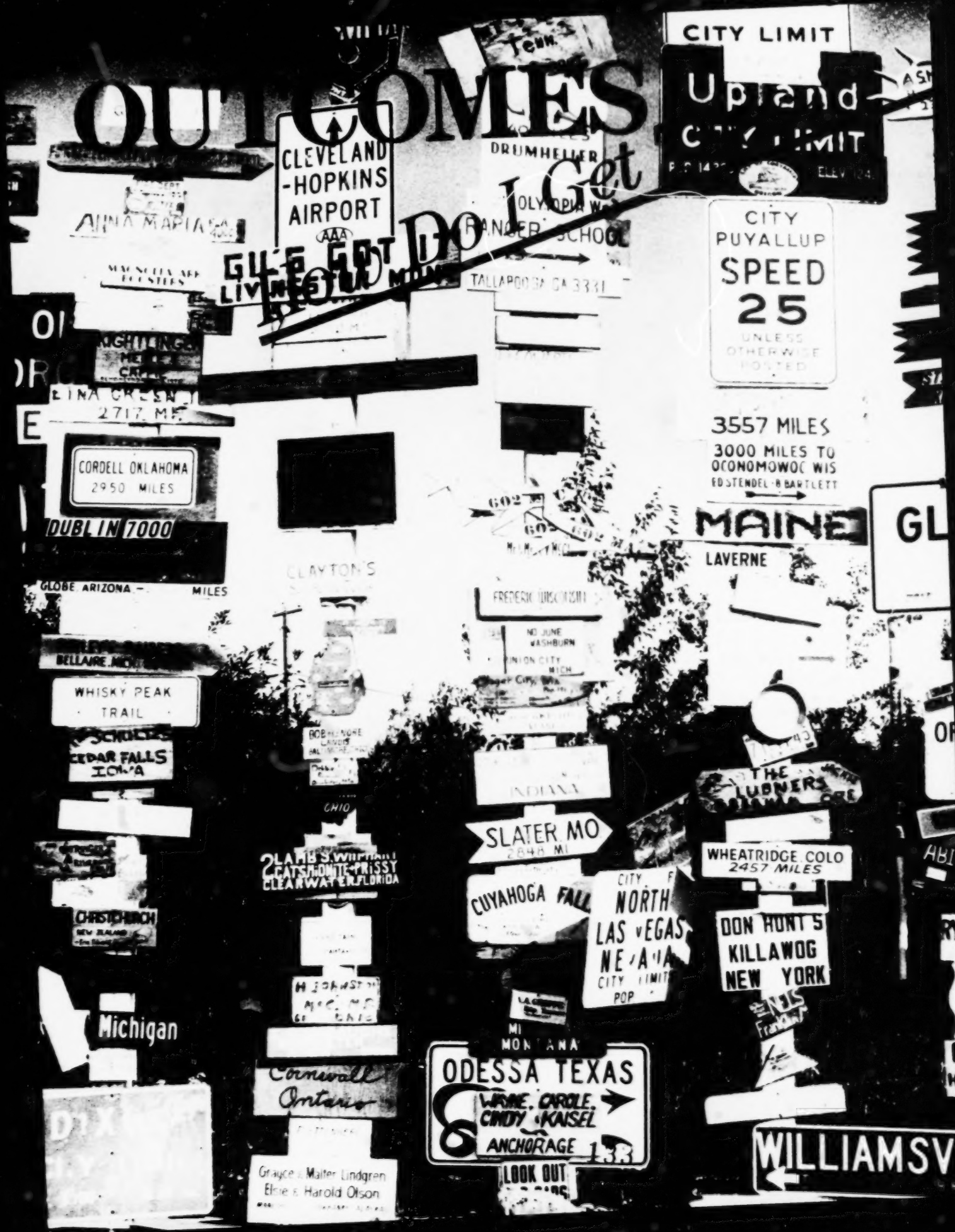


Fig. 2


OUTCOMES
How Do I Get
CITY LIMIT
Upland
CLEVELAND
-HOPKINS
AIRPORT
DRUMHELLER
CITY
PUYALLUP
SPEED
25
UNLESS
OTHERWISE
POSTED
3557 MILES
3000 MILES TO
OCONOMOWOC WIS
ED STENDEL · B BARTLETT
CORDELL OKLAHOMA
2950 MILES
DUBLIN 7000
MAINE
LAVERNE
GLOBE ARIZONA
MILES
CLAYTON'S
FREDERIC WISCONSIN
NO JUNE
WASHBURN
UNION CITY
MICH
WHISKY PEAK
TRAIL
CEDAR FALLS
IOWA
INDIANA
THE
LUBNERS
SLATER MO
2848 MI
WHEATRIDGE COLO
2457 MILES
CUYAHOGA FALL
NORTH
LAS VEGAS
NEVADA
CITY LIMIT
POP
DON HUNT'S
KILLAWOG
NEW YORK
CHRISTCHURCH
Michigan
MONTANA
ODESSA TEXAS
WAYNE, CAROLE,
CINDY & KAISEL
ANCHORAGE
Cornwall
Ontario
Grayce & Malter Lindgren
Elsie & Harold Olson
LOOK OUT
WILLIAMSVI

Fact Sheet

OUTCOMES: *How Do I Get There?*

Running Time: 22:28 minutes

Documentary Segments (3):

1. ***GIANT EGG:*** Designing a giant chicken egg is more difficult than it sounds. **Can human plus computer achieve the design power of a hen?** 136
2. ***CATHEDRAL:*** After nearly 100 years, the Cathedral Church of St. John the Divine in New York City is still unfinished, and construction will continue into the twenty-first century. **How do you manage a process that will most likely not be completed in your lifetime?** 141
3. ***DEMOLITION:*** The Loizeaux family combines awesome destructive force with kid-glove delicacy: They're a demolition company. **How do they blow up a building while leaving neighbors and neighborhood intact?** 146

Comedy Segment (1):

4. THE SANDWICH: Hired to paint an advertising billboard, "Bill the Sign Painter" brings some new perspectives to the job. **The finished product is not the desired outcome, but will it do the trick?** 134

Math Concepts:

GIANT EGG: Equilateral triangles and hexagons, ratio proportion, probability, irregular surfaces, tessellation, ordering and sequencing.

CATHEDRAL: Geometry of a cube rectangle, ratio proportion, scale and modeling.

DEMOLITION: Distance rate time, volume, vectors, formulas.

Problem-Solving Concepts:

Working backward; checking results against conditions; establishing methods; achieving goals; thinking linearly.

HOW TO SPOT AN OUTCOMES PROBLEM

You know it's an outcomes problem if . . .

- ***The destination or objective is known.*** *What's missing is the way there.*
- ***You must choose from many possible routes.*** *Mazes are classic outcomes problems. The information is given; the destination known. What is needed is to determine the right path given many choices.*
- ***It's a problem of procedure or method, not information or situation.*** *You have the knowledge and data with which to find the most effective or efficient path. Chess is an outcomes problem; poker an information problem.*
- ***It helps to work in reverse—tracing the path from the destination to the start.*** *Working backward suggests going back over old ground, but in the case of outcomes problems, it is often the most efficient way to work.*

X=

Throughout the chapter, the X = sign indicates pages containing exercises and problems for classroom use and discussion.

WHICH WAY TO CUYAHOGA? In order to know how to get there, you first have to know where you're going. In problem solving, "outcomes" places emphasis on planning one's route to a known goal.

BEST COPY AVAILABLE

OUTCOMES

How Do I Get There?

When we know where we want to go, we can direct our attention, take aim and work toward our goal. A gold medal for an Olympic athlete, a published book for an author, a degree for a college student, election for a politician: All these are outcomes. They are not reached passively; they require us to figure out how to get there and work at it. An outcomes problem is one of process. *We know the desired answer or goal; we don't know the route to get there.*

In terms of problem solving, outcomes are the beginning and the end of strategy formulation: A problem solver begins with the desired outcome as the result to be accomplished, and gears his or her strategy to achieve that end.

Telephone systems, transatlantic airline routes and computer software are masterpieces of outcomes-oriented problem solving. Each is characterized by an origin, a known destination and a set of conditions that must be met along the way. The path from start to finish can be labyrinthian and awesomely complex.

In our fast-moving, mobile society, finding solutions to routing problems is extremely important. A trucker hauling merchandise from California to Louisiana, for example, can't just hop in the truck and take off. Among other things, he must know what route he's going to take, how long it will take to get there, how long he can drive in a single shift and whether there are any special requirements for the load he is carrying. Similarly, an overnight courier service has precious little time to spare. Routes must be well planned to provide efficient service and to minimize costs.

Not all outcomes problems are big. Painting a picture, hanging a picture, taking a picture, selling a picture—these are all outcomes problems. Perhaps the most common of all problem situations, the outcomes problem is also one of the subtlest. It never asks for an answer. It asks for a *solution*.

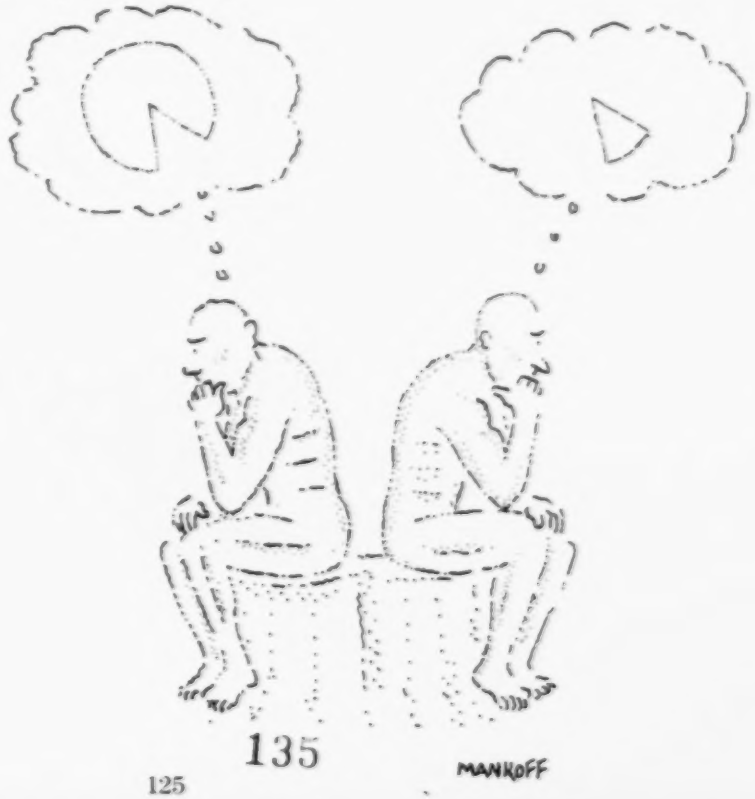
Why **Outcomes**?

Traditional textbook problems are usually solved when a student finds a numerical result like a sum, product or value of a variable. Such exercises are good at developing computational skills, but they represent artificial problem situations—the necessary information is provided, and the result is the answer. This is often perceived by the student as normal problem solving.

It isn't. In fact, this type of problem is as unusual in mathematics as it is in life. More common is a setup where the outcome is known in advance and the problem is how to get there. (On a level closer to the classroom, one example is the mathematical proof; one starts with what is to be proved and works backward to establish the procedure.)

For instance, the desired outcome might be a new bicycle. In the textbook version, the problem might read like this, asking for a numerical result:

- John wants a new bicycle that costs $75. He can earn $2 an hour working part-time at the drugstore. If he works 10 hours a week, how long will it take to earn enough money to buy the bicycle?



But the student might be thinking about John's *real* problem—a maze of decisions, conditions and potential blind alleys:

- How can I make $75? Maybe I can get a part-time job. Where? Maybe at the drugstore. Suppose they don't need anyone—what then? I'll try the fast-food restaurant. How much do they pay? Would the hours they need someone fit my school schedule? Could I get someone to give me a lift to work and back?

Developing a problem-solving approach that includes the ability to work backward from the outcome to the beginning is the bridge between the classroom and the kinds of problems students will meet in later life.

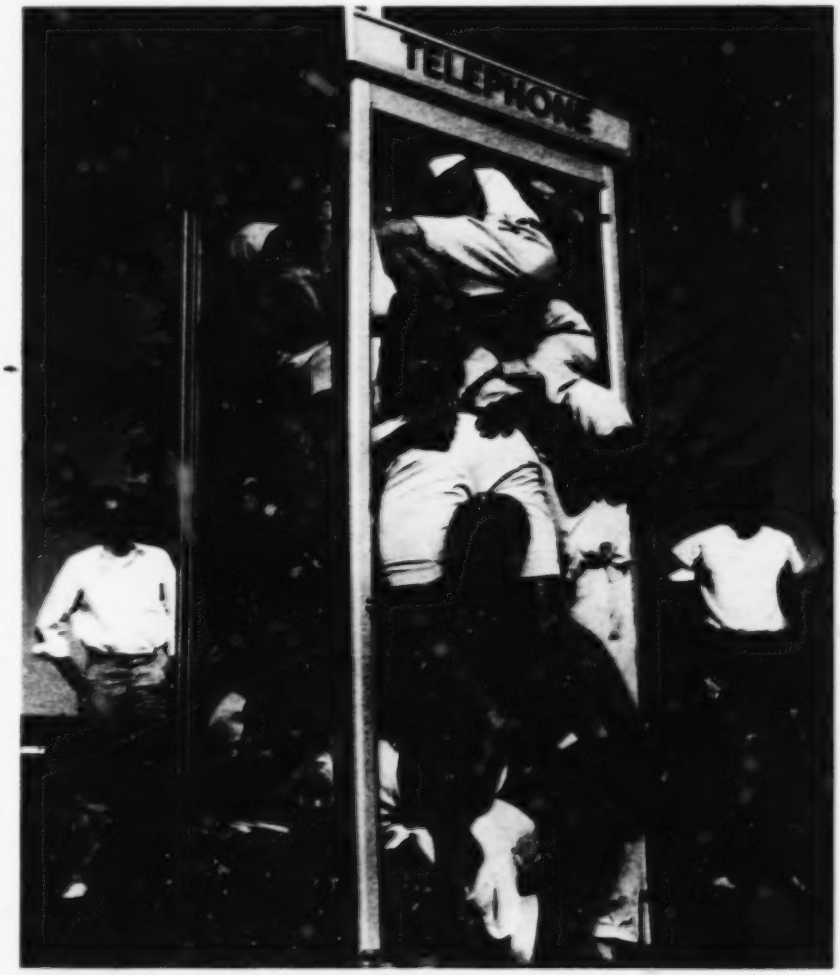
Teaching Outcomes

There is very little in a student's life that can't be brought into play in teaching outcomes. Anyone who has ever said, "I want to do. . ." is talking about a desired outcome.

Outcomes focus on the solution, not the answer. There is a distinct difference between solutions and answers. Answers, as students think of them, are results, but outcomes are usually the very first thing a problem solver must consider. The solution is then the road to the result, not the end of the road.

Outcomes flag contradictions. If the result of a problem is known in advance, it can be tested against the other information given. If the two don't fit, the problem can't be solved under the stated conditions, and it would seem wise to plot another course. Example: You want to install wall-to-wall carpeting in a 9-by 12-foot room (outcome). You have 16 square feet of carpeting (given information). Solution?

Outcomes teach a step-by-step approach to problem solving. Starting with the result, the problem solver works backward, step by step, to reach the starting conditions. In this chain of cause and effect, each link is important: A misstep can lead the problem solver in the wrong direction and sometimes to an outcome very different from the one desired. ■



"IT'S FOR YOU": If your goal is to stuff as many people as possible into a phone booth, you probably don't need to do much in the way of planning. But think again—how do you pack them in and still leave them breathing space?

Outcomes in the Classroom

Students can expand their problem-solving skills by using outcomes strategies to attack their math problems.

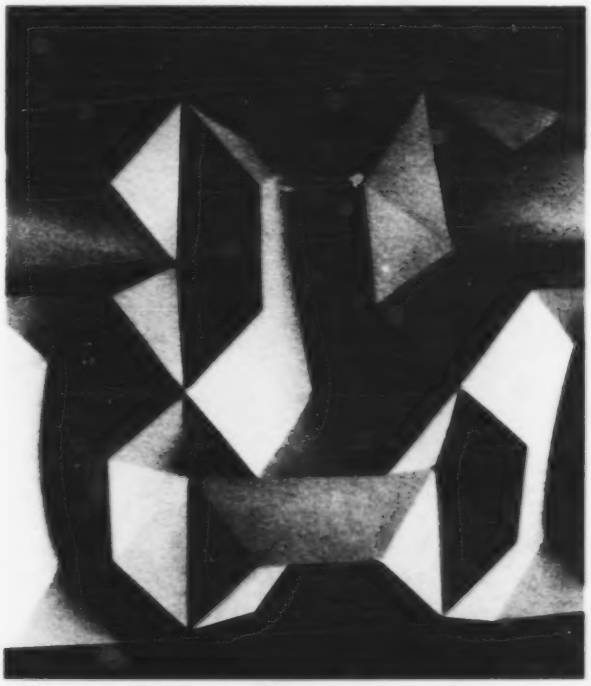
Working Backward: Starting with a result and working back to a given set of conditions may not be easy at first, but will come with a bit of practice. This technique is especially useful for problems with a "one-in, one-out" sequence of events, where each step produces a unique result. (Mazes are classic examples where working backward is the fastest way to plot a route.)

Solving for Variables: Often, working backward is the fastest way to the solution of an equation. For example, if $x + 5 = 7$, you subtract 5 from both sides of the equation to solve for x. Another way of saying it: "The outcome is 7. How much do I have to add to 5 to get there?

Spotting Contradictions: There will always be some problems that turn out to be of the "square peg/round hole" variety: You can't get to the solution you want with the materials you're given. Find the value of x if x is an integer and $x + 4 = 6.5$. Working backward lets you spot contradictions and inconsistencies between the goal and the data given.

Probability: Sometimes, although a single outcome is clear, the problem can be choosing among the many possible paths to that goal. When Ron Resch had finally solved the problem of how to build the Vegreville Egg, he faced one further challenge: how to color the egg in traditional Ukrainian patterns. Each of the 3,512 facets could be one of three colors. How many possible alternatives does this imply?

Geometry: In each of the three documentary segments, geometry plays an important part in solving their outcomes problems. Ron Resch is conducting what he has called "pure research in geometry" when trying to find a way to tessellate an egg. (His folded paper "experiments" in geometry may inspire students to similar investigations.) When Alan Bird decides which stones to cut from the limestone blocks, he, too, relies on his knowledge of solid geometry to minimize waste. Finally, vector geometry plays a role in helping the Loizeauxs understand how a building will fall. ■



PAPER PATTERNS: Ron Resch's work on the design of the Vegreville Easter egg (see story in this chapter) was an outgrowth of his interest in geometry. Above, one of his experiments in paper folding.

For Openers

Ask your students the following questions after they've seen the OUTCOMES film:

- **What do you think the film means by "outcomes?" How are outcomes important, and how can we use them in problem solving?**
- **What are some goals you've set for yourself? How do you go about achieving them?**
- **How do the characters in the film solve their problems by working backward from the desired outcome?**
  1. **What is the desired outcome in the segment on the Vegreville Egg? Why was it so difficult to achieve?**
  2. **Why do you think the segment on the Cathedral of St. John the Divine was included in OUTCOMES? What is unusual about James Bambridge's goal?**
  3. **What is the desired outcome in the segment on demolition? Is it just knocking down a building, or is there more to be kept in mind?**
  4. **How would an outcomes strategy have made "Bill the Sign Painter's" job an easier one?**

Behind the Scenes: OUTCOMES On Film

We hadn't been researching long before we came up with a theme for the OUTCOMES film—construction. Construction is a good outcomes problem. Builders have a clear picture of their goal; their challenge is to find the most direct and efficient path to that goal. Once we had a theme, we started looking for examples that were out of the ordinary—unique construction problems that would grab and hold the attention of student viewers.

Egg: In advisor Philip Davis' book, *The Mathematical Experience*, we found a photograph labeled "The Vegreville, Alberta Easter Egg." The picture shows a polyhedral egg with 3,512 visible facets, standing 31 feet tall. Who built this thing? Why? When we tracked down the designer and builder, computer scientist Ron Resch, and listened to his intriguing story, we knew it was right for CHALLENGE. His segment focuses on geometry and computer modeling. He also talks about the mistakes and frustrations experienced in designing this geometric first—the tessellation of an egg.

Cathedral: We found our second story right in our own backyard. Construction has resumed on the Cathedral Church of St. John the Divine in New York City, begun almost 100 years ago. Two factors intrigued us about this construction problem: First, in the world's most modern city, this cathedral is being built with methods and techniques that are centuries old; second, few of the people working on the cathedral can expect to see its completion. The job of finishing what will be the world's largest Gothic cathedral may take another 60 years. This segment focuses on spatial relations, scale drawing, geometry of the rectangle and ratio and proportion.

Demolition: We couldn't resist this construction problem in reverse. Controlled Demolition, Inc., run by the Loizeaux family, is world famous for "blowing down" buildings without blowing up the neighborhood. After weeks of negotiations, they allowed us to film them preparing a building for demolition as well as the demolition itself. Concepts in this segment include distance/rate/time, volume and vectors.

Sandwich: "Bill the Sign Painter" brings a certain flair to the job, but ultimately suffers from an inability to see the "Big Picture." This comedy sequence shows by negative example that it pays to know what you're solving for before you start solving. ■

GETTING STARTED: **Thinking OUTCOMES**

How do you prepare students to think in terms of outcomes? There are a few basic skills people call on to solve this type of problem, and these skills can be taught. The exercises in this chapter place an emphasis on working backward, starting with the result or goal and backtracking to the first step in order to find the best route.

***Before* viewing the OUTCOMES film, try the following exercises with your students to get them thinking about the topic.**

1. Draw the arrangement of circles as shown in Figure 1 on the board or demonstrate with actual coins. Ask your students how, by moving only three circles or coins, they can make the arrangement point in the opposite direction, as in Figure 2.

Hint: This problem can best be solved by working backward. It may help to have them make a drawing of what the solution should look like next to the original arrangement. What do they notice is the same about both arrangements? What's different?

The circles (or coins) that should be moved to accomplish the sought-after outcome are outlined in Figure 2.

Fig. 1

Fig. 2

2. A rich Texas rancher owned an "L"-shaped piece of property known as El Rancho Peligroso (see Fig. 3). On reading his will upon his death, his four children learned that he had left them the ranch provided they could figure out how to divide it into 4 pieces of equal size and shape. How can they fulfill the dictates of his will?

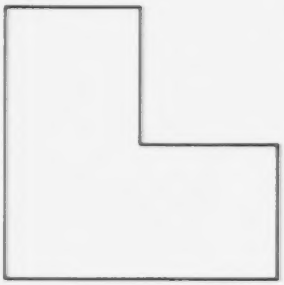


Fig. 3

Again, students should be advised that the best way to solve the problem is to work backward. The goal is given; how do you get there? (See Fig. 4 for the solution.)

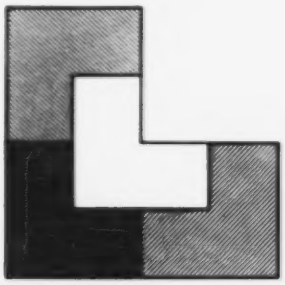


Fig. 4

Stalking an **Outcomes** Problem

Making Up Your Own Problems

The following is a hypothetical conversation between teacher and students. It is intended to suggest ways to encourage students to make connections between problem-solving concepts in the film, their own experience and their knowledge of mathematics.

"

- ***Now we're going to spend the rest of the class making up outcomes problems. But first, can anyone tell me what "outcomes" means?***

  It's how it all comes out in the end.

- ***Like "they all lived happily ever after?" What's so important about that?***

  I'm not sure.

- ***Okay. Let's talk about the film we just saw. What was the outcome for the demolition people?***

  They blew up the building so that it fell straight down.

- ***Right. And that didn't just happen by itself. They made it happen; this was the whole reason they were there. So, when we talk about outcomes in solving problems, we're talking about something you make happen—a set goal. You start with the answer. You solve them by figuring out how to get there. And that brings me to outcomes in math problems.***

  I was afraid we'd get there sooner or later. Can't we talk about the film some more?

- ***All right. Suppose you tell me why the computer artist was in the OUTCOMES film. What was his goal?***

  He wanted to design a giant Easter Egg.

- ***And what was his problem?***

  No one had ever done it before. He didn't know how. Wait a minute. . . I think I get it. With an outcomes problem, you have to figure out how to do it.

- ***That's it! And what about the cathedral?***

  They're trying to figure out how to cut stones to fit the plans they've already drawn up. Kind of like a jigsaw puzzle where you know the shape of the hole you need to fill and have to find the right piece to fill it.

- ***Good! And now, I'm afraid we can't put it off any longer. What's the outcome in a math problem?***

  The answer.

- ***So, can you think of a math problem where you might start with the answer and figure out how to get there?***

  Umm. . .

- ***Maybe it would be easier if, instead of trying to think of a "math problem," you think about something people do that involves numbers.***

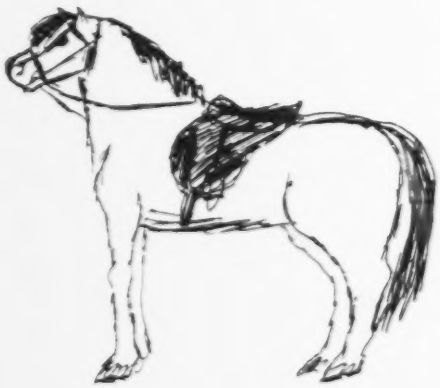
  I know. The supermarket checkout people have to figure out how to make change. Their cash registers tell them how much money to give you, but they have to figure out how to do it.

- ***Good. If I give you $3.67 in change, I have to say, "what combination of coins and bills do I have to put together in order to get to $3.67?" I know where I want to go; my problem is how to get there. That's a great example of an outcomes problem.***

  Let's try to think of some more. You might be surprised at the outcome. . .

"

Elaine is about to practice jumping her horse for an upcoming show. First she must set up a course of obstacles for her horse to jump, for her horse to get the maximum amount of exercise and do well in the show she must have a course exactly like the one in the show. She knows that; every jump is different, all are between 3ft and 3ft6", there are a total of 25 paces her horse must take, there are 10 jumps. What is a possible course?



Trinity Wood
Howard Bishop Middle School
Gainesville, Florida

Following Through: Problem Ideas from the Film

Students can make up their own problems based on the OUTCOMES film. Here are some ideas to get them started.

Vegreville Egg: Computer artist Ron Resch fills the surface of an egg with regular geometric figures to build the Vegreville Egg.

- Draw a square and fill it with equilateral triangles. You can decide how big to make the triangles, but make sure they're all the same size, and they fill the square evenly. (Suggestion: Use graph paper; use one square as a unit of measurement.) Find the area of the square; then take a small section—say, 4 by 4 squares. How many equilateral triangles of the size you chose will fit in that area? Use your findings to calculate how many triangles will fill the whole area. Then, try other figures, like quadrilaterals or pentagons.

- What kinds of shapes might you use to tessellate a circle? A triangle? A trapezoid? Draw a picture showing how these shapes might fit.

Cathedral: To minimize waste, stonemasons at the Cathedral of St. John the Divine must figure out how to get the most from each block of limestone.

- Design a structure to be built from a set of building blocks. Then make an architect's drawing of each face of your proposed structure. Provide measurements wherever possible. How many blocks and what shapes will you require? Then, trace drawings with another student. Using the drawing as a guide, try building the structure. How successful were you? Did you estimate your needs correctly? Did you over- or under-estimate? What refinements should be made in the drawing or the structure itself?

Demolition: The Controlled Demolition team must place its explosive charges very carefully to make sure the Newhouse Hotel falls just the way they want it to.

- Using a large number of dominoes, plan how you would arrange the dominoes so that by pushing one over, all the rest would fall over in sequence. Make up a pattern on paper first. It can be as complicated as you want, with twists and turns and even ramps where dominoes would fall a distance to set off a chain reaction below. Now try out your plan. Does it work the way you planned? If not, what adjustments would you make?

- In a similar exercise, experiment with a house of cards. See if you can figure out, by making a drawing first, how you might make the house of cards fall straight down instead of tumbling to one side or the other.

DOMINO THEORY: Setting up the thousands of dominoes in this display, and getting them to fall as planned, takes a clear picture of the intended effect. . .and patience.

Outcomes With Class

Making Up Your Own Problems

This hypothetical dialogue between teacher and students provides examples of how to draw students into the process of devising and solving their own outcomes problems.

"

♦ ***Can anyone think of a problem that involves outcomes? No? Let me put it another way. . . Is there anything coming up that you really want to do? Something that involves making plans?***

I want to watch a rock concert on TV tomorrow night, but my parents say I have to finish all my chores first.

♦ ***And you're thinking you might not be able to do that?***

Tomorrow's Thursday. I have a lot of things to do on Thursday.

♦ ***Good. I mean, it's a good outcomes problem. One of the ways we can solve it is by working backward from the goal to the beginning. So. . . What time does the show start?***

Nine o'clock.

♦ ***And what do you have to do before your parents will let you watch it?***

Well, I have to do my homework, do the dinner dishes, practice the violin, babysit while my parents go to a lecture or something. . .

♦ ***Whoa! Wait a minute. This is complicated. Tell you what—let's start by making up a chart. Why don't you come to the blackboard and write down all the things you have to do and about how long each one will take?***

Job	Time Needed
dinner dishes	15 minutes
homework	2 hours
pick up dry cleaning	20 minutes
babysit	45 mintues
practice violin	1 hour
return Margie's bike	20 minutes

♦ ***You do have a problem. But let's try to solve it. But we have a little more to do with this chart first. You have to pick up the dry cleaning. What time does the store close?***

Six o'clock.

♦ ***That means you have to do it before then. And you have to do the dinner dishes. But don't you have to eat dinner first?***

I forgot about that.

♦ ***Let's get that on the chart, too. We'll add a separate column for specific time requirements, where you know them.***

Job	Time Needed	Time Restrictions
pick up dry cleaning	20 min.	before 6:00
eat dinner	30 min.	6:30–7:00
dinner dishes	15 min.	7:00–7:15
homework	2 hours	
babysit	45 min.	8:15–9:00
practice violin	1 hour	
return bike	20 min.	

♦ ***Okay. Now we'll start at nine o'clock and work backward. You have to sit with the baby from 8:15 to 9. Couldn't you do something else at the same time, such as your homework, or practicing the violin?***

I can't practice the violin, because the baby cries whenever I start to play—I don't blame her.

"Working backward can be helpful because problems are often easier to solve in one direction than another."

John R. Hayes, *The Complete Problem Solver*

♦ ***All right. But you can do some of your homework. Now you have an hour before your parents go out. More homework? That means you've done how much homework?***

I did 45 minutes plus 60 minutes. I still have about 15 minutes to go.

♦ ***That's all right. We'll get that in somewhere else. Now we're back to 7:00, and that's when you'll do the dishes, right?***

And we eat from 6:30 to 7:00. . . this is getting awfully close.

♦ ***Let's finish that homework now, shall we? This takes us back to 6:00, when you'll be on your way back from the dry cleaner.***

So I'll leave for the dry cleaner at 5:25. . .when I finish practicing the violin!

♦ ***Good. Where does that leave us?***

At 4:30. That's when I get home from school on Thursdays after band practice.

♦ ***Perfect! So it looks as though you'll be able to watch the concert after all. Now let's put it all together. This will be your "road map" through the day.***

Schedule for Thursday

go home from school	4:00
practice violin	4:30
pick up dry cleaning	5:30
homework	6:00–6:30
eat dinner	6:30
do dishes	7:00
homework	7:15–9:00
watch the baby	8:15–9:00
concert	9:00

♦ ***You've even built into your schedule a little extra time for your homework.***

Wait a minute! We forgot about Margie's bicycle! I told her I'd return it tomorrow, and she lives all the way on the other side of town.

♦ ***I don't see any extra time, do you?***

I'll have to do it before I go to school. Oh, no! That means I'll have to get up at 5:30 in the morning!

♦ ***I hope the concert is worth it. Maybe you'd better start planning for tomorrow tonight and get a good night's sleep. I think you're going to need it.***

3 Cats
1 box of food for each cat every week
$1.37 per box
How much money for 1 year?

Scott Troyanek
Walled Lake Junior High School
Walled Lake, Michigan

On the Lighter Side

If "Bill the Sign Painter" had an outcomes-oriented approach to problem solving, he never would have painted himself into a corner.

Bill is a capable—albeit literal—artist, as his work shows. The problem is he doesn't take the desired outcome into account. So, in each picture, Bill applies his technical proficiency toward the wrong end. He paints sandwiches of the wrong size, the wrong perspective and, finally, no sandwich at all, but an empty plate. Sometimes the means to the end become the end itself.

A Comedy of Errors: The Sandwich

● *The Goal:* A picture of one of Sam's Super Sandwiches to complete a billboard.

● *Wrong Turn #1:* A perfect picture, but much too small.

Bill—The Burger King

■ One of Sam's Super Sandwiches is 6 in × 6 in × 4 in. What is the volume of the sandwich? If a model were constructed with a volume 8 times that of the Super Sandwich, how tall would it be? What if it had a volume 27 times that of the Super Sandwich?

■ Bill has been hired to paint a billboard for Harry's Hamburgers. He's learned his lesson from Sam, so he decides to paint the hamburger's top view as many times as possible on a sign. If the painted burger has a radius of 1 ft, find the number of hamburgers he can paint on a 16-ft × 20-ft sign. (*Hint:* the hamburgers touch each other.) What area of the sign is not burgers? Use $\pi = 3.1416$.

● *Wrong Turn #2:* The picture's the right size, but the perspective's wrong.

● *A Surprise Finish:* The picture isn't exactly what Sam had in mind, but he has to admit Bill's arrived at the desired outcome—attracting customers.

■ Bill decides to paint the burger from a side view. What is the scale of Bill's sign if the real burger is 1.5 in tall and the painted one is 9 in tall? 2 ft tall?

■ The hamburgers need to be packed into a box a half-dozen at a time. The dimensions of each hamburger are as follows: radius, 2 in; height, 1.5 in. What size box is needed for the half-dozen?

■ How many burgers will fit into boxes of the following dimensions?

- 1 ft sq
- 8 in × 8 in × 9.5 in
- 5 in × 5 in × 8 in
- 4 in × 4 in × 9 in

Hatching a Giant Egg on a Computer

"We didn't know what was wrong—our math, our geometry or the program we were using. There was no way at all for us to determine even whether building the egg was geometrically possible."

When computer artist Ron Resch set out to generate a computer model for a "monumental" chicken egg, he figured it would be an easy job. "I thought if I took two domes and maybe wrapped them around into a kind of barrel with a bulge in it and tied the three structures together, I would have an egg," he explains. With such a clear outcome in mind, he never suspected that it would take a year for him and graduate assistants Robert McDermott and Jim Blinn to develop the pioneering mathematical formulas and sophisticated computer technology necessary to build the egg.

The Vegreville, Alberta, Chamber of Commerce had commissioned the giant Easter egg in 1973 in order to celebrate the 1974 centennial of the Royal Canadian Mounted Police. The egg was chosen to honor the Ukrainian settlers of the region, who traditionally decorate eggs with geometric patterns at Eastertime.

But the town's ambitions presented many practical problems: Could a giant egg be constructed within their modest budget? Could it be made light enough not to sink into the ground? Could it be constructed of material that could stand up to the rough Canadian winters without requiring expensive maintenance?

HARD SHELL: The egg begins to take shape. To figure out how to engineer that shape, computer artist Ron Resch spent six frustrating months grappling with the geometry of the irregularly curved surface before scrapping everything for an entirely new approach—interconnecting equilateral triangles.

The solutions to the last two problems came from Permaloy Enterprises, a Utah firm that anodizes a lightweight aluminum which requires no maintenance for at least 75 years. The actual design problems proved more difficult to solve. Two architectural and engineering firms took on the commission, only to abandon it when their standard geometrical approaches could not be applied to the egg's natural form. Then someone at Permaloy recommended Ron Resch, at that time a professor of computer science at the University of Utah, whose talents and experience in art, design and computer technology seemed perfectly suited to the task.

Defining Geometry

Resch thought that only small changes would be needed in his existing computer simulations to make them conform to an egg shape. After six frustrating months of work, he, McDermott and Blinn were forced to abandon these optimistic assumptions. "We didn't know what was wrong," Resch recalls, "our math, our geometry or the program we were using. There was no way at all for us to determine even whether building the egg was geometrically possible."

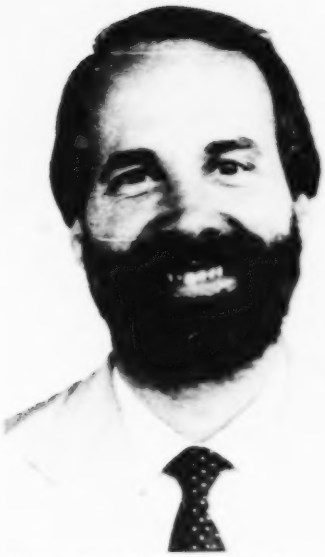
Although the original grant had been spent and the centennial past, Resch did not abandon the project—or the puzzle. "I threw out all the existing computer programs, and then undertook what I never would have done in the beginning," he says. "That was to start with no tools whatsoever, just a problem and some experience with geometry and say 'I have to develop mathematical models for that geometry.' "

Resch's solution was to simplify his original system, which was based on folded paper, by eliminating the coffers, or crevices, and leaving only a subset of the system—equilateral triangles.

Follow that Curve

The problem still wasn't solved for Resch, however. In order to follow the changing curve of an egg, he added a second design, the three-pointed star (an equilateral hexagon composed of three isosceles triangles whose bases form a fourth interior triangle). The design stays constant except for the dimensions of these interior triangles, which change to make the stars narrower or wider. This pulls the design around the egg's curves. The sides of the equilateral triangles are all 12 inches long, as are the sides of each point of each star; thus every joint in the egg is also one foot long. This uniformity distinguishes the Vegreville egg mathematically: According to Resch, it may be the only structure ever built using a constant module on a three-dimensional surface.

HONORING TRADITION: Vegreville's large Ukrainian community follows the time-honored custom of decorating Easter eggs with traditional patterns. The design of the giant egg is an authentic Ukrainian pattern.

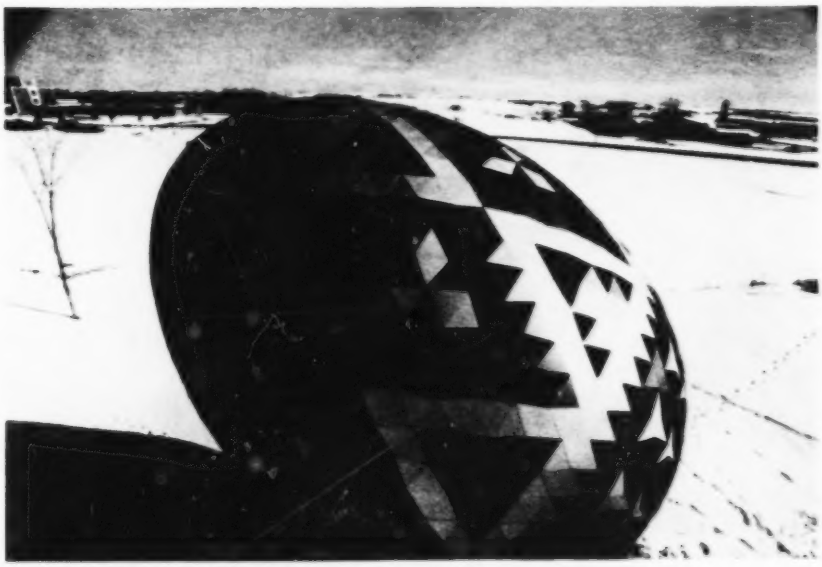


Computer artist Ron Resch.

Nuts and Bolts

"It was only after we defined the geometry that I worked with a Ukrainian pattern expert," he recalls, "and then we worked a pattern into the existing geometry." Resch used a computer-controlled plotter for the delicate and arduous task of cutting the 3,512 separate triangles out of sheets of aluminum. Then, with the help of the citizens of Vegreville, he assembled the egg on the site with 6,978 nuts and bolts, though no connections are visible on the egg's exterior.

The 2,000-pound exterior is supported by a 3,000-pound interior system of 177 struts that look like an open umbrella. Balanced precisely on its 27,000-pound steel-and-concrete base, with a ball bearing between the egg and its base, the colorful 31-foot-tall and 18-foot-wide Vegreville Egg monument now turns gently in the wind. ■



THE ENORMOUS EGG: Held together by 6,978 nuts and bolts, the Vegreville Easter Egg weighs 5,000 pounds, rests on a 27,000-lb. base and is carefully balanced so it turns gently in the wind.

Further Readings

Dixon, Robert. "The Drawing-Out of an Egg." *New Scientist*, 29 July 1982.

Dixon describes how the shape of different types of eggs relates to the lifestyles of the birds that lay them. He also discusses the geometry of an egg, and shows in simple diagrams how to construct an egg using a compass and a straight edge. Interesting and accessible to junior and senior high-school students.

Gardner, Martin. "Fun with Eggs: Uncooked, Cooked and Mathematical." *Scientific American*, April 1980.

In this *Mathematical Games* column, CHALLENGE advisor Martin Gardner discusses the geometry of the egg, and provides tricks and exercises that divulge its physical properties. He describes how an ellipse and some Cartesian ovals can be constructed with two pins and a piece of thread, as well as how to balance an egg with a cork, a bottle and two forks. Entertaining and informative.

Hutchinson for Leapfrogs. *Tiles*. London: Hutchinson & Company, 1976.

A part of the *Network Mathematics* pamphlets series, this 16-page booklet provides a range of activities and exercises for junior and senior high-school students. Starting with squares and rectangles and progressing to triangles and hexagons, it introduces the idea of covering a shape with smaller geometric shapes. Also in this series, *Cubes* and *Systems* contain activities that relate to the egg segment in this film.

Tessellating an Egg

The basic mathematical problem that faced Ron Resch in the construction of the Vegreville Egg was that of tiling or *tessellating* a curved surface.

In the purest mathematical sense, a tessellation is the repeated use of a shape, such as a polygon, to fill a surface without gaps or overlapping.

Tiling can also be thought of as completely covering a given portion of a surface with a given set of polygons. For example, kitchen floors and counters are often tiled with squares or hexagons, or with a combination of polygons.

In tiling a *flat* surface, the sum of the angles around any given point must be 360°. It is possible to tile the plane with an equilateral triangle (in fact with any triangle) because the sum of the angles of a triangle is 180°. Similarly, regular hexagons (which are composed of equilateral triangles) will tessellate a plane (see Figs. 1 and 2).

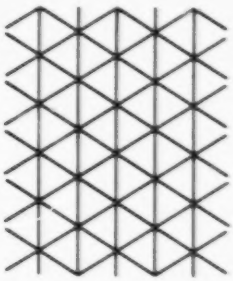


Fig. 1

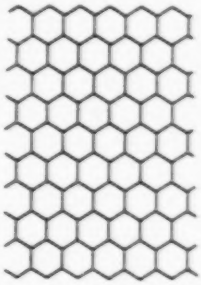


Fig. 2

In tiling any *curved* surface, the sum of the angles around a given point must be slightly greater than 360°. Working with a shape such as an egg is further complicated because the curvature of the surface of the egg is constantly changing; therefore, the sum of the angles around any given point must also change.

Ron Resch solved this difficulty by thinking about the egg as though it were tessellated with equilateral hexagons and equilateral triangles. Although the lengths of the sides of the hexagons and equilateral triangles remained the same, the vertex angles of the hexagons were increased slightly in each successive row to accommodate the changing curvature of the egg.

Figures 3 and 4 show the plane tessellated with equilateral hexagons of two different, but constant, shapes. The lengths of the sides of the hexagons in these two figures are the same, but the size of the angles is slightly different in each figure. Similarly, in the egg in Figure 5, each row contains hexagons having sides of equal lengths, but angles of slightly different sizes.

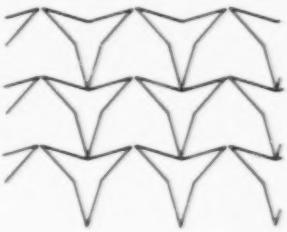


Fig. 3

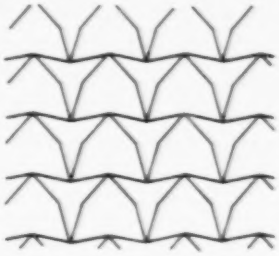


Fig. 4

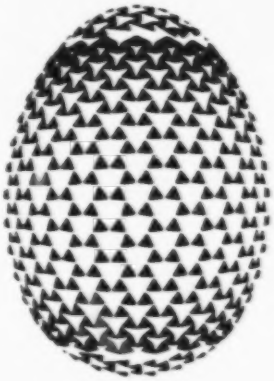


Fig. 5

Exercises:

1. Construct a regular (equilateral) triangle, square and hexagon, each having sides 1-in long. Using these polygons as patterns, make at least 12 tiles of each polygon.

   a. Try to fit two squares and three equilateral triangles together so that they share a common point. Try to make your pattern cover the plane. Compare your pattern with that of others. Sketch your results. Are all patterns the same?

   b. Do the same with (1) two triangles and two hexagons; (2) four triangles and a hexagon; and (3) a triangle, two squares and a hexagon. Again, sketch your results.

2. Construct a regular octagon (8-sided figure) and a regular dodecagon (12-sided figure) each having sides the same length as the figures you made in Exercise 1. Again, make several tiles and try to tessellate the plane using (1) octagons and squares; (2) dodecagons, triangles and squares; and (3) hexagons and dodecagons.

There are 3 "regular" tessellations (those in which only 1 regular polygon is used), and there are 8 "semiregular" tessellations (those in which more than one regular polygon is used). Did you find them all?

3. It is not possible to tessellate the plane with a regular pentagon since each angle is 144° and small gaps will be left if pentagons are put together. Figures 6 and 7 show interesting patterns of tessellations of pentagons "with gaps." Construct a regular pentagon and make your own tessellations of regular pentagons with gaps.

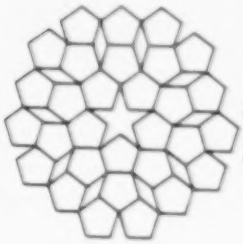


Fig. 6

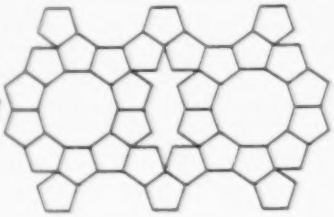


Fig. 7

4. Figure 8 shows a tessellation of the plane by a nonregular pentagon. Construct the pentagon that will result in such a tessellation. (*Hint:* Notice that there is a [nonregular] hexagon formed by the sides of the pentagons. These hexagons are, in turn, formed by rhombuses. Notice also that the sides of the pentagon are perpendicular to the sides of the hexagon.)

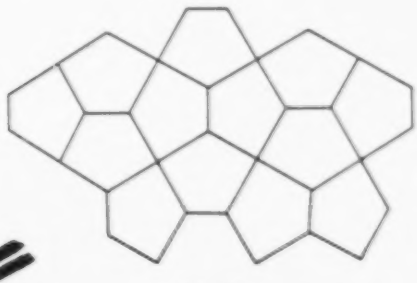


Fig. 8

Tessellating with a Computer

This exercise requires use of a computer with the LOGO programming language.

(These procedures were written in Apple LOGO.)

```
TO SUPERHEX :SIDE :ANGLE
HT PU SETPOS [0 30] PD
IF :ANGLE = 50 [STOP]
HEXAN :SIDE :ANGLE
CLEARTEXT
WAIT 120
CS
SUPERHEX :SIDE :ANGLE+2
END
TO HEXAN :SIDE :ANGLE
REPEAT 3 [HEXA :SIDE :ANGLE LT (60 - :ANGLE)]
END
TO HEXA :SIDE :ANGLE
RT :ANGLE FD :SIDE
RT (180 -2*:ANGLE) FD :SIDE
END
```

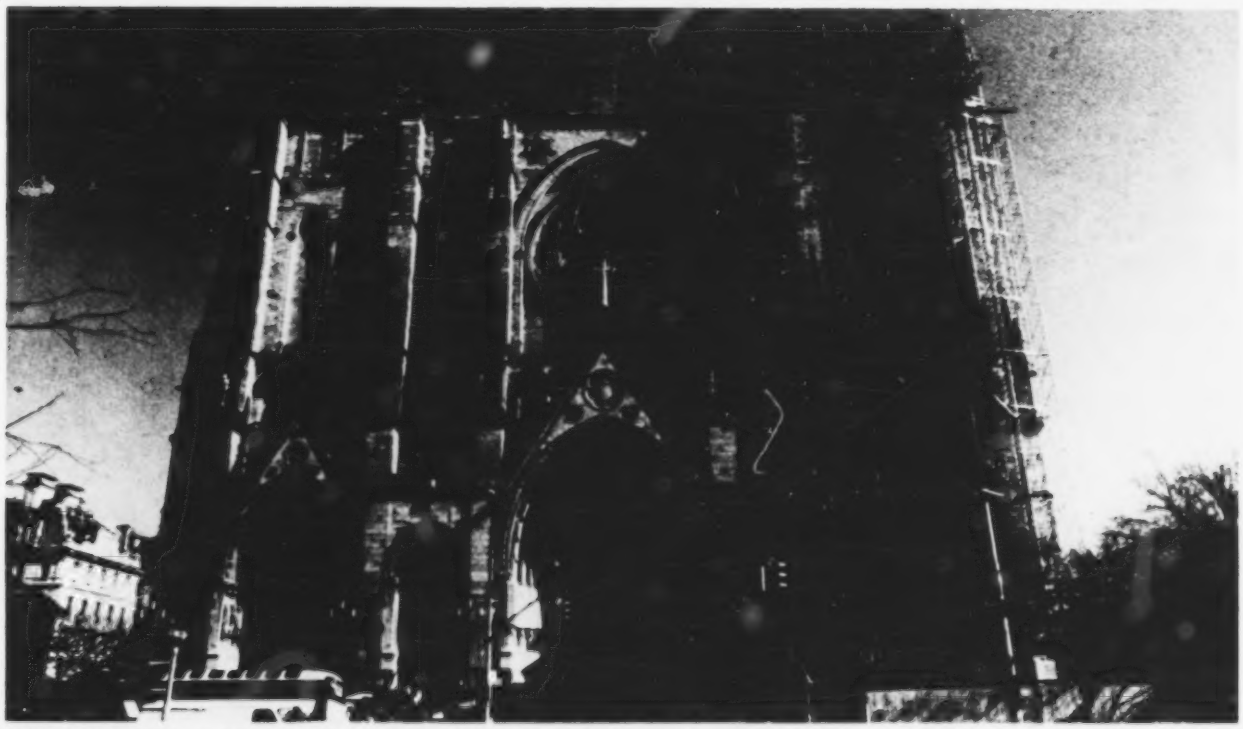
Run the procedure SUPERHEX using any length as input for :SIDE and 2 as input for :ANGLE. This procedure illustrates the formation of a hexagon with sides of constant length and constantly changing angles.

Write a procedure that will tessellate any of the hexagons formed by the procedure HEXAN on the plane of the screen.

ELECTRONIC EGG: Resch's computer model of the Ukrainian Easter Egg, its pattern created by tessellating equilateral triangles of different colors across the surface.

Building a Cathedral: A Gothic Tale

"It should all come together like a giant jigsaw puzzle."



STILL BUILDING: At New York City's Cathedral of St. John the Divine, construction marches along generation after generation. Begun in 1892, the cathedral won't be finished until about 2045.

When you walk into the sanctuary of the Cathedral Church of St. John the Divine in New York City, you find yourself within a man-made mountain. Building such a powerful structure requires vision and discipline, an eye trained to dimensions and proportions and lots of patience.

Construction of the world's largest Gothic cathedral began in 1892. Now, almost 100 years later, there are at least 60 years of work left to go.

For master builder James Bambridge, master stonemason Alan Bird and 12 workshop apprentices, the goal since 1979 has been to finish the two Gothic-style towers and a gallery on the cathedral's west front. Approximately 24,000 stones are needed for the job. Each stone must be individually designed and hand-carved before it is hoisted to the tower and set in place.

Very few cathedrals are the product of a single master builder or plan: In this way, the Cathedral Church of St. John the Divine follows tradition. Before he died in 1942, architect Ralph Adams Cram made some drawings at a scale of 1/8 inch to 1 foot, or 1:96, for the two front towers. These drawings provided a general outline for the work to be done, but Cram's designs were never converted into actual working drawings, which are done to a scale of 1/2 inch to 1 foot, or 1:24. This allows each individual stone to be numbered and detailed sufficiently for full-size reproduction.

Cram died in 1942, shortly after World War II brought a long-term halt to work on the cathedral. When work resumed in 1979, Bambridge hired John Doran, the only surviving architect from Cram's old firm, to make the working drawings he needed to complete the towers. Doran was able to reconstitute only five of Cram's drawings before he died in late 1979. As a result, Bambridge has taken on the task of rescaling the rest of Cram's drawings himself, in addition to overseeing the cutting and setting of the stones for the towers.

Breaking It Down

Working toward a clearly established outcome, Bambridge breaks down the overall task of completing each tower into workable subparts: drafting the working drawings; making full-scale molds or patterns for each stone; cutting and setting the stone.

The apprentices who do the actual stonework are responsible for translating Bambridge's two-dimensional drawings into three-dimensional stones. To ensure that every detail is carried out correctly, two apprentices take Bambridge's drawings and render them to full size. Another apprentice then slips a sheet of zinc under the drawing and, using a metal stylus, taps through the paper along the lines to punch small holes in the zinc. The templates (molds or patterns), which guide the actual cutting of the stones, are then cut from the zinc. A zinc template is made for each face of each stone.

The *template* is like a dress pattern, with the punched lines indicating work to be done on the stone. Using the templates, stonecutters trace the outline of each face before they actually begin cutting. Precision is essential: The mortar allows only 1/16th of an inch leeway.

STONE CUTTING: An apprentice stone cutter (above) makes a zinc template following the architect's drawings. This template will be used to guide the cutting of the stone. For each face, another template.

PASSING ON A TRADITION: Master builder James Bambridge (left) supervises the construction. The project will continue into the next century, but he must always keep the outcome—the finished cathedral—firmly in mind, even though it will be his assistants who will see to its completion.

Blockbusters

Alan Bird, master stonemason, is the one who determines how and when each stone should be cut. When the 10-ton blocks of raw limestone arrive, Bird must make several critical decisions. First, he asks himself, "Which stones can be cut now?" Time and resources will be wasted if, for example, cutters prepare stones for the sixteenth row when the masons are only working on the third row or if only unskilled apprentices are available to work on stones requiring elaborate carvings. Since limestone is a sedimentary rock, it is important to cut along a natural bed or layer in the rock which will then be set so the rock's layers are parallel to the ground to prevent flaking.

Finally, Bird asks himself, "Which stones need to be cut in order to keep everyone occupied?" There are three work groups: those who cut the large stones with machine saws, those who carve standard nonornate stones with mechanical gougers and those who use chisels and hammers for exacting handcarving. It takes from one to two weeks to carve each stone. Setting the stones is much like bricklaying. After a stone has been hoisted up onto the tower scaffolding, Bambridge and Bird make sure it is placed in its proper orientation as set out by the working drawing.

It is an enormously complex endeavor to build a Gothic-style cathedral tower. Knowing the outcome is just the beginning. But as Bird says, if everyone does his or her job well, "it should all come together like a giant jigsaw puzzle." But what's different about this jigsaw puzzle is that few, if any, of those who are working on putting its pieces together today will live to see it completed. They must be satisfied with being part of the history of the solution. ■

Further Readings

Cohalan, Michael. "Tension in the Cathedral." *Science 81*, December 1981.

This article describes engineering experiments that have determined how the trademarks of Gothic design—ribs in Gothic vaults, flying buttresses, pinnacles—keep a cathedral standing. It includes a sidebar on the Cathedral Church of St. John the Divine.

Donhauser, Peter L. "The Rebirth of a Medieval Craft." *Museum*, November/December 1982.

This article gives additional background on Alan Bird and James Bambridge, the two men featured in this film segment, as well as a useful summary of the history of the building and rebuilding of the Cathedral Church of St. John the Divine.

Leedy, Walter C., Jr. "Fan Vaulting." *Scientific American*, February 1983.

Excellent article describing the purposes for and construction of Gothic fan vaulting. Good line drawings; a nice supplement to the *Science 81* article.

Wickersham, The Rev. George W., II. *The Cathedral Church of St. John the Divine.* New York: C. Harrison Conroy Co.

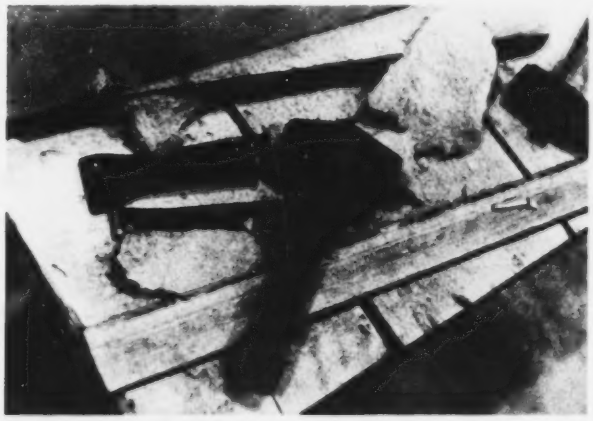
This beautifully photographed, 60-page booklet describes the history of the Cathedral, and gives a complete "walking tour" (with lots of photos) through the Cathedral and its grounds. It also contains a chart of the dimensions of the Cathedral, compared with other European cathedrals. (Available from the Cathedral Church of St. John the Divine, 1047 Amsterdam Avenue at 112th Street, New York NY 10025.)

Cutting It Down to Size

When Alan Bird, master stonemason at the Cathedral Church of St. John the Divine, decides which stones to cut from 60 cubic feet of limestone, he is concerned with three mathematical concepts: space-packing principles or relationships; the minimization of the volume of wasted materials; and geometrical dissections.

Bird's task is similar to trying to pack clothes into a suitcase or books into a cardboard box. In the case of a box, it is the box that has a fixed volume—you must fill it selectively with books of varying sizes. Since the combined volume of books may be greater than the volume of the box, decisions must be made as to which books to pack and which to leave behind. In very rough terms, each attempted arrangement or packing is a geometrical dissection of the box's volume. The quality of your packing effort can be judged by how much wasted space there is.

The major difference between your packing efforts and the stonecutting efforts of the master stonemason is that you can try again if you are unhappy with the results (books left out that you want to include) or the amount of wasted space (space into which none of the remaining books will fit). If Alan Bird's calculations or estimations are wrong, he has wasted an expensive piece of limestone and valuable time.



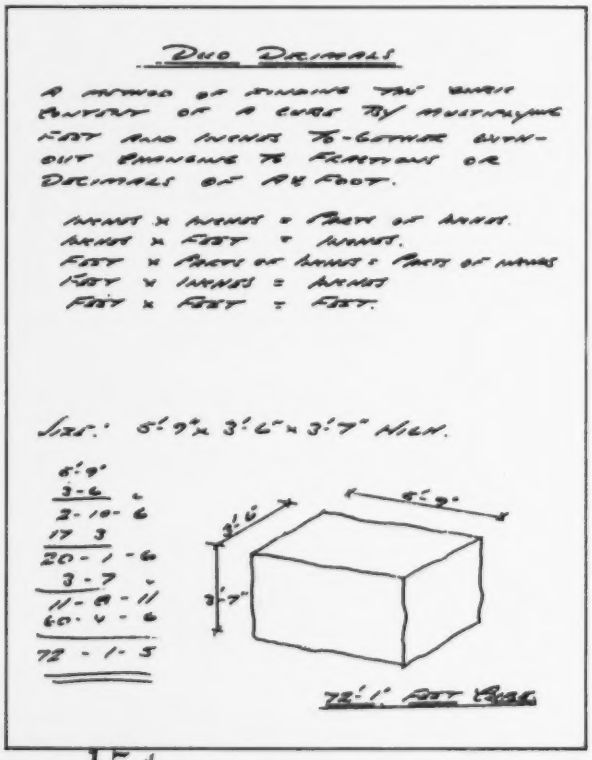
TOOLS OF THE TRADE: Above, a stone cutter's tools. Right: Over the centuries, stone masons and master builders like Bambridge have developed a shortcut to multiplying feet and inches by feet and inches. See if you can figure out how they do their "uncommon" denominator multiplication based on Bambridge's notes. It helps to know your "12-times" table *very* well.

Wasted Space

Cutting a given length of rope into desired segments is also similar to Bird's task. If the rope is 12 meters long and you want to create as many 5-meter lengths as you can, simple calculations using division reveal that you can create two 5-meter lengths with a waste factor of 2 meters of rope. In contrast, if you want to create 4-meter lengths, there will be no waste.

Mathematicians study a related problem when they investigate different ways in which spheres of identical sizes can be piled or packed. Wasted space is inevitable, yet it can be minimized. You can do some investigations of sphere-packing, rope-cutting and box- or suitcase-packing on your own, with the task always being to minimize wasted space while filling the space with what you need.

Though these problems may seem hypothetical, the related mathematics do have "real-world" applications as any traveler or package designer can attest to. In addition, research scientists use similar techniques to understand the building blocks of matter such as crystal structure, and artists use geometric patterns to create mosaics or wallpaper and floor-tiling designs.



Exercises:

1. Suppose you have a circular disk with a point placed at a random location in the disk's interior. Your task is cut off as large a section of the disk as possible using a single straight cut which must pass through the given point. Investigate different strategies using the different situations given in Figures 1–3.

Fig. 1 Fig. 2 Fig. 3

a. Using Figure 1, what is your conclusion if the random point happens to be the center of the disk? Using Figures 2 and 3, can you establish a general procedure for making the *best* cut? If a hint is necessary, first draw the diameter of the disk which passes through the random point.

b. Repeat the task and investigations using a square or rectangle in place of the circular disk. Can you generalize your cutting strategy to any polygonal region or even to any convex region? Figures 4–6 will get you started:

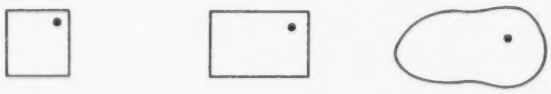


Fig. 4 Fig. 5 Fig. 6

2. *Soma Cubes* are a recreational puzzle invented by Piet Hein while listening to a lecture on quantum physics. The 7 pieces represent the 7 irregular shapes that can be created using no more than 4 cubes, with the sole restriction being that they join at their faces. A set of inexpensive Soma Cubes can be constructed using glue and some 27 small wood blocks or sugar cubes.

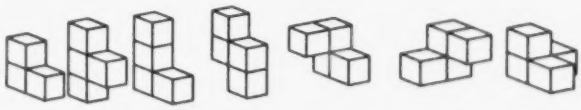


Fig. 7

a. Since a total of 27 cubes are used to make the 7 shapes, can you discover how the shapes can be assembled to form a solid $3 \times 3 \times 3$ cube? That is, the shapes represent a possible dissection of the larger cube.

b. You have a 3 in $\times$ 3 in $\times$ 3 in cube painted red on all 6 faces. You want to cut this into 1 in $\times$ 1 in $\times$ 1 in cubes.

1. How many cubes are there?
2. What is the minimum number of cuts it will take regardless of the rearrangement?
3. How many cubes have 4 painted faces? 3 painted? 2 painted? 1 painted? None painted?

(**Note:** This exercise can be easily modified to use either tangrams, pentominoes and polominoes. The surface to be packed could be designated portions of a chessboard.)

"Do not start at the beginning and follow systematically step by step to the end goal... try to work backward to find the solution path to the problem....This way, not only is the answer obtained, but you derive a new path to a solution. The solution path is as important as the answer and, in problems where the goal is specified, the path is the solution."

Moshe F. Rubinstein, *Patterns of Problem Solving*

Bringing Down the House

"We just set it into free fall and then utilize the weight and construction of a structure to pull itself apart."

In the 1940s, Jack Loizeaux began his pioneering career by studying the differences between explosive demolition and the conventional wrecking ball. He concluded that explosive demolition is generally faster, safer and more convenient. When demolition takes place over a period of seconds rather than months, it is much easier to assure that no one will be in or around the building being demolished. It is also easier to protect nearby buildings by sealing doors, windows and air systems with tape to keep the dust out. The fact that explosive demolition is quicker means that it can also be cheaper, since fewer workers are involved, and demolition cleanup usually takes a matter of hours, not weeks or months.

The demolition of the Newhouse Hotel, standing at Fourth and Main in Salt Lake City, Utah, took place on a Sunday. Jack Loizeaux explains: "We normally blast on Sunday mornings because that's when pedestrian and vehicular traffic is at a minimum. And that gives us an entire day to clean up. . ."

With his sons Mark and Doug, Loizeaux runs the family company, Controlled Demolition, Inc. They use "a little bit of explosive and a lot of gravity to pull a building down." Carefully placed explosives literally kick a building's feet out from under it by shattering the structural members that hold it up. "We just get it into a free fall," he explains, "and then utilize the weight and construction of a structure to pull itself apart."

QUIET EXPLOSION: The 12-story Newhouse Hotel in Salt Lake City comes tumbling down—neatly and safely, thanks to the careful planning of CDI, the demolition company. CDI uses the building's stored energy, gravity, 900 sequenced explosive charges and a great deal of math to blow down the building without blowing up the neighborhood.

A Matter of Timing

Although the actual demolition lasts only a few seconds, it sometimes takes weeks of preparation. The Loizeauxs begin by analyzing the outcome they want. Engineers help them plan the amount of explosive required and the locations for the charges. Particularly in a downtown area, it is not enough just to bring the building down. It must come down within strict limits so as not to damage surrounding structures or streets.

To do this, the explosives are placed and timed to go off in an exact sequence at precisely timed intervals. They must all be wired together; even the wire connecting them is carefully measured to ensure the proper electrical resistances and precise delay. "When we press our button," Loizeaux says, "[all the charges] go off only milliseconds apart."

After the first charge "kicks out" the supports of the lowest floor, the ceiling above—with nothing to hold it up and a great deal of weight pushing it down—begins to cave in and weakens the floor above. At this critical instant, other supports are eliminated, removing the floor's last resistance to gravity's pull.

The manner in which a building was constructed determines how it will be demolished. Reinforced-concrete buildings are relatively simple to bring down, because they are very heavy and very strong; ironically, their strength actually helps during demolition.

Buildings with a skeleton or frame of steel beams, like the Newhouse Hotel, present a different set of problems. When the supports are blown out on a steel-frame building and the floor begins to sag, the rivets suffer more stress than they were designed to resist. When the floor beam moves as much as 15 degrees, the rivets

CDI's Jack Loiseaux.

usually fail — they are sheared through by the leverage of the falling floor—and the beams tear loose, leaving the column standing upright by itself. This presents the Loizeauxs with a problem: The column could fall at any time, with results difficult to predict.

A Neat Pile of Rubble

The Newhouse Hotel is an L-shaped building, 12 stories tall with masonry exterior walls, held up by structural steel columns and beams. Its perimeter walls pose the biggest problem to safety because they tend to collapse outward.

The Loizeauxs' solution to the problem of pulling down the columns is to strap the building with 3/4-inch steel cables. "We attach the cables to the outside columns and tie them around the next row of columns," he says. "Then, when we remove the support of the second row of columns by blasting, the weight is transferred to the cables. This pulls the outer walls in on top of the pile."

Loizeaux estimates that each 11-foot story will be condensed into about 2 feet of rubble after the blast, because a building is typically 15 percent structural material—the rest is space.

When the button was pushed on the Newhouse Hotel, there began an elaborate and sustained series of explosions: 900 individual charges, organized into carefully planned groups, went off in a precisely timed sequence, milliseconds apart. The old hotel shuddered briefly, then seemed to freeze in space. After a few seconds, it began to sag, then sink, then crumble—caving in on itself. Less than 12 seconds later, the Newhouse Hotel had become a large cloud of dust and a neat pile of rubble. ■

Further Readings

Gordon, J.E. *Structures: Or Why Things Don't Fall Down*. New York: Da Capo Press, 1978.

An excellent and witty introduction to basic engineering principles, this book describes the theory behind structures and their materials. Bridges, beams, flying buttresses, metal fatigue, joints, fastenings and design are all defined and analyzed. Plenty of line drawings.

Hellman, Hal. "The Demolition Family: Bringing Down the House." *Geo*, October 1979.

Interesting article about the Loizeaux family and their company, Controlled Demolition, Inc. (CDI). The article gives background on the development of CDI's engineering technique and information on how that technique can help reduce a 12-story building to a "neat pile of rubble."

Doug Loiseaux oversees the demolition of the Newhouse Hotel.

Support Systems

Until the late 1800s, the walls of buildings supported the floors and roof above them. The height of a building was limited by the weight which the walls would support; the higher the building, the more weight the walls had to bear so the thicker they had to be. The granite walls of some of Europe's cathedrals are over 20 feet thick so they can support the massive structures above them.

In 1883, a new concept emerged called "skeleton construction," and the world's first skyscraper was built (the Home Insurance Building in Chicago). It was 10 stories tall. In skeleton construction, a frame, or skeleton, of steel girders and beams supports the weight of a building. Today, most buildings over four stories tall have a skeleton of steel.

Exercises:

1. Use 25 straws. Thread them together with wire or string and build a structure. Compare the structures built by different students. Which are more sturdy than others? What shape is the most rigid? Would that always be true? Why?

2. Look at Figures 1–4. Assume they are sides of cubes.

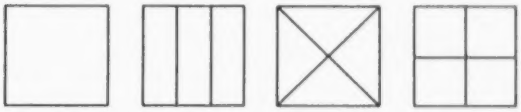


Fig. 1 Fig. 2 Fig. 3 Fig. 4

a. Which figures can change their shape while maintaining the lengths of the sides and supports as well as the points of attachment?

b. How does this ability affect the strength of the figures?

3. Look at Figures 5–8.

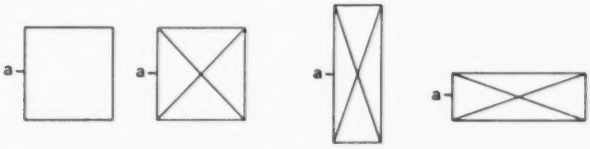
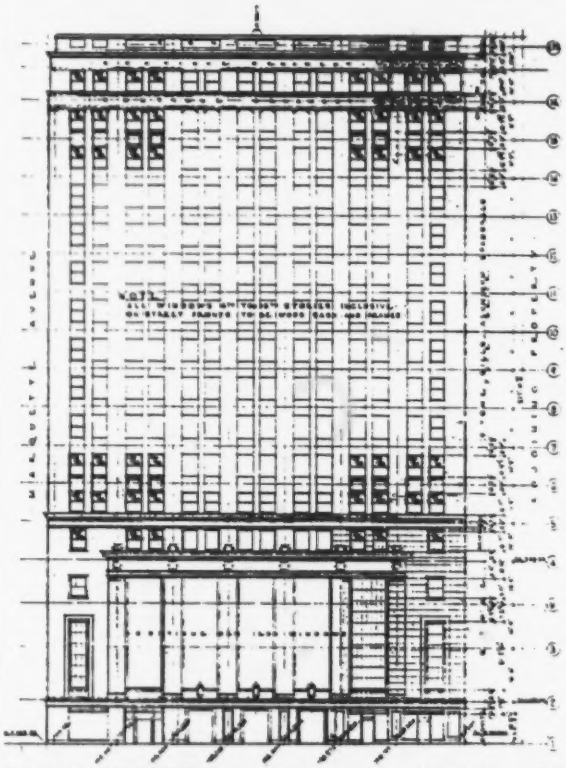


Fig. 5 Fig. 6 Fig. 7 Fig. 8

a. Which box would support the most weight?

b. If support *a* were cut, which box would hold the most weight? Why?

c. What effect does the length of the supports and the angles at which they are joined have on the strength and stability of the figures?



DEMOLITION TOOL: In preparing for a demolition, CDI starts with a thorough analysis of the building, right down to original architect's blueprints. Each building has its own structural weak points; taking advantage of them with specially timed and placed explosive charges is an integral part of the strategy.

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Vectors

There is a branch of physics that gives specific answers to questions about what structures can support the greatest weight and why. This branch is called *vector analysis*. Vectors are formed using a combination of arithmetic, algebra and geometry, and are critically important to those whose job it is to plan the building—or demolition—of skyscrapers, bridges, roadways, etc.

Using the theory of vectors, we can predict which structures will hold up the best and why. In Figure 1, for example, when the support *a* is cut, a triangular support remains. The force pushing down on the structure is distributed down the hypotenuse (*h*) of this triangle. The amount of weight *h* must support, and the amount that is supported by the remaining side (*j*), is determined by the angle at which the supports are joined. The smaller the angle *e*, the more weight the structure will support because more weight is taken by side *j*.

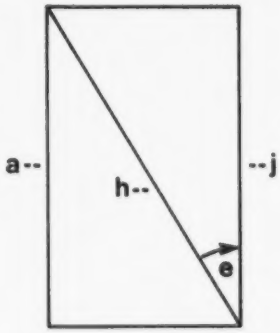


Fig. 1

The small triangles above Figures 2 and 3 are vectors: They diagram the distribution of the forces on the structures. The length of a vector is the amount of force it represents. Side *z* of the vector triangle represents the total weight pushing down on the structure, while side *y* depicts the amount of force pressing on side *h* of the structure. (Line *y* here is the perpendicular from point *a* to the hypotenuse.) You can see that vector *y* is shorter for Figure 2 than for Figure 3. What does this mean about the weight each figure can support?

Angle *e* of Figure 2 is also smaller than the same angle of Figure 3, and the angle of the vector triangle is the same as that of the figure to which it relates.

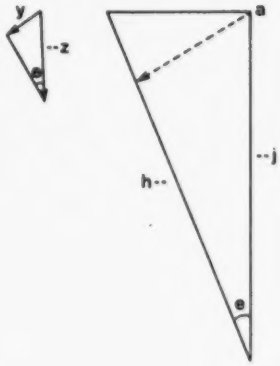


Fig. 2

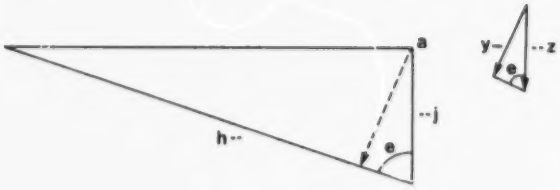


Fig. 3

(For a series of simple exercises on vectors, see page 115.)

Water, Water Everywhere

You have an unlimited supply of water and have been asked to obtain a precise quantity of water by filling and emptying two or three containers of given measurements. For example, if you want to obtain 42 gallons of water using a 50-gallon jar and a 4-gallon jar, you would fill the 50-gallon jar first and *from this*, fill the 4-gallon jar. Then, empty the 4-gallon jar and fill it again from the larger jar. This will leave exactly 42 gallons of water in the 50-gallon jar.

Now try your hand at the following:

	You Have			You Want
1.	29 gal jar,	3 gal jar		20 gal of water
2.	21 gal jar,	127 gal jar,	3 gal jar	100 gal water
3.	14 " "	163 " "	25 " "	99 " "
4.	18 " "	43 " "	10 " "	5 " "
5.	9 " "	42 " "	6 " "	21 " "
6.	20 " "	59 " "	4 " "	31 " "
7.	23 " "	49 " "	3 " "	20 " "
8.	15 " "	39 " "	3 " "	18 " "
9.	28 " "	76 " "	3 " "	25 " "
10.	18 " "	48 " "	4 " "	22 " "
11.	14 " "	36 " "	8 " "	6 " "
12.	5 " "	25 " "	10 " "	0 " "
13.	3 " "	65 " "	29 " "	3 " "

Crunching Numbers

1. You are given a number to start with and a number to arrive at as the desired outcome. Using only the legal moves given in each case, try to arrive at the outcome by the least moves possible.

For example: You start with the number 99 and are asked to arrive at the number 100, using only the legal moves of *adding 11* and *subtracting 7*. If you subtract 7, add 11, subtract 7, add 11 and subtract 7, you will arrive at 100. (What happens if you change the order of your operations?) Now, try these:

Start	Legal Moves	Desired Outcome
99	add 11, subtract 7	101
17	add 11, subtract 7	19
25	multiply by 2, subtract 7	65
135	multiply by 2, divide by 3	60
321	add 11, divide by 3	54

2. Place the numbers 1, 2, 3, 4, 5 and 6 in the circles in Figure 1 so that the *sum* of the numbers on each side is the same. What is the smallest sum possible? What is the largest sum possible?

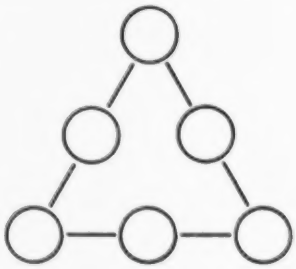


Fig. 1

3. Start with 4-unit squares made of 12 toothpicks as in Figure 2. Then, carry out the following instructions to reach the given outcome.

a. Remove 2 toothpicks leaving 2 squares of different sizes.

b. Move 3 toothpicks to form 3 identical squares.

c. Move 4 toothpicks to form 3 identical squares.

d. Move 2 toothpicks to form 7 squares which are not all identical.

e. Move 4 toothpicks to form 10 squares which are not all identical.

Fig. 2

Teacher's Notes

(see opposite page)

Going in Circles

Answers:

1. .01, .06, .070, .101, .20 = .144
2. .01, .02, .06, .083, .09, 0.1, .101, .199, .2 = .863
3. .38 (nothing larger for next move); .030 (nothing smaller for previous move).

Arrow Dynamics

There are only five different possible actions that can be taken at any given stage of the problem: to invert arrows 1 and 2; 2 and 3; 3 and 4; 4 and 5; or 5 and 6.

The order in which the actions are performed makes no difference. For instance, inverting arrows 3 and 4 and then inverting 4 and 5 gives the same result as inverting 4 and 5 and then inverting 3 and 4. The same is true for any set of three or more actions in a sequence. Thus, you do not have to deal with ordered sets of actions but only with unordered sets. (This means that all the different orderings or permutations of a given unordered set of action sequences are equivalent, greatly reducing the number of possible solutions to be considered.)

Two of the five possible actions—inverting arrows 1 and 2 and inverting arrows 5 and 6—can be ruled out at the start, since arrows 1 and 6 are already in the right position. So the number of possible actions to be considered can be reduced to three.

There are six different action sequences (see Fig. 1) that all achieve the goal in the smallest number of steps (three). The solutions differ only in the order with which the three actions are applied.

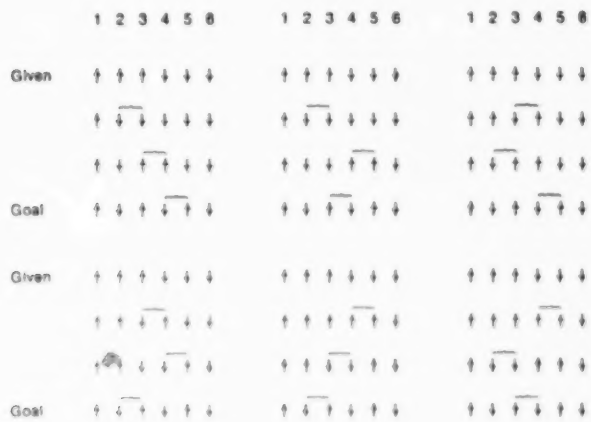


Fig. 1

Going in Circles

Use the diagram in Figure 1 to arrive at the solutions to the following problems. Don't count the circles for "start" and "end"; you must always move to a larger number.

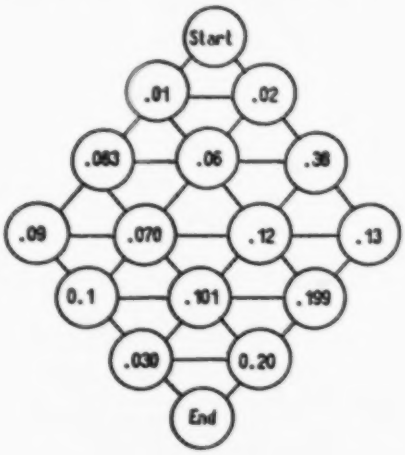


Fig. 1

1. Find the path with the smallest sum total through the maze.

2. Find the longest path with the largest total through the maze.

3. Which circles will never be used? Explain.

4. Design your own number maze using Figure 2. Give it to another student to work on.

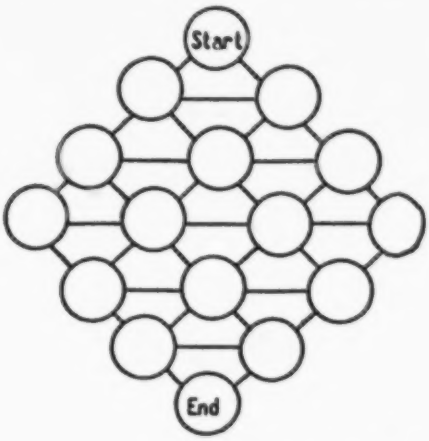


Fig. 2

Arrow Dynamics

You're given six arrows in a row, the left three pointing up and the right three pointing down (Fig. 1). Your goal is to transform them into an alternating up-down sequence (Fig. 2), using the minimum possible number of actions. *You're allowed to invert any two adjacent arrows.* This means that you cannot invert one arrow at a time, but must invert two arrows at a time, and these two arrows must be next to each other.

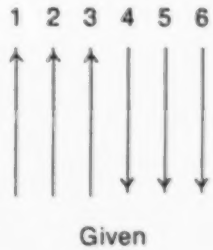


Fig. 1

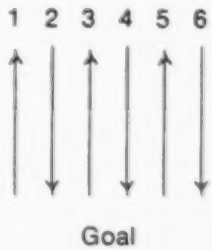


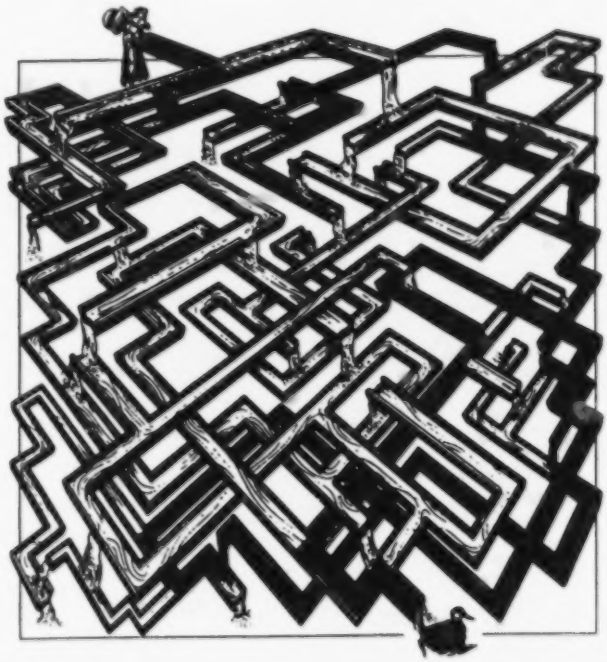
Fig. 2

Teacher's Notes

(see opposite page)

Aqueduck

Solution:



"You Turn"

Answer:

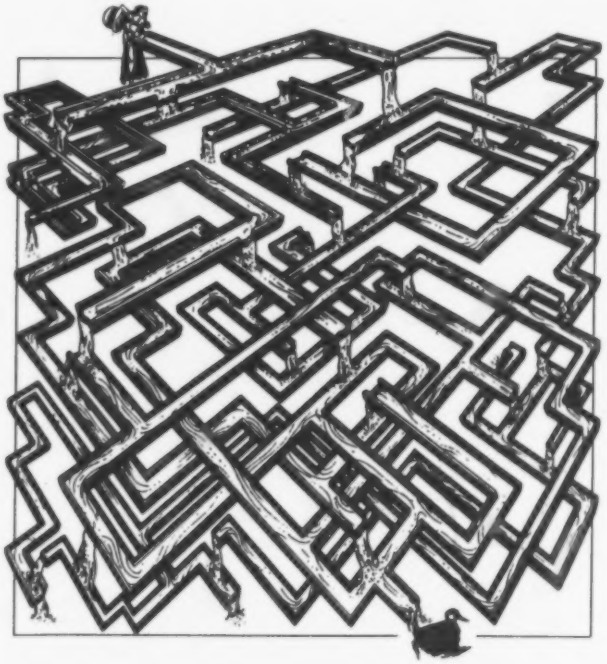
The correct, legal (if somewhat roundabout) route is: 10 (One Way, right); 11 (straight ahead or right; you should have gone right); 4 (left turn); 5 (straight ahead, in this case); 6 (One Way, left); 13 (straight or left; left); 12 (straight or right; right); 19 (One Way, left); 18 (One Way, right); 25 (straight); 32 (One Way, left); 31 (straight only); 30 (One Way, right); 37 (straight or right; right); 38 (a legal parking space). . .at last!

"Some problems are posed in such a way that you are given the final conditions of an action and are asked for a condition that occurred earlier. Or in working toward a solution, you may be able to determine how an action ends and then work backward to a desired intermediate condition faster than starting from the beginning. When either of these conditions exist, working backward *is a valuable strategy to employ."*

The Ohio Department of Education, *Problem Solving. . . A Basic Mathematics Goal*

Aqueduck

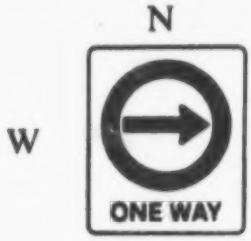
Part of solving mazes is working backward. But here's one where working backward will cause trouble if you forget that water can't travel uphill.



"You Turn"

It's 10:00 p.m. and, as usual, you're having a little trouble parking the car. You've criss-crossed the neighborhood a dozen times already, and if you don't find a spot soon, you'll run out of gas.

Each of the boxes in Figure 1 represents an intersection. Beginning in the blank square at the bottom of the grid, and using the traffic signs as a guide, *find the shortest legal route to a parking space.* No U-turns or diagonal moves are allowed. And remember: Entering the same intersection from different directions yields different results. For example:



Entering from the east, you'd have to turn north; from the west, south; from the north, west; and from the south, east. The arrows can be misleading.

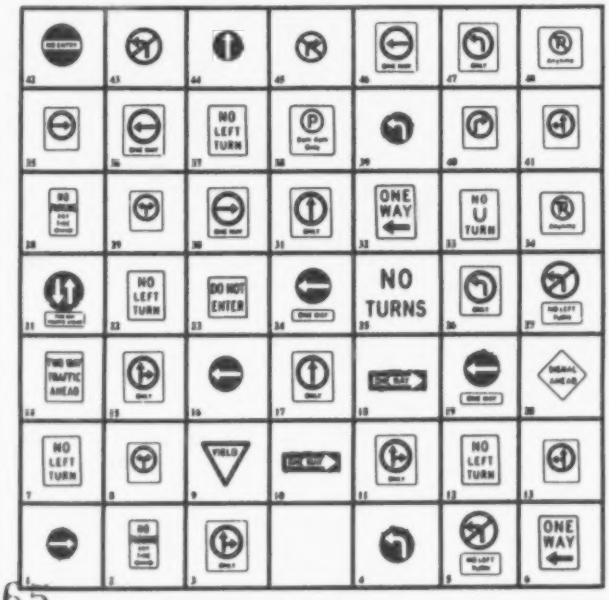


Fig. 1

Teacher's Notes

(see opposite page)

Air Traffic Control

This problem is a modern version of an old type exemplified by the classic (and more complex) "Seven Bridges of Konigsberg." (For a discussion of this problem, see *Mathematics: A Human Endeavor* by Harold Jacobs.)

A good way to start is by drawing a diagram representing all the routes between each point (see Fig. 1).

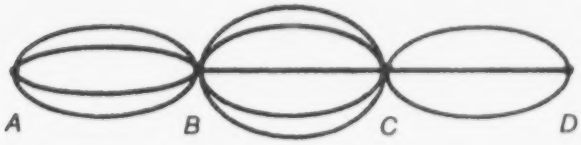


Fig. 1

From the diagram, you can see that for each of the four routes from A to B, there are five possible routes from B to C, making 4×5 or 20 possible routes from A to C. For each of these 20 routes, there are three routes from C to D. Thus, there are 20×3 or 60 possible routes from A to D.

You can figure the possible number of routes for the return trip the same way, *but*, don't forget that for each leg of the return trip, one of the possible routes has already been taken on the trip over. Thus, from D to C there are two possible routes (one less than the original number of three); from C to B there are four, and from B to A there are three, for a total of $2 \times 4 \times 3$, or 24 possible routes back.

Let R stand for the total number of routes. R = the number of routes forward times the number of routes back. So,

$R = (4 \times 5 \times 3) \times (2 \times 4 \times 3)$
$R = 60 \times 24$
$R = 1{,}440.$

You could make 1,440 trips from A to D and back without taking the same route on any section of the return trip.

A Weighty Problem

The solution is to divide the coins into three piles on each weighing. Two of the piles will be equal in weight; the third will contain the heavier coin.

First, weigh two sets of eight coins against each other. If one pan is heavier than the other, then it contains the heavy coin; if not, the heavy coin is in the third pile.

Take the pile that contains the heavy coin. Partition it into three (unequal) parts by weighing two sets of three coins against each other. No matter which pile contains the heavy coin, it will take you only one more weighing to find it.

Total number of weighings: *three*.

For Further Discussion:
What happens when you divide the coins into two groups, instead of three, at each weighing?

Stringing Them Along

In thinking about outcomes problems, it is necessary for students to recognize the difference between *linear* and *circular* arrangements. By visualizing possible outcomes, they will come to realize that arrangements that might have been possible linearly are repetitive when arranged circularly. They might have a tendency to think, for instance, that

is different from

forgetting that the two ends are attached.

There are only three possible arrangements that meet the conditions of this outcomes problem.

For Further Discussion:
To make this problem even more fun, and more advanced, increase the number of beads and add another color.

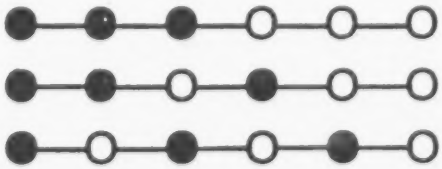


Fig. 1

Air Traffic Control

There are four possible air routes from A to B; five possible routes from B to C; and three possible routes from C to D.

How many different trips can be taken from A to D and back without taking the same route on any section of the return trip?

A Weighty Problem

You have a pile of 24 coins: 23 of them have the same weight; 1 is heavier than the others. You have a balance scale, which will compare the weights of any 2 sets of coins.

How do you find the heavier coin in the minimum number of weighings? **Hint:** A balance scale has three possible outcomes:

- The left pan is heavier.
- The right pan is heavier.
- The two pans are equal.

Stringing Them Along

Three black and three white beads are arranged on a string, forming a circular bracelet. How many different arrangements of black and white beads are possible?

"When you have trouble solving a problem head-on, it is often useful to try to work backward."

John R. Hayes, *The Complete Problem Solver*

Teacher's Notes

(see opposite page)

Double Their Money

There is only one way to solve the problem: You must start with the outcome and work backward. This is possible because the players all finish with the same amount of money. Label the loser of the first game P_1; the loser of the second P_2; and the loser of the third P_3.

WORKING BACKWARD

Game 3:
At the end of Game 3, each player had \$8. Working backward to the end of Game 2, we find that:

1. The stakes of P_1 and P_2 would have been half of their holdings at the end of Game 3 (both won Game 3 and thus doubled their stakes).

- P_1 had \$4.
- P_2 had \$4.

2. P_3, obligated to double the holdings of P_1 and P_2, lost \$8 in Game 3.

3. Therefore, P_3 had \$16 at the end of Game 2.

Game 2:
Keep working backward. P_2 lost Game 2 and would have had to pay P_1 and P_3. So, at the end of Game 1:

1. P_1 and P_3 held half of their stakes at the end of Game 2.

- P_1 had \$2.
- P_3 had \$8.

2. P_2 had to double the stakes of the other two players, paying out a total of \$10.

3. Therefore, P_2 had \$14 at the end of Game 1.

Game 1:
Work backward one more time to the beginning of Game 1, this time taking P_1 as the loser of the first game. Using the same approach, you'll find that P_1 started the game with \$14, P_2 with \$7 and P_3 with \$4.

	P_1	P_2	P_3
End of Game 3	\$ 8	\$ 8	\$ 8
End of Game 2	4	4	16
End of Game 1	2	14	8
Beginning	13	7	4

Fig. 1

Working backward to solve a doubling-game problem

For Further Discussion:
Is the order in which the players won and lost important to solving the problem? Explain.

Suppose the three players ended up with *different* amounts of money? Now what can you say about the order in which the players won and lost?

Building a Number Triangle

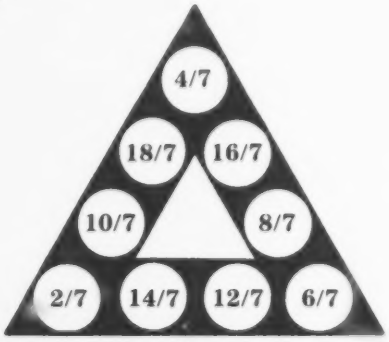
Students should notice that the numbers given are 2/7 times the first nine counting numbers, and that 34/7 is 2/7(17). Thus, the problem is to find an arrangement of 1, 2, 3, 4, 5, 6, 7, 8 and 9 so that the sum on each side is 17.

Since the sum of each side is 17, the sum of the three sides is 3×17, or 51. Corner numbers always are added twice. Since the sum $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$ is 45, the corners must total $51 - 9$, or 6. Thus, the only possible corner numbers are 1, 2 and 3.

Students might find it useful to use the following to find the remaining numbers by guess and test.

$17 - (1 + 1) = 14$ (Must be $5 + 9$ or $6 + 8$)
$17 - (2 + 3) = 12$ (Must be $4 + 8$ or $5 + 7$)
$17 - (1 + 3) = 13$ (Must be $4 + 9$, $5 + 8$ or $6 + 7$)

Answer:



168

Double Their Money

Three people play a game in which one person loses and two people win each game. The one who loses must double the amount of money that each of the other two players has at that time. The three players agree to play three games. At the end of the three games, each player has lost one game and each person has $8. What was the original stake of each player?

Building a Number Triangle

Using the fractions 2/7, 4/7, 6/7, 8/7, 10/7, 12/7, 14/7, 16/7 and 18/7, each only once, place them in the triangle of circles in Figure 1 such that the sum on each side is 34/7.

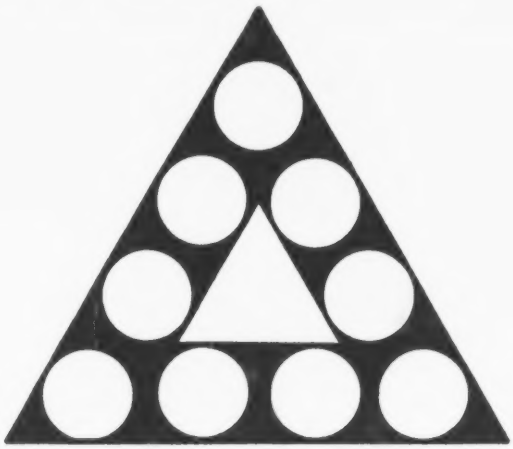


Fig. 1

Suggestions for students:

- Keep a log. Record every trial—errors as well as successes.
- Try to find patterns.
- Record the observations. How many corner possibilities are there? What is the sum of all the terms?

Abbott and Costello's Arithmetic Outcomes

Abbott and Costello were very pleased with their solutions to the problems below. They were sure they were right because they checked and double-checked their answers (#2 and #3 represent "checks"!). The problem, of course, is that their original premises were all wrong to begin with.

1.
$$\begin{array}{r} 13 \\ 13 \\ 13 \\ 13 \\ 13 \\ +\ 13 \\ \hline 28 \end{array}$$

2.
$$\begin{array}{r} 13 \\ \times\ 7 \\ \hline 21 \\ 7 \\ \hline 28 \end{array}$$

3.
$$\begin{array}{r} 13 \\ 7\overline{)28} \\ 7 \\ \hline 21 \end{array}$$

Can you figure out how their reasoning went?

There's Method in This Madness

Here is a new way to multiply—a correct way, too. Try to figure out the examples; then do the problem. But show your work: This is a clear case where it's *how* you get there that counts.

Example 1:
$$\begin{array}{r} 24 \\ \times\ 37 \\ \hline 148 \\ 74 \\ \hline 888 \end{array}$$

Example 2:
$$\begin{array}{r} 371 \\ \times\ 49 \\ \hline 49 \\ 343 \\ 147 \\ \hline 18179 \end{array}$$

Problem:
$$\begin{array}{r} 73 \\ \times\ 42 \\ \hline \end{array}$$

MEN
I Do

Fact Sheet

MANAGEMENT: *What Do I Do Next?*

Running Time: 21:08 minutes

Documentary Segments (1):

1. ***CANOE:*** Carving a 200-pound canoe from a 3-ton log in the traditional manner of the Makah Indians requires careful planning and 14 weeks of work and worry. **But is it shipshape?** You can't be sure until it's finished and you take it on a shakedown cruise. 174

Comedy Segment:

2. ***THE OUTING:*** The Peters family is planning a day at the beach. **Are they trying to pack too much "fun, sun and recreation" into one day?** 172

Math Concepts:

CANOE BUILDING: Symmetry, sequencing and ordering, volume, ratio and proportion

Problem-Solving Concepts:

Subproblems and subgoals; alternative solution paths; feedback; overview.

HOW TO SPOT A MANAGEMENT PROBLEM

You know it's a MANAGEMENT problem if. . .

- ***The objective is known.*** *You have to assemble the details, organize and reorganize them, keep your eye on the destination and figure out the best route to get there.*
- ***You can't solve it in one fell swoop.*** *Building a skyscraper, baking a cake or traveling to China all require step-by-step planning.*
- ***You have to monitor the problem as you solve it.*** *Management problems require you to get* feedback *on your progress and to make adjustments as you go along.*
- ***You worry as much about what can go wrong as how to make things come out right.*** *Management problems require contingency planning, anticipating mistakes and preparing remedies in advance.*
- ***It can't be solved once and for all.*** *Big complex systems, such as New York's traffic or the national budget, require constant adjustment to keep them in optimal balance.*

Throughout the chapter, the $X =$ sign indicates pages containing exercises and problems for classroom use and discussion.

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MANAGEMENT

What Do I Do Next?

There is a story about NASA's Viking project to explore Mars. Before the rocket left the launch pad, the project staff had compiled two phone directory-size books: The first volume was called "problems that can be fixed" and the second, "problems that cannot be fixed." Under the pressure of the space shot, when a problem did crop up, they could quickly find out if they had options or if the problem was unmanageable.

Not all problems are as complex and as long-term as space shots, but most require good management skills—planning, organizing, analyzing and just plain worrying about what can go wrong. Whether the problem is building a canoe, designing a city or controlling its traffic, writing a computer program, devising a budget, fixing a car or planning a party, management is a critical skill.

The world is full of exciting and important problems that can't be solved in a day. They may be too complex or too subtle; they may not be solvable at all. Management is the way to find out if they are solvable—and if they are, to cut them down to size, to organize all the pieces and keep on top of them. The big-city traffic engineer knows he'll never "solve" the problem of keeping the traffic moving smoothly through the city; his job is to keep on top of things, to develop large-scale, long-term plans for meeting the general traffic demands of the city and at the same time to fight traffic jams, street by street, hour by hour as they happen. The manager of a household budget is in the same position. A budget must change to meet changing times. Inflation may increase, or one spouse may get a raise or lose a job, family priorities may change or emergencies might arise.

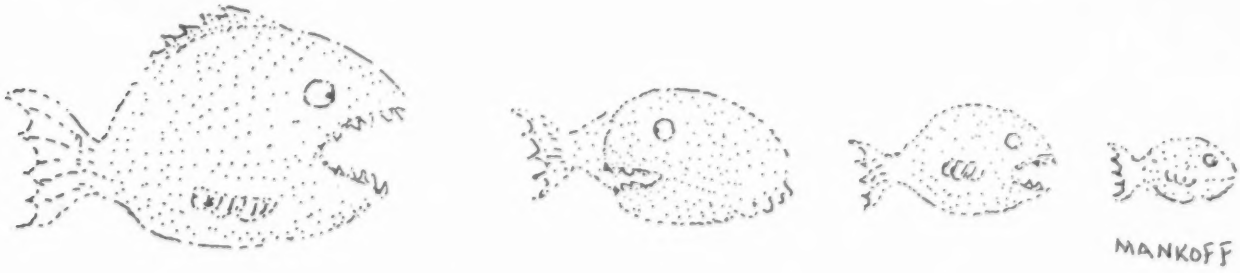
Why Management?

We rely on problem management to tackle the big challenges and to keep complex systems in balance. The economy, businesses, personal health, a touring company of a Broadway show all require management, constant monitoring, planning for contingencies and adjusting to feedback.

We rely on management to control problems over time. If a problem can't be solved in a single effort, it will probably change during the solution process. Building an office complex or traveling to China must be planned out well in advance. But plans must be flexible to take into account countless contingencies that might arise—the unavailability of materials, wage increases, the overbooking of airplanes, a change in the currency exchange rates, etc.

We rely on management for room to maneuver. Good managers anticipate obstacles and plan for alternative routes to the goal. Just as a commuter will take an alternative route after hearing radio reports of a traffic jam on his usual route, a good manager reviews options in advance and is willing to take a detour if its the best route to his goal.

The most common errors students make in solving problems are management errors. They read the problem through quickly, and start right in scribbling down numbers and symbols, even the obligatory formulas, with little or no regard for purpose or direction. In their eagerness to *finish* the problem, they spend little time looking for ways to solve it correctly and most effectively. Teachers report that even the best problem solvers often have difficulty in organizing complex problems and sequencing operations.



'Excuse me, but I'm confused. Do I eat the second fish after he has eaten the first fish, or do I eat the second fish and then eat the first fish, or do you eat me as the second fish is eating the first fish, or do you eat me, the second fish, and the first fish in that order?"

Teaching **Management**

Management skills are important in solving word problems in the math classroom; they are indispensable in the larger world. The Scholastic Aptitude Test, Graduate Record Examination, Law School Admission Test and Civil Service Exams all contain numerous word problems. Students who can unravel and manage difficult word problems will perform better on these important tests, which might affect their futures. In just such a way, and in many others, management skills carry over into the world outside school. Doctors and nurses, auto mechanics, technical writers, cooks, police and farmers all need management skills to perform their jobs well, to untangle complex problems, to develop options or alternatives and to evaluate their own performance in choosing a solution process and executing a solution.

Management breaks big problems down to size. Big problems can be cut up into a number of subproblems. It's like putting together a jigsaw puzzle. The skilled solver sorts the pile into edge pieces, sky pieces, red pieces, etc. Then, the puzzle can be framed, and areas filled in. Trying out hundreds of pieces at random, without a management plan, would leave the pieces lying around forever. Management helps solvers take a long journey with map in hand—a map which can show them how to proceed one step at a time.

Management provides a roadmap to the goal. Management provides an overview and helps the problem solver identify opportunities, and safe routes, to skirt roadblocks and avoid hazards and blind alleys. If the path to a solution is found to be blocked, the careful manager has already plotted alternative routes so that more time won't be wasted along the way. Management provides a way of gauging whether a chosen solution path is really the best and most efficient route to close the distance to the goal.

THIS IS MANAGEMENT? Solving a management problem is a lot like juggling—you have to keep a lot of balls in the air at once.

Management relies on feedback. At any point in the solution process, it is the task of the good manager to look for feedback and monitor progress at every turn. In such a way, problems and solution paths are refined and choices narrowed down. Planning and thinking ahead is the key to management.

Management provides choices. There's more than one solution to most big problems and more than one route to that solution. Management identifies options and assesses the best solution. Good managers generate alternatives, instead of being limited by what may appear to be the only solution. ■

Management in the Classroom

As Lance Wilkie manages the building of his canoe, he draws on some important mathematical techniques to help him.

Symmetry: Symmetry is useful in solving many mathematical problems. In geometry, for instance, detecting rotation symmetry in plane and solid figures can aid students in understanding the mysteries of certain kinds of proofs. Detecting symmetrical patterns in arithmetic and algebra is equally important. Through an understanding of the concept of symmetry, students can extend their mathematical knowledge to the study of physics and chemistry and the enjoyment of music and art.

Ratio and Proportion: In order to derive the measurements for the canoe under construction, Wilkie measures canoes in museums. Then, he scales the measurements proportionally to find the length and width of the canoe he should carve from a given log.

Curves: Wilkie carves the curve of the canoe hull with a straight-edge adze. This successive approximation is similar to curve stitching—an activity introduced in the nineteenth century for children. By drawing or lacing string from tacks on a board, a series of lines are created outlining a curve. Students can create designs from a simple parabola to more artful and intriguing mathematical curves.

Word Problems: Word problems have been a staple of mathematics since ancient Egyptian times. For almost as long, they have been the bane of the math student's existence. When students protest that they don't understand a word problem, teachers usually respond by asking them to analyze it, to pinpoint what part of the problem confuses them, to describe the elements of the problem in a way that clarifies the relationship and indicates how they might attempt a solution. In effect, they ask students to *manage* the problem.

Teaching management skills means helping students tackle problems complex enough to demand a number of operations. Students may profit from just talking through a complex problem in class. By removing the pressure to *solve* the problem, students can concentrate their effort on analyzing and comprehending it. The class can draw up a solution plan, indicating what operations are necessary to solve it and listing them in sequence. Textbook problems are usually limited in scope. Practice in tackling more challenging management problems may require closing books and giving students an exercise to be solved in the course of several class periods. For instance, give them an imaginary budget of $100 and ask them to plan a class activity, a field day or whatever they choose. In order to develop a consensus, they will have to develop alternative solutions, choose among them (not only by preference, but by researching costs) and negotiate one solution for the class.

Upstream/Downstream Problems: This classic type of word problem continues to stump students, often because they fail to visualize the relationship at the core of problems of this type. At the end of the MANAGEMENT film, Wilkie rows his canoe downstream to the ocean. Use this image as the basis for discussing the adding and subtracting of rates of speed and exploring the underlying concept of the Upstream/Downstream problem. ■

"Do not commit yourself too early to a course of action. You may find it hard to break away from the path, find it may be the wrong one. Search for a number of paths simultaneously and use signs of progress to guide you to the path that appears most plausible."

Moshe F. Rubinstein, *Patterns of Problem Solving*

Behind the Scenes: MANAGEMENT on Film

Management is a tough concept to bring to life on film. We needed to find a problem with a number of elements, one which was solved in stages over time and demanded the steady care of an attentive problem solver. There are thousands of interesting problems that answer this description, and there were times when we thought we'd researched them all. But all those management problems had one thing in common: They fell flat as stories for film. They were fascinating, but too technical; or, they had so many elements that following them on film would prove unwieldy; or, they reinforced the stereotype of management as a matter of bossing people around.

We knew immediately when we had found the right management story. It had all the essential problem-solving elements, and it was a great film subject. It had an intriguing character, beautiful scenery, suspense and the mystery of an ancient craft.

Canoe Building: When we found Lance Wilkie making canoes in the traditional manner of his Makah Indian ancestors, we knew we had the way to bring management to life on screen. Over a period of 14 weeks, while Lance carved the canoe, we visited him four separate times to film critical stages of the process and to have him explain what he was doing and thinking. In between times, we kept our fingers crossed. We could only know if Lance had managed his problem right on the last day of filming. If the canoe was a success, it would be visible: It would float upright, evenly balanced in the water. If not, then we'd have that on film, too.

Wilkie didn't let us down. The canoe floated just right—hard evidence of his skills as a craftsman and manager. The canoe segment also brings alive the mathematical concepts from proportion (as Wilkie determines the measurements of his canoe from museum models) to successive approximation (as he carves the canoe's curved sides with a straight-edge adze). Most important for us, it shows that math and problem solving are not the exclusive property of specialists and professionals. They belong outdoors with men and women who work with their hands as much as they belong to those in high technology fields.

The Outing: In planning for a "short," daytrip to the beach, the Peters family struggles with a management problem of their own making. The questions that should be asked at the outset about priorities, available space and ordering are cast aside. What the Peters are left with is another kind of management problem—how to rescue something from nothing and still enjoy a day in the sun. ■

GETTING STARTED: **Thinking MANAGEMENT**

The problems presented in this chapter are designed to help students appreciate and deal with the many aspects of problem management. Here are some exercises to get them started in thinking "management" *before* they see the film.

1. How do you go about putting together a jigsaw puzzle? Describe, step by step, how you take a jumble of pieces and make them into a picture. How important is the picture on the box? What do you learn from it? Do you work on the edge pieces first, or sort the pieces by color? What do you do when you get stumped?

After discussing how a jigsaw puzzle is put together, try doing one. Did you follow your own description of the process? What, if anything, was different? Do you approach every jigsaw puzzle the same way, or tailor your strategy to the puzzle?

For Openers

Here are some questions to ask your students *after* they've seen the MANAGEMENT film.

- What is meant by a "management" problem?
- What kinds of problems in your own life might qualify as management problems? How might you best go about solving these multilevel problems?
- How is building a dugout canoe a management problem? How does Lance Wilkie use management skills to solve the problem?
- How does the Peters family manage—or mismanage—a simple daytrip to the beach? What might they have done differently to end up at the beach rather than in the backyard?

2. Given a 4-liter unmarked container, a 7-liter unmarked container and an unlimited supply of water, try to obtain an accurate measure of 5 liters of water? What is the minimum number of pourings necessary?

4-liter container **7-liter container**

Plan your solution step by step (pour by pour).

Here's *one* way to solve the problem:

- Fill the smaller, 4-liter container.
- Pour those 4 liters into the large 7-liter container.
- Fill the small container again.
- Fill the larger container from the small container (1 liter remaining).
- Empty the large container.
- Pour 1 liter from the small container into the large container.
- Fill the small container.
- Pour the small container into the large container (4 liters + 1 liter = 5 liters).

Minimum number of pourings necessary: 8.

Stalking a **Management** Problem

Making Up Your Own Problems

The following is a hypothetical conversation between teacher and students. It is intended to suggest ways to encourage students to make connections between problem-solving concepts in the film, their own experience and their knowledge of mathematics.

"

♦ ***Before we try some management problems of our own, let's review what makes a problem a management problem.***

If it's too big or too long.

♦ ***Can you give us a for-instance?***

Well, Lance's canoe is complicated and it takes a long time to carve it, so he has to pay attention all the time.

♦ ***I think that's most important. Big problems need to be cut down to size and organized into steps that we can do one at a time. We use management to do that.***

Lance is always worrying about the canoe, and saying things like "if I don't do that, then this will happen" or "if this happens, then I'll have to do that or some other thing will go wrong."

♦ ***Good. He has many alternatives, and he knows that there isn't just one way to solve his problem, so he has to pick and choose. There's something else, too. How does he cut his problem down to manageable size?***

He divides it up, like first carving the outside, then drilling the holes and carving the inside, then steaming it, then putting on the bow and stern pieces.

♦ ***Excellent. These are subproblems. He doesn't try to accomplish his task in one fell swoop, but in a series of stages. But at the same time, he keeps the big picture in mind. It would defeat his purpose if he made a beautiful bow piece that was too big for the canoe. Well, I think we have a pretty good grasp of why making the canoe is a management problem, but can you think of other management problems or problem managers from your own experience?***

Well, we have to write a 15-page paper in English and I think that's big enough to be a management problem.

♦ ***Sure sounds like it. Have you broken your management problem down?***

Not yet, but there's a lot to do: first, pick a topic; then, research; outline the paper; write it; copy it out neatly or type it.

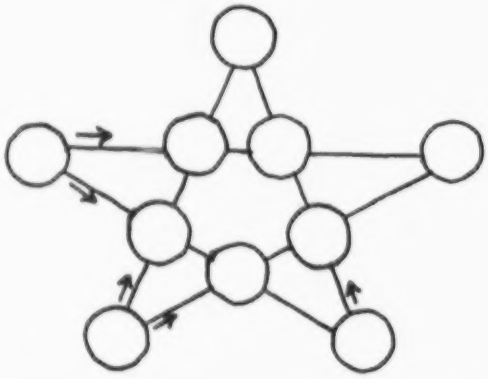
Problem:

The wealthy owner of a southern mansion tells his gardener that he wishes for five rows of peach trees with four trees in each row to be planted in the front courtyard.

The gardener orders twenty peach trees, but only ten arrive. There being no way to order more trees in time to appease the owner, the gardener has to find a way to comply with his command.

How does he do this?

Christopher Paul Gildner
J.A. Fair Junior High School
Little Rock, Arkansas



173

♦ ***Any one else have a management problem? What about in school here? Can you think of any problem managers?***

What about the woman who runs the cafeteria?

♦ ***What about her? What are the elements of her management problem?***

Well, she has to figure out the menus and order the food. And she's in charge of all those people who serve the food and clean the cafeteria.

♦ ***Anyone else?***

The principal.

♦ ***And what kind of things does he manage?***

He hires teachers and orders supplies and keeps track of the money, and all that stuff.

♦ ***All that stuff, indeed. He meets with parents and the school board and oversees class assignments and schedules and buses. Let's pretend we're the principal and work out a bus schedule and route for this class.***

But that's not the way the buses work.

♦ ***I know, but working out a bus schedule for the whole school is tougher than you think, and it might take us weeks to get it done. So, we'll start small. How are we going to organize this problem? Let's make a list.***

We need to know who goes on the bus. And where they live. We probably need a map for that.

♦ ***Fortunately, I came prepared. Here's a map of the town. But we're not done with the list. If we were the principal, what goals would we set?***

We would want to set up stops so that if two or three people lived close together we could make one stop and get them all.

♦ ***Anything else?***

Well, we'd have to find the shortest route.

♦ ***And that is where the math comes in! Okay, what shall we do first?***

WHAT'S WRONG WITH THIS PICTURE? Good management starts with a clear objective and an uncluttered view of the tasks that will achieve it.

Following Through: Problem Ideas from the Film

The MANAGEMENT film shows that solving a complicated problem places a premium on planning. Here are some examples from the film on which to draw.

Canoe Building: Wilkie must plan each stage of canoemaking, planning ahead toward his final goal, looking ahead to what might go wrong and keeping an eye out for unexpected problems and opportunities.

- Draw a flowchart of a simple task, such as locating and purchasing a copy of Euclid's *Elements*. A flowchart is a diagram that gives a sequence of instructions for a given activity. In order to draw a flowchart, you will have to understand the structure of the activity and break it down into subtasks and alternative actions.

- In one respect, making a canoe is a problem in subtracting volume. Wilkie takes a 3-ton log and subtracts wood until he has a canoe weighing about 200 pounds.

  1. How many pounds of wood are carved away?

  2. What percent of the weight of the original log is the finished canoe? What percent is "wasted" wood?

- Wilkie determines the desired measurement of his canoe by measuring canoes in museums and deriving his measurements proportionally. The canoe in the museum is 39 feet long and 12 feet wide.

  3. If Wilkie wants his canoe to be 13 feet long, proportionally how wide should he plan on making it?

  4. If Wilkie's canoe can be no more than 9 feet wide, find the maximum length he should plan on making it.

- Wilkie finishes his canoe and is off for a trial run. He rows downstream to the ocean. In a flat calm, Wilkie can row 6 miles per hour. Wilkie travels the 2 miles to the ocean in 15 minutes.

  5. How fast is the current in the stream running?

  6. How long will it take him to row back upstream?

"Good problem solvers have learned that analyzing complex problems and ideas consists of breaking the ideas into smaller steps. They have learned to attack a problem by starting at a point where they can make some sense of it, and then proceeding from there."

Arthur Whimbey and Jack Lochhead, *Problem Solving and Comprehension*

■ The square ABCD, in Figure 1, is circumscribed around a circle with a radius of 6 inches. The circle has been cut out.

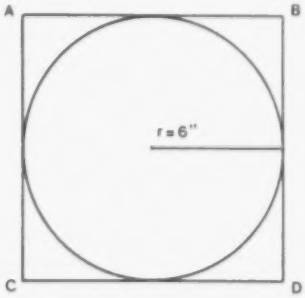


Fig. 1

7. Find the area of the remaining figure. (Express the answer to the nearest tenth of a square inch; use $\pi = 3.14$.)

■ A cone with a radius of 5 inches is placed in a cylinder with the same radius and with a height of 10 inches (Fig. 2).

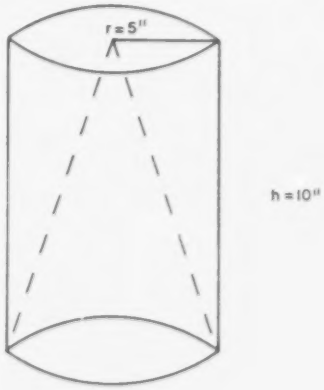


Fig. 2

8. Subtracting the volume of the cone, how much space is left over?

■ Getting the right tasks in the right order is an important part of managing a problem. Take the instructions for a simple task, such as the recipe for making a pizza, and mix up the steps. Have another student, or group of students, try to put the steps in the right order. This exercise can be made even more challenging by having the second group of students diagram the process they used to arrive at their solution, including budgeting the activity and shopping for groceries.

GETTING A HANDLE ON A BIKE: French comedian Jacques Tati has an objective—a bicycle—and all the parts to achieve it. His management problem is to define the steps and put them in the right order.

Answers:

1. 5800 lbs

2. 3.33%; 96.66%

3. $\frac{39}{12} = \frac{13}{n}$

$n = 4$ ft wide

4. $\frac{39}{12} = \frac{n}{9}$

$n = 29.25$ ft long

5. $\frac{6m}{\text{hr}} = \frac{x}{.25}$

$x = 1.5$ miles in 15 minutes if current is zero. Current is running $2 - 1.5 = .5$ miles in 15 minutes, or 2 mph.

6. 6 mph − 2 mph = 4 mph

2 mi ÷ 4 mph = 30 minutes

7. Area of the square = 144 sq in

Area of circle = 113.04 sq in

144 − 113.04 = 30.96 sq in

Expressed to the nearest tenth of a square inch, 31.0 sq in.

8. Volume of a cone $\frac{\pi}{3} r^2h$ or $1.047r^2h$

Volume of a cylinder = πr^2h

$\pi r^2h - \frac{\pi}{3} r^2h = 524.25$ cu in.

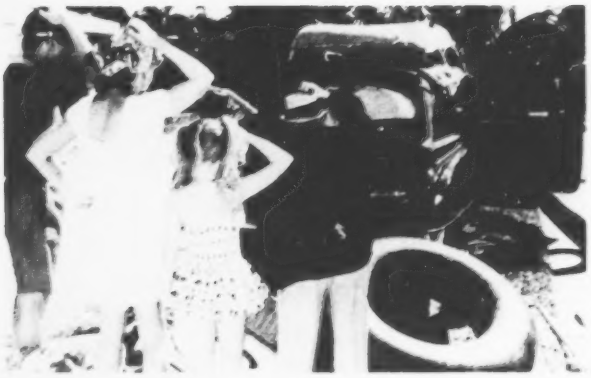
On the Lighter Side

"If at first you don't succeed, try, try again." As a problem-solving maxim, it leaves something to be desired. It should read: "If at first you don't succeed, *think it over* and try again."

Wendell Peters' management technique can only be described as helter-skelter. To give him credit, he does manage the problem by breaking it down into two subgoals: Let's (1) pack the car and (2) go. But he's inundated with equipment and he just keeps shoving it in the car, until nothing else (including his family) fits! He never establishes any priorities in packing the assortment of "materiel." Do they really need to take a bushel of oranges for a day at the beach? Nor does he show any grasp of order or 3-dimensional space. When his plans fall flat, the family finds "fun, sun and recreation" in its own backyard. And when the comedy's over, perhaps students will find ways of applying it on their home turf. They may see some advantages to looking before they leap.

A Comedy of Errors: The Outing

Today's American Family knows that meticulous planning and careful preparation are the cornerstones of a successful outing.

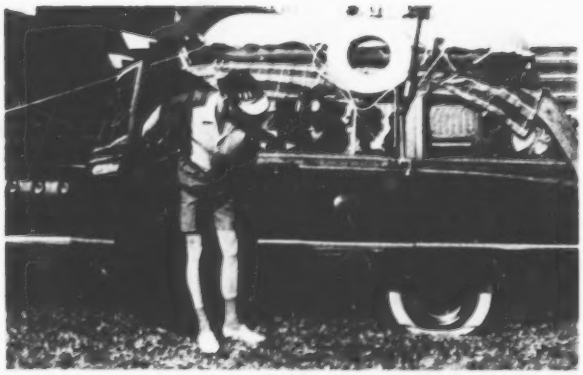


Here, Dad selects only those things they'll need for that 10-mile drive to the beach. . . .You know, he's just making a big mess here. This is *not* the way you pack a car.

Packing It In

■ Wendell Peters is packing for a trip to the beach with his wife, three children and Grandpa. He's gathered all the equipment on the lawn. Help him get organized. Make three lists under the headings of "necessary," "unnecessary" and "desirable" with the following items:

5 beach chairs
6 towels
2 inflatable boats
7 lifejackets
3 beachballs
2 innertubes
1 bushel of oranges
1 volleyball net and ball
1 badminton set, including net, 6 racquets and 4 shuttlecocks
1 picnic hamper, with 5 knives, forks and spoons; 5 plates and glasses; 7 sandwiches; 4 peaches and 10 cans of soda and juice; 2 dozen napkins
suntan lotion
bug spray
camping stove
4 paddles
book of tide tables



Mom, Dad, kids and Grandpa, all set to go for a day of fun, sun and recreation. . . .Oh no, I mean what are we going to do with this?

Today's American Family often finds that rest and recreation are no further away than its own backyard. And so we say goodbye for this week to The American Family.

- It's 2:30 and the car is all packed. Wendell and his family are ready to leave for the beach when the tire goes flat. It will take Wendell 15 minutes to unpack the car; 30 minutes to fix the flat; 35 minutes to repack the car; and 20 minutes to drive to the beach. What time will the Peters family get to the beach?

- The Peters have three children: Tom, the oldest; Loretta in the middle; and Skippy, who's four years old. Grandpa is six times older than Tom and 15 times older than Skippy, who is half Loretta's age. How old are Tom, Loretta, Skippy and Grandpa?

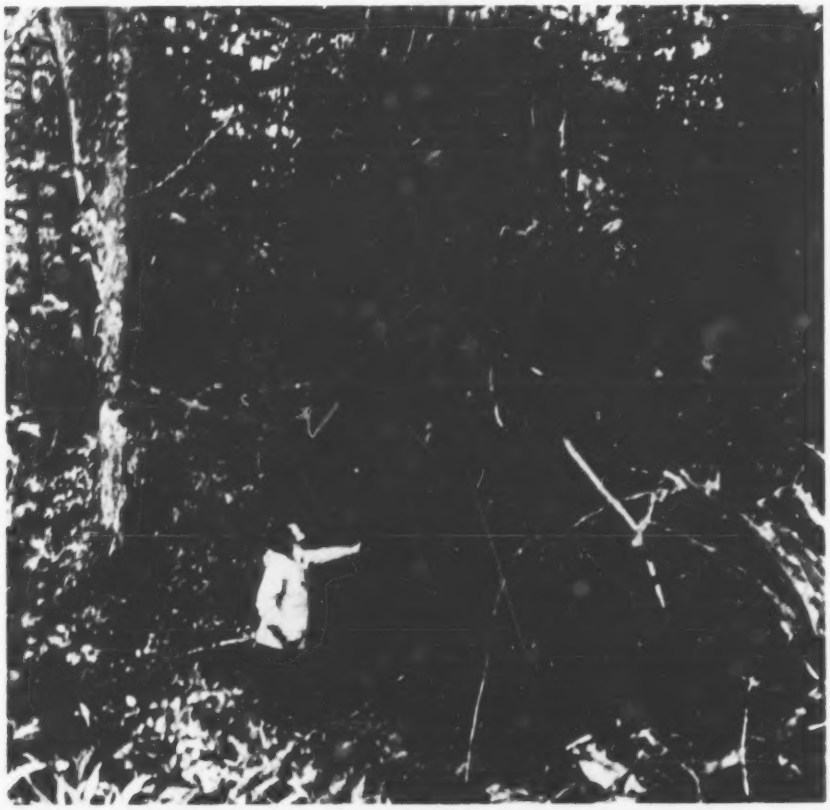
Canoe Building: A Balancing Act

"I have an idea in my mind of what the finished product should look like. But there are a lot of problems to deal with along the way. Solving each one ensures that the canoe is going to come out the way I want it to."

Lance Wilkie, a 26-year-old Makah Indian living on a reservation on the west coast of Washington State, is perpetuating an ancient craft. Although he is a salmon fisherman by trade, his most satisfying work is making dugout canoes using the traditional methods and materials of his ancestors.

The canoes range from 10 to 37 feet in length. The canoe's ultimate use—salmon fishing or seal hunting—determines the size of the log Wilkie chooses.

After choosing and cutting a tall cedar tree, Wilkie turns the log in search of sound wood. The strongest wide-grained wood, called "heartwood," will eventually form the bottom of the canoe. Now, the heartwood is facing up and Wilkie scores it crosswise at one foot intervals, using a chainsaw. As a labor-saving device, the chainsaw is one of Wilkie's few concessions to modernity. The rest of the work—the actual craft of making a canoe—is executed with hand tools like those used for hundreds of years.



BIRTH OF A CANOE: The creation of a canoe begins with a tree. Lance Wilkie, working in the tradition of his Makah ancestors, picks a stately cedar and fells it himself. From the tree he'll cut a 3-ton log and begin to carve out the wood, first using a chainsaw to block out large pieces of wood, then only hand tools and an eye to symmetry.

Standing astride the log, Wilkie wields a shipwright's adze to level the bottom. An adze looks something like a firefighter's axe with its blade turned crosswise. It is grasped with both hands and swung like a pendulum between the legs. Properly used, it strikes its razor-sharp edge between the chainsaw cuts and pries off a large chunk of wood. It is an efficient but dangerous tool.

Using the flat bottom as his starting point, Wilkie then carves the outside of the canoe. "[Some] people will do half of the outside, and then try to do the inside," Wilkie explains. "Then they'll end up with the problem of needing to go somewhere else in the wood where they've already committed themselves."

Good problem managers, like Wilkie, develop a logical sequence of steps to arrive at a solution before beginning to solve a problem. Diving into a complex problem with no strategy for solving it sometimes creates problems that could have been avoided by looking ahead.

Ship Shape

Carving the hull is a critical stage: Both sides of the canoe must be mirror images of each other. If they aren't, the canoe will be unbalanced and lean and/or pull to one side.

Determining the curve of the hull is equally important. A hull that is too round is easily unbalanced; a hull that is too flat will handle stiffly. To ensure a balanced hull, Wilkie uses a chalkline stretched from one end of the log to the other to strike the "centerline." This is his *axis of symmetry*. Wood is not a uniform material, however, so Wilkie must also rely on how the wood looks and feels from each side of the canoe to know exactly how much wood to remove.

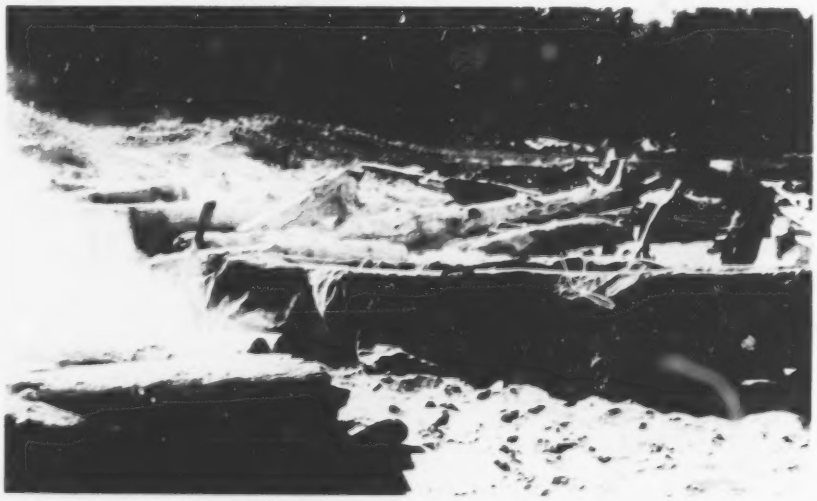
Wilkie roughs out the hull into a basic "V" shape. "You can see where the point of the bow is; where the point of the stern is. You're looking at the finished shell with the wood still left inside."

Square Pegs in Round Holes

Before he begins hollowing out the canoe, Wilkie drills about 50 holes into the bottom and sides of the canoe. It sounds crazy, but it is actually a brilliant management technique, enabling him to monitor the thickness of the hull. If the wood is thicker on one side than the other, it will also be heavier, and the canoe will tip to one side.

Wilkie drills the holes from the outside, boring in to the depth of a standard auger bit. He then pounds square pegs of red cedar into the round holes. The pegs are cut to the same length as the thickness of the sides he needs, but shorter than the holes. The sharp edges of the square pegs grip into the soft wood of the round holes, ensuring a watertight bond. Three sets of pegs are inserted: one row along the bottom centerline;

CARVING OUT A CANOE: While hollowing out the canoe, Wilkie keeps an eye out for the holes he has drilled in the sides and bottom. The holes are fitted with pegs measured to the desired thickness of the hull. When Wilkie reaches the pegs, he stops carving.



A FIERY TRANSFORMATION: Lance puts the canoe through its baptism of fire: Steaming is the point at which "the piece of wood becomes a canoe." He puts heated rocks into the water-filled vessel, then covers the canoe with canvas. The steam makes the wood pliable, and Wilkie forces the sides out gently with sticks.

another halfway up the sides; and a third right around the gunwhale (the upper edge of the boat's sides).

As he hollows out the canoe, he keeps an eye out for the drill holes; when he reaches one, he begins carving more carefully. When he reaches the pegs, he knows to stop carving. As a system of gauging thickness, pegs are quite accurate, and more convenient than stopping every few minutes to make clumsy calculations with calipers* or other devices that measure thickness.

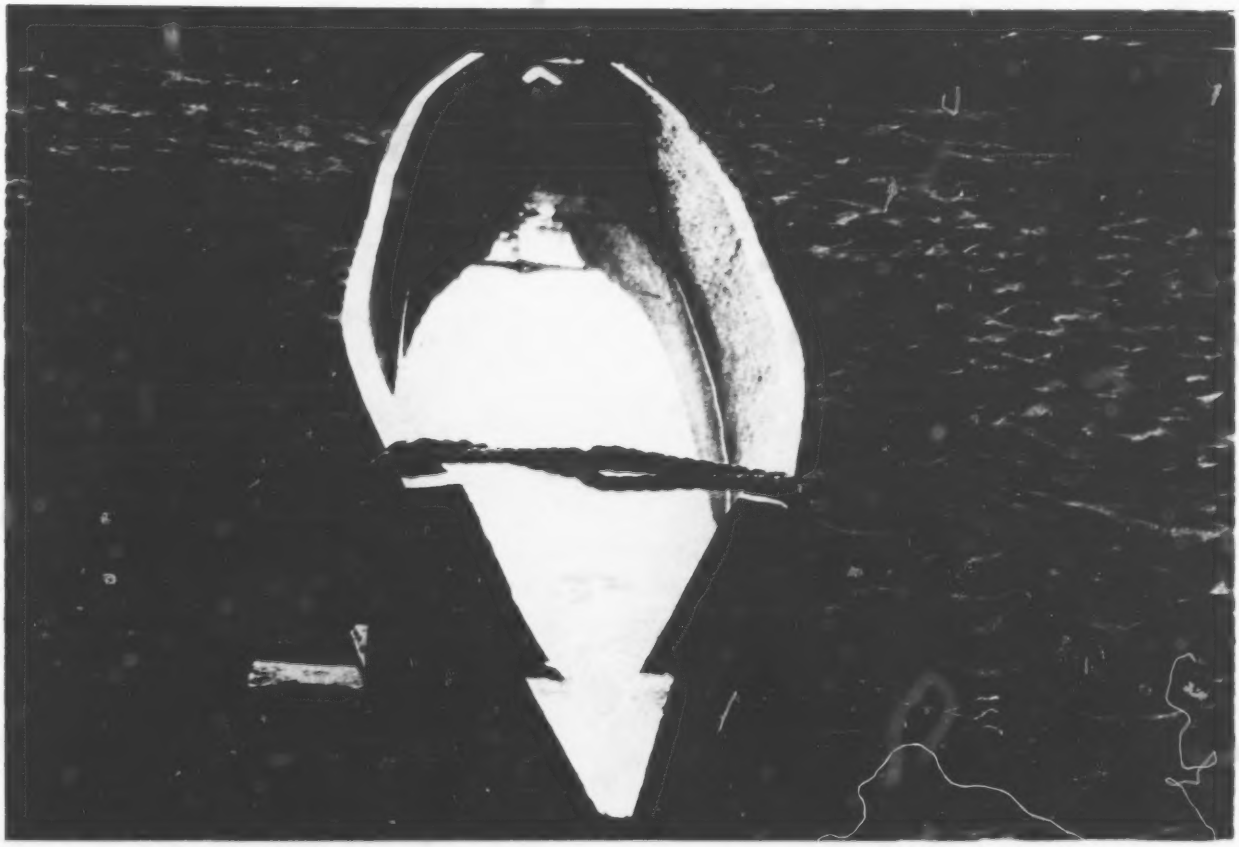
*A measuring instrument consisting of two prongs or jaws that can be adjusted to determine the relative thickness, diameter or distance between surfaces.

As he hollows out the canoe, Wilkie leaves a large section of wood in the center. This center block absorbs the shocks of adzing (carving). "Without that center block in there," he claims, "there's a good possibility of splitting the canoe."

Fire and Water

The canoe achieves its final shape—flared sides and flat bottoms and the tensile strength to support that shape—not by carving, but by cooking: Wilkie "steams" the wood into shape. From a stream, he collects solid, rounded igneous rocks which won't crack or shatter when heated. The rocks and canoe are brought to the beach, which is literally out his front door. There he builds three fires: one fire to heat the rocks; and two others to warm the outside of the canoe. When the rocks are hot enough, Wilkie pours fresh water into the canoe. The heated rocks go in next, and the canoe is covered with canvas. Soon, the water in the canoe is boiling.

Once Wilkie sees that the sides of the canoe are expanding naturally from the heat—about a quarter of an inch—he knows he can add *gentle* force. Using a series of spreading sticks, which are slightly longer than the width of the canoe, he forces the sides outward bit by bit until the



NOW IT'S A CANOE: The almost-finished canoe, just before the bow piece is added. From start to finish, it has taken Wilkie about 14 weeks to make the 16-foot, 200-pound canoe. Will it float upright? He won't know until he takes it out on the water—the final test of his management skills.

proper shape and flare are achieved. This is done slowly—over a period of several hours—to prevent cracking the wood. As the canoe spreads, Wilkie adds slightly longer sticks. The canoe stretches from about 25 inches wide to about 38 inches wide at the end of the process. Wilkie will then install permanent cedar thwarts to keep the sides pushed out.

"When I push the sides out, it all meshes together as a nice shape," Wilkie explains, "with the bow and stern higher than the middle." To Wilkie, the steaming process is the real moment of drama: "It's the point at which the piece of wood becomes a canoe." The tension created in the wood by the newly curved sides actually makes the canoe stronger as the thwarts hold the wood back from returning to its original shape.

Bow and Stern

The bow and stern pieces of the canoe are added last, carved from a separate block of cedar. If the entire hull were carved from a single log, the grain in the bow and stern would be weak. In that case, "you could knock off the bow and stern pieces just by bumping them," Wilkie explains. He makes paper patterns or metal *templates* in the shape of a bird or a wolf's head (in the tradition of his ancestors). He then carves the pieces, matching the curves of the bow and stern pieces with the curve of the hull. Getting this right can take a week. "It gets very frustrating when you're fitting the two surfaces together; the thing keeps changing on you," Wilkie says. Scarf joints (two pieces of wood notched to fit together) and wooden pegs lock the bow and stern pieces to the body of the canoe.

After a final adzing of the outside to give the canoe texture, cutting out any knots and patching them, Wilkie makes the thwarts and lashes them in. He finishes off the canoe by charring and oiling it to seal it and make it watertight.

About 14 weeks after starting the project, Wilkie hauls the 16-foot, 200-pound finished canoe to the water for the final test. If he has been correct in his measurements, in locating the centerline and carving out the wood symmetrically, the canoe will be balanced in the water. "Everything has got to work right," explains Wilkie. "It doesn't have to be perfect, it just has to be within mathematical limits; as long as it's uniform throughout the canoe."

Why is each step in the canoe-making process managed the way it is? Wilkie answers: "You're dealing with 6,000 years of history—it's a finished form; it's an art form. I don't know how to explain it any differently."

Wilkie grabs his hand-fashioned paddle and pushes off from the shore. The canoe glides silently—and in perfect balance—through the water. ■

Further Readings

Adney, Edwin Tappan and Chapelli, Howard I. *The Bark Canoes and Skin Boats of North America*. Washington, D.C.: Smithsonian Institution Press, 1983.

For students interested in all types of North American Indian canoes, this is a beautiful and comprehensive study of bark and skin canoes. Lots of black and white photos; detailed line drawings help explain different construction techniques.

Fichter, George. *How to Build an Indian Canoe*. New York: David McKay Company, 1977.

Although this 90-page book does not cover the building of the dugout canoe seen in the film, it does give a simple, well-illustrated description of how various other North American Indian tribes constructed their strong, watertight bark and skin canoes.

McPhee, John. "The Survival of the Bark Canoe"—Parts I and II. *The New Yorker*, 24 February and 3 March 1975.

An extremely thorough study of one man—Henry Vaillancourt—and his construction of birch-bark canoes. This article takes the reader meticulously through the entire process, recording every detail. It also gives historical background on the Indians who first made these canoes and how the process was "rediscovered" in the early 1900s.

Makah Cultural and Research Center, P.O. Box 95, Neah Bay WA 98357.

The Makah Cultural Center houses almost all the archaeological artifacts of the Makah Indian Tribe. For more information on the Makah Indian Nation, their history and their dugout canoes, write directly to the Center.

Mirror-Images

To transform a three-ton cedar log into a 200-pound canoe that is streamlined, seaworthy and balanced, Lance Wilkie uses a series of problem-solving tools and techniques, including the mathematical concept of *symmetry*. Creating symmetry is critical to the success or failure of the project. Fourteen weeks of effort finally come down to how well Wilkie has been able to make one side of the canoe the mirror image of the other.

In mathematics, a figure in a plane is symmetric if you can locate a line in the plane that separates the figure into two identical halves; one half being the exact reflection of the other.

A more precise mathematical definition is possible. Two *points* are symmetric with respect to a line if the line is the perpendicular bisector of the line segment joining the two points. A *geometric figure* is symmetric with respect to a line if every point on the figure has a corresponding point, also on the figure, such that the pair of points are symmetric with respect to that same line.

The same essential principles of symmetry hold for three-dimensional objects, except that a *plane of symmetry* separates the object into two "mirror-identical" halves.

A mirror can be used to test candidates for *lines of symmetry* on geometric figures. When a mirror is placed along the possible line of symmetry, the exposed half of the figure and its reflection should look just like the original figure.

The concept of symmetry has numerous applications. Symmetry is very important to mathematicians in their study of graphs, functions, distributions and algebraic groups. Scientists have discovered naturally occurring symmetry in the fields of biology, chemistry, biochemistry, physics and crystallography.

The following exercises will give students practice in recognizing lines and planes of symmetry.

Exercises:

1. Cut a tennis ball exactly in half and lay it on a mirror. A complete tennis ball should be seen. Try the same experiment with an apple, banana, pear or other fruit. The importance of finding the correct plane of symmetry will become apparent—*unless* your objective is to create new and strange varieties of fruit.

2. Ask students to collect objects that they can test for possible lines or planes of symmetry. Pictures of flags of different nations and corporate logos collected from magazines and newspapers can give students good practice with the concept of symmetry.

3. Adapt the techniques that Lance Wilkie employs by carving a large cucumber into a "canoe." Eyeball the basic structure of the cucumber to decide where it would be best to make the cuts and how much of the pulp to cut out. Use toothpicks as Wilkie uses wooden pegs, to gauge thickness. When completed, subject your *cucumber canoe* to the "trial by water" to see if it is balanced and will float. (Note: Unfortunately, the steaming process is not useful when building cucumber canoes!)

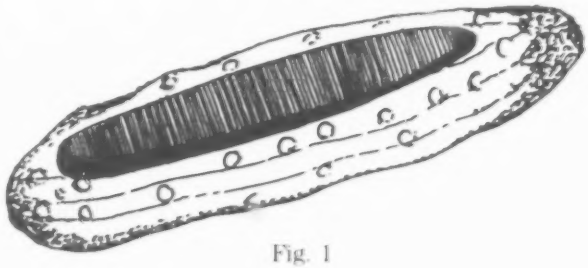


Fig. 1

X=

Exercises with a Mirror:

4. Determine which letters of the alphabet have lines of symmetry. How many have more than one line of symmetry? Can you find a word with either a horizontal or a vertical line of symmetry?

ABCDEFGHIJKLM
NOPQRSTUVWXYZ

Fig. 2

5. Try to create figures **a-f** (Fig. 3) by moving the mirror's edge on the first figure. What other figures are possible? Sketch them and have other students try to find them.

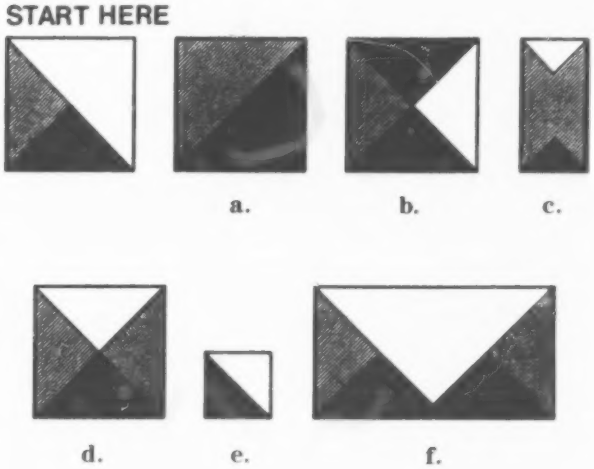


Fig. 3

6. Try to create figures **a–d** (Fig. 4) by moving the mirror's edge on the first figure.

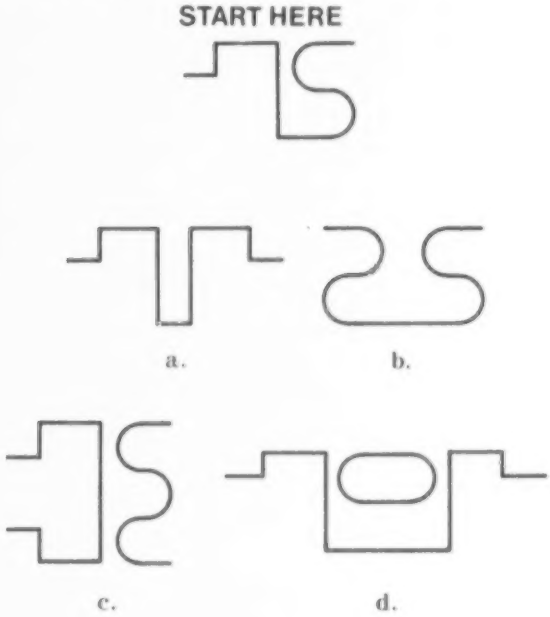


Fig. 4

7. Use a mirror to locate possible lines of symmetry for the figures **a–h** (Fig. 5). (Some figures may have more than one line of symmetry, and some none.)

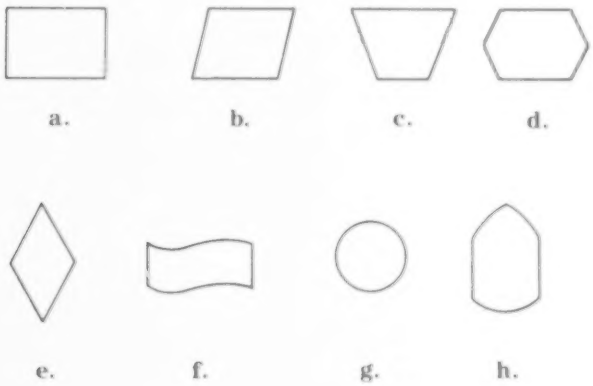


Fig. 5

189

Following Orders

One of the most important problems a manager has to solve is what sequence of problem-solving steps to use. Very often, the manager must guess and test a number of possible strategies before arriving at a sequence that will work. At each step, he should evaluate what his last step accomplished and how close he is to the correct answer. In this exercise, you will have the opportunity to try being a problem-solving manager yourself. One of the best things about this kind of problem is that by following the rules, you can immediately prove whether your answer is right or wrong.

Exercises:

1. Consider the following program of instructions.

Step 1: First Run: Choose a 2-digit number, which is neither a multiple of 10 nor a multiple of 11, and write it in Box A (Fig. 1).

Step 2: Reverse the digits of the number in Box A and write the number in Box B.

Step 3: If the number in Box A is larger than the number in Box B, skip steps 4 and 5 and go to step 6. If not, go to step 4.

Step 4: Subtract the number in Box A from the number in Box B and write your answer in Box C.

Step 5: Skip step 6 and go to step 7.

Step 6: Subtract the number in Box B from the number in Box A and write your answer in Box C.

Step 7: Subtract the smaller digit of the number in Box A from the larger digit in that box and write your answer in Box D.

Step 8: Divide the number in Box C by the number in Box D and write your answer in Box E.

"Identify the subgoal and pursue it. Upon completion, survey again the original problem. Its solution may now become apparent, or a new subgoal closer to the solution may appear."

The Ohio Department of Education, *Problem Solving . . . A Basic Mathematics Goal*

X=

	1st run	2nd run	3rd run
Box A			
B			
C			
D			
E			

Fig. 1

Run the program 3 times, choosing a different number for Box A on each run. (No matter what 2-digit number you choose for Box A, the number 9 should appear in Box E.)

2. Beginning with the number 6, perform the following operations in the given sequence in order to obtain the answer 23.

a. Square the number.
b. Add 3.
c. Multiply by 2.
d. Subtract 9.
e. Divide by 3.

3. Beginning again with the number 6, rearrange the sequence of the 5 operations in #2 in order to obtain each of the following answers:

a. 27
b. 6
c. 5
d. 9
e. 0

4. Using the symbols $+$, $-$, $\times$ and $\div$, as well as parentheses, obtain a true statement in each of the following exercises:

(Example: Given 7 3 5 = 22, you can write
$7 + (3 \times 5) = 22$.
Remember your order of operations.)

a.(1) 4 3 6 = 13
(2) 4 3 6 = 1
(3) 4 3 6 = 36
(4) 4 3 6 = 2

b.(1) 12 3 1 = 3
(2) 12 3 1 = 6
(3) 12 3 1 = 15
(4) 12 3 1 = 24

c.(1) 8 4^2 = 24
(2) 8 4^2 = 16
(3) 8 4^2 = 4
(4) 8 4^2 = $\frac{1}{2}$

Managing a Word Problem

When it comes to word problems, students tend to jump right in and perform operations, often without stopping to understand and analyze the relationships represented by the problem.

Managing a word problem implies approaching the problem systematically, investigating it in an orderly fashion, breaking it into subproblems or tasks as necessary, and exploring alternative solutions or solution processes.

A box that is 4 feet by 4 feet by 4 feet is packed with cans that are 2 feet high and have a diameter of 6 inches (see Fig. 1). When the box is fully packed with cans, how much space is wasted in the box?

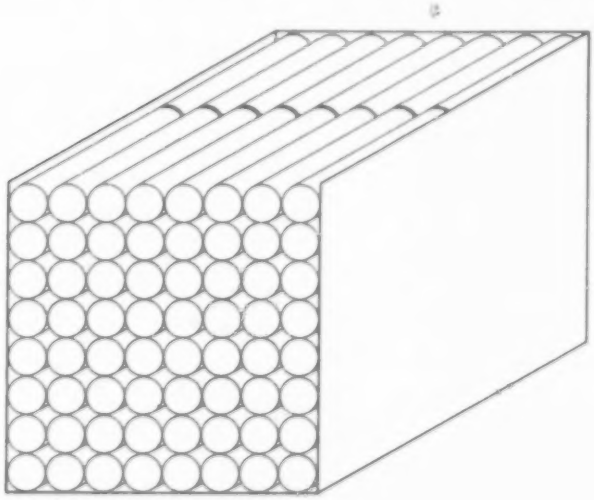


Fig. 1

In order to compute the wasted space, it will be necessary to find the total volume of the box less the volume of all the cans that can be placed in the box. The subproblems break down as follows:

Subproblem 1: Volume of box
Subproblem 2: Number of cans that can be placed in box
Subproblem 3: Volume of all the cans
Subproblem 4: Volume of box − volume of cans = wasted space

(Find formulas necessary to solve problem.)

Subproblem 1: Volume of the box = length × width × height = 4 ft × 4 ft × 4 ft = 64 cu ft.

Subproblem 2: Number of cans in the box. Diameter of cans = 6 in or 1/2 ft

a. $\frac{4 \text{ ft}}{1/2 \text{ ft}} = 8$ cans on a side

$8 \times 8 = 64$ cans

b. Because the cans are 2 ft high and the box is 4 ft high, 2 layers of cans will fit in the box.

Total number of cans in box = 128.

"When confronted with a problem, we sometimes can see that its resolution will require a series of actions, each dependent upon the preceding one. We may also face a problem whose specific solution is just beyond reach, but we have strong feelings that certain steps will either produce the solution or give us the additional material to find an answer. In each instance, identifying a subgoal is a way of making the problem smaller and getting started."

The Ohio Department of Education, *Problem Solving . . . A Basic Mathematics Goal*

Subproblem 3: Volume of all the cans

Volume of a cylinder =
$h = \pi r^2$
$h = 2$ ft
$\pi = 3.14$
$r = \frac{d}{2} = 3$ in $= 1/4$ ft

Volume of can $= 2(3.14)(1/4)^2 = \frac{1.57}{4}$

Volume of all cans $= 128 \frac{(1.57)}{4} = 50.14$.

Subproblem 4: Wasted space = volume of box − volume of cans
Wasted space = 64 − 50.14 = 13.86 cu ft.

For Further Practice:
Give your students some word problems from their textbook; omit the numbers given in the problems. Instead of having them solve the problems, ask them to write a plan for each, describing how they would "manage" the solution of the problem.

Managing a Number-Search Problem

Sometimes problems cannot be solved directly; they have to be closed in on with successive tries. Many students fall down in managing this kind of trial-and-error process. They fail to keep track of their successes and failures so they cannot use that information as feedback to improve performance in successive trials.

Here's a trial-and-error problem.

Start with the number 99. By adding 11 and subtracting 7 (as many times as necessary), try to get to 100.

Talk through the problem with students, asking them to share aloud their solution process, and to compare strategies. Then, show them the diagram in Figure 1 as one way they might see the solution process.

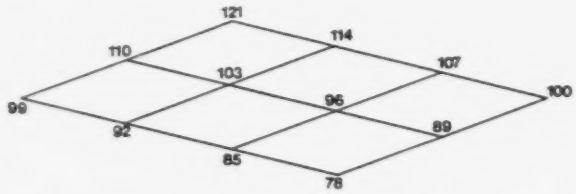


Fig. 1

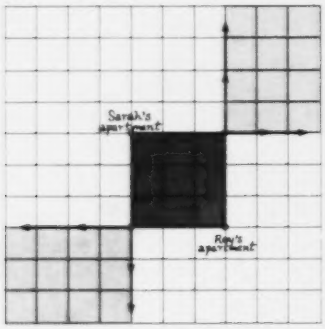
This diagram shows the problem can be solved in a minimum of 5 moves. It also shows there are at least 10 ways to get from 99 to 100. See if your students can find all 10.

For Further Practice:
You can tailor the problem for your class by changing the kind of legal moves that can be made. Use other mathematical operations such as multiplication, subtraction, squaring or even functions.

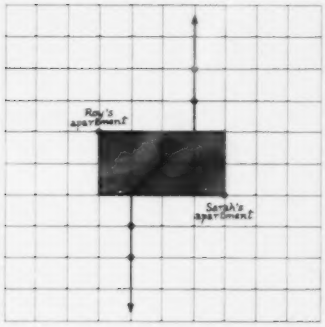
Equidistant

There is a range of locations that meet the requirements of each problem. Most of these locations may be too far away to be practical, but a good problem manager will get a picture of several possible solutions before deciding on a preferred one.

Reasonable solution space:



Solution #1



Solution #2

Oasis

There's no direct way to solve this problem, which signals a need for a management approach. As is, the 5 full, 5 half-full and 5 empty canteens cannot be shared equally among the adventurers.

Subdivide the problem into two parts: How much water will each man carry (7.5 canteens − 3 = 2.5) and how many canteens (15 − 3 = 5)? Each man will end up carrying 2.5 canteens of water distributed among 5 canteens.

Next, outline some alternative ways in which 2.5 canteens of water can be distributed among 5 canteens: 5 half-full canteens; 3 half-full canteens, 1 full and 1 empty, etc.

Equidistant

Sarah and Roy live in a city, where going anywhere means following a grid-like pattern of streets. All blocks are of equal length.

One Saturday morning, Sarah calls up Roy, and asks, "What do you say we meet for lunch?"

"Okay," says Roy, "but every time we get together, I always end up having to walk farther than you. Let's find somewhere to meet that's the same distance from both of our apartments."

"That sounds fair," Sarah agrees. "Where should we meet?"

1. Find all the points that are the same distance from Sarah's and Roy's houses on the map in Figure 1.

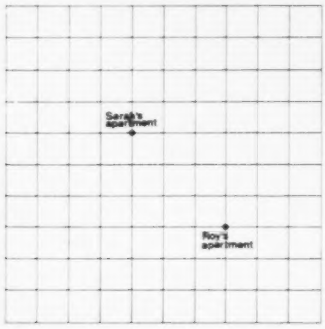


Fig. 1

2. What if Roy moves to an apartment building in a slightly different location? Find all the points that are equidistant from both apartments on the map in Figure 2.

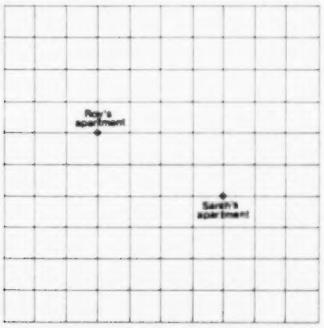


Fig. 2

Oasis

Josh, Robert and Brad are left stranded in the middle of the desert when their camels, frightened by a mirage, throw them and run away. The three decide they have a greater chance for survival if each one sets out in a different direction. This way it is more likely that at least one of them will pass by a sufficient number of oases where canteens can be filled to enable him to reach civilization.

They have 5 canteens full of water, 5 canteens half-full of water and 5 empty canteens. All the canteens are the same size. Since each man wants to carry as many canteens as possible (to collect water should he find an oasis), and each wants to set out with as much water as the others, they need to divide the supply of water and canteens equally among them. How can they do it?

Teacher's Notes

(see opposite page)

A Weighty Problem

There is often more than one way to break a problem into parts. This problem offers an opportunity to see how different management strategies can lead down different solution paths. Here are just two ways in which to solve this "weighty problem."

1. Replacement: Looking at picture B, you see that the weight of a cube equals the weight of a bolt plus the weight of a nail. Replace each of the cubes in the first picture with a bolt and a nail; you have 3 bolts and 2 nails opposite 8 nails. Take 2 nails from each side to keep the operation symmetrical. You are left with 3 bolts opposite 6 nails, which means that the weight of 1 bolt equals the weight of 2 nails. When you refer back to the second picture, replacing the bolt with two nails, you will find the cube balanced by three nails.

2. Elimination: You may decide to try to put cubes on one side of the scale and nails on the other. First, you have to find a way to get rid of the bolt in picture A. The information in picture B tells you that you can replace the bolt and nail on the left side with a cube. Add a nail to each side in picture A. The balance is preserved. Then you can replace the bolt and nail on the left with a cube. This leaves 3 cubes balanced by 9 nails. Therefore, the weight of 1 cube equals the weight of 3 nails.

Can your students come up with some other solution paths?

Take Me to the River

The first step in solving this problem is to identify the key elements—the *assumptions* and *problems*—involved.

The first assumption: There is no way to accomplish the task in a single crossing. In fact, the problem entails making many crossings with different passengers in each crossing.

Perhaps the most important subproblem of all is the *goose*, who quite clearly causes trouble for just about everyone on the boat. Therefore, separating the goose from the fox and the corn becomes critical to solving the problem.

One strategy for managing this problem is to devise a way of trying out solutions and keeping track of trials. Illustrations, or even movable cutouts, can be valuable tools to problem solvers in shedding new light on confusing situations. Here's one way students might see the solution:

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Step 6:

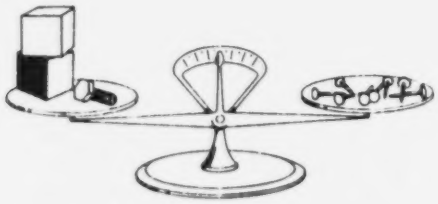
Step 7:

What do the river crossers do now that they're together again?

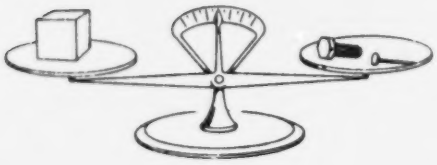
A Weighty Problem

How many nails will balance the cube?

A.



B.



C.

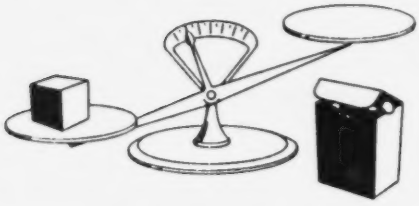


Fig. 1

An important part of managing a problem is devising options or alternatives and evaluating which is most effective in solving the problem. There are a number of ways to solve this problem. Try to come up with more than one path to a solution. Then compare them. Which was most efficient?

Take Me to the River

A man, a fox, a goose and some corn are on one side of a river. The man needs to get himself, the animals and the corn across the river using a boat, which will carry him and only one other item at a time. The fox will eat the goose if they are left alone together, and the goose will eat the corn if left alone with it. How can the crossing be made?

Trying to be Square

The number of rows into which the band leader has tried to fit the musicians must be known before the problem can be solved. This is the *subgoal* of the problem.

x = number of rows (on the band leader's first attempt)

In the first case, the total number of people is the number of people in the square plus the 24 people left over:

$x^2 + 24$ = total number of people

For the second case, the band leader adds a row, but is shy 25 people. So:

$(x + 1)^2 - 25$ = total number of people

Therefore:

$(x + 1)^2 - 25 = x^2 + 24.$

Solving for x, you get:

$x^2 + 2x - 24 = x^2 + 24$
$2x = 48$
$x = 24$

The band leader tried 24 rows and columns on the first try (25 on the second).

There are 576 members of the band.

For More Practice:
Have students make up and solve problems similar to this where the band is made up of less than 100 members; 100-200 members, 200-300 members; etc.

Two Trains Runnin'

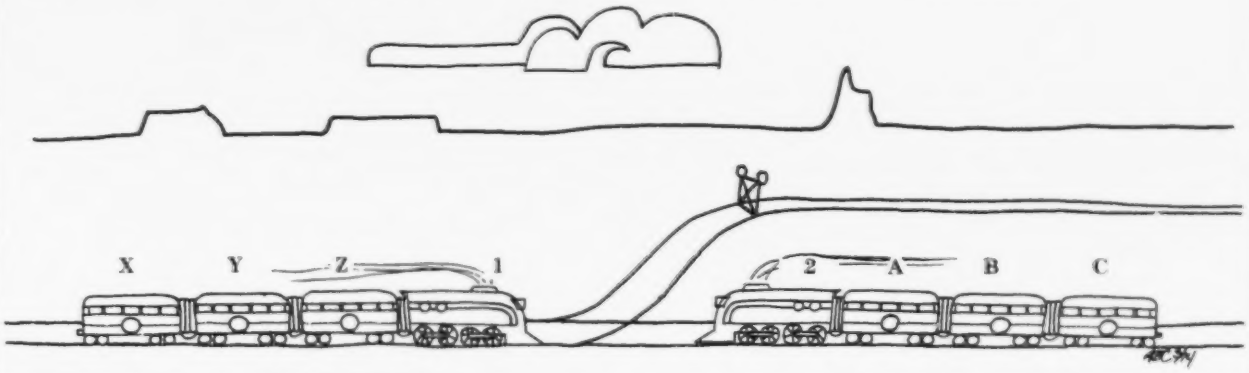
The solution steps are:

2ABC to left
2ABC to S (BC on siding)
2A to left (leaving BC on siding)
2A to right
XYZ1 to S (picks up BC on siding)
XYZ1BC to left
2A to left
2A to S (2A on siding)
XYZ1BC to right (past siding)
2A to left (off siding)
XYZ1BC to S (BC on siding)
XYZ1 to left
XYZ1 to right (past siding)
2A to S (picks up BC on siding)
2ABC to left

(Note: If students are having trouble figuring out the solution to this problem, they might try the same problem but with 2 cars for each train and with the siding holding only 1 car.)

Two Trains Runnin'

Two engines with 3 cars each are going in opposite directions on the same track. Near the point where they meet is a siding which will hold only 2 cars *or* 1 engine and 1 car at a time. How can the trains and their cars pass each other and continue on their way? (Note: Each engine may push or pull cars.)



Trying to be Square

The leader of a marching band wants her band members to march in the form of a perfect square for a special halftime number at the big game. During rehearsal, she arranges them in rows and columns to form a square. The first time she tries it, she ends up with 24 people left over. So she adds a row, but finds she is now 25 people short. How many band members are there altogether?

"The process of decomposing a problem into manageable subgoals is clearly an important aspect of planning in problem solving."

James G. Greeno, *Problem Solving and Education*

Teacher's Notes

(see opposite page)

A Parking Nightmare

The "parking nightmare" problem is a variant of the classic "magic squares" problem. Because this version is so complex, it is best done with the class as a whole. You may want to have your students make cutouts so they can try various alternatives.

The correct answer is *41 moves:* 13 DOWN – 14 DOWN – 11 RIGHT – 12 RIGHT – 18 UP – 19 UP – 24 UP – 34 UP – 35 UP – 33 RIGHT – 28 DOWN – 29 DOWN – 15 DOWN – ELROY'S CAR RIGHT – 8 DOWN – 9 DOWN – 10 LEFT – 16 UP – 17 UP – ELROY'S CAR RIGHT – 15 UP – 28 UP – 29 UP – 33 LEFT – 34 DOWN – 35 DOWN – 24 DOWN – ELROY'S CAR RIGHT – 16 DOWN – 17 DOWN – 2 DOWN – 3 DOWN – 4 LEFT – 5 LEFT – 18 UP – 19 UP – 11 LEFT – 12 LEFT – 13 UP – 14 UP – ELROY'S CAR RIGHT AND **OUT**. Count 'em.

For Further Practice:
As a follow-up, you might ask students what would happen to the solution process if Elroy could not only move his car forward and straight back, but could turn it as well. After all, he does have the keys to *his* car.

Extracurricular Activity

Kalah, or What Do I Do Next?

Kalah is an old African game which requires a change of strategy after every move. It is a good exercise in changing plans and reexamining subgoals every step of the way.

You can use an egg carton to make a Kalah board (see Fig. 1). Cut the lid of the carton in half and fasten one half to each end of the egg carton bottom to form each player's Kalah (pit).

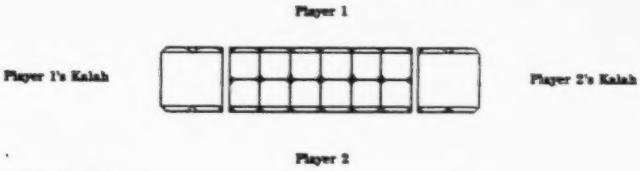


Fig. 1

The Action:
Two players sit on opposite sides of the board. The 6 pits on a player's side of the board belong to that player, as does the Kalah to the player's right. At the start of play, 3 beans are placed in each pit, but none are placed in either Kalah. (*Note:* Kalah may also be played with different numbers of beans in the pits to start.)

Play begins by the first player picking up all the beans in one of his pits and placing them, one by one, in each successive pit around the board to the right—including, if he has enough, his own Kalah—and on into his opponent's pits. Once a bean lands in Kalah, it stays there. If a player's last bean lands in his own Kalah, that player gets another turn. If a player's last bean lands in an empty pit on his *own side* of the board, he captures all of the beans in his opponent's pit directly opposite his pit and places them, along with the capturing bean, in his own Kalah. The object of the game is to capture all of your opponent's beans in your Kalah.

Once you have played a few games, discuss strategies you have found helpful as well as problems you may have had. Identify the *subgoals* that lead to a winning strategy.

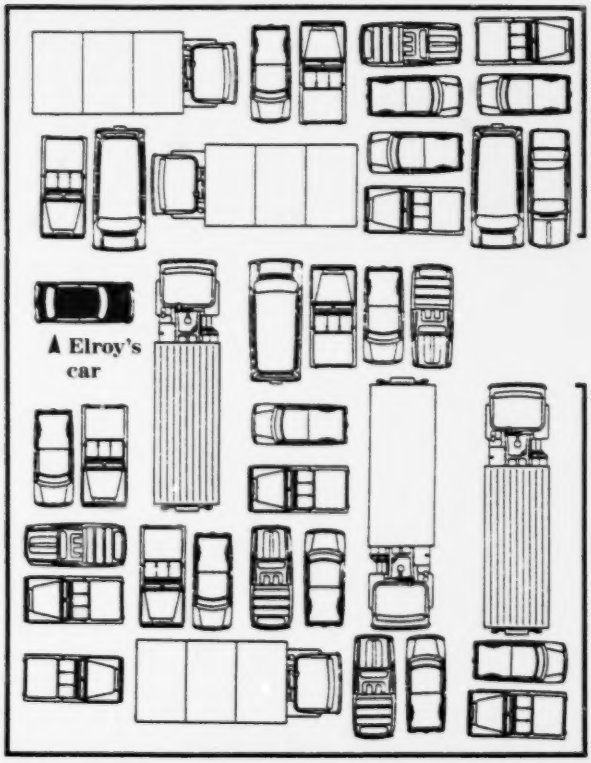
A Parking Nightmare

Elroy parked his car in a downtown lot early this morning. When he came back at 2 p.m., it was completely boxed in.

It was the attendant's first day on the job, and he had forgotten to get keys from the car owners, so even though they had left their cars unlocked, they couldn't be started. Unfortunately for Elroy and the parking attendant, all the cars in the lot were equipped with anti-theft devices so the steering wheels were locked and the cars could only be moved straight forward or straight back.

With a lot of ingenuity and careful step-by-step management of the problem, Elroy and the attendant got his car out in 41 moves. How did they do it?

(Hint: One move equals the straight forward or backward rolling of a car *at least* one car's length. There are no trick moves, though some vehicles are moved more than once.)



Elroy's Revenge

Elroy has been hired to design and construct a parking lot in front of the new supermarket in his neighborhood. After his terrible experience being stuck in the parking lot, Elroy wants to be careful to design a lot that not only holds the greatest number of cars, but also is easy to get in and out of. The land measures 54.2 meters by 158.9 meters. Show Elroy how to design the best layout for the parking lot.

As part of the preplanning for this project, make a list of considerations that should go into the design. For instance:

- The average car is 5 m × 2 m.
- The land must be fairly level.
- There must be enough room between cars for doors to open.
- Cars must be able to get in and out.
- Should the lanes be one way only?
- How wide should the lanes be?
- How well does each design fit into the shape of the given area?

Try out some possibilities.

Evaluate the parking lot designs you come up with. How might they be improved?

ESTIMATION

Am I Close?

Fact Sheet

ESTIMATION: *Am I Close?*

Running Time: 21:04 minutes

Documentary Segments (3):

1. ***SHARKS:*** **How many sharks are there in the Bimini Lagoon?** Two estimates are better than one. ... 207
2. ***CHINA:*** On July 1, 1982 the population of China was 1,008,175,288. **How accurate is the figure?** ... 214
3. ***AVALANCHE:*** Predicting avalanches requires gauging probabilities based on long experience and lots of data. **What do avalanche forecasters then do with their predictions?** ... 220

Comedy Segment: ... 204

4. ***THE BAKERY:*** **"Bill the Baker" knows that his watch is in one of 360 pies, but how will he narrow the search?**

Math Concepts:

SHARKS: Transect sampling, tag and recapture, ratio and proportion, statistics, percentages, mathematical modeling

CHINA: Standard error, statistics, measurement, percentages, large numbers (exponents, powers)

AVALANCHE: Probability, measurement, angles

Problem-Solving Concepts:

Educated guessing; scale of problem; appropriate (useful) answer; constraints of problem; check test data; provisional solutions; expertise.

HOW TO SPOT AN ESTIMATION PROBLEM

You can tell it's an ESTIMATION problem if . . .

■ ***An exact answer is not necessary.*** *In an estimation problem, you don't need to know the answer down to the last detail. "Close" is good enough.*

■ ***The problem must be solved within limits and constraints.*** *If you have limited resources to apply to the problem—of time, money, or energy, for instance—a good estimate is often better than an exact answer that comes too late or costs too much.*

■ ***The problem or problem situation is familiar or routine.*** *An estimate may be all you need. Everyone has a supply of "rules of thumb," which become more accurate with practice and growing expertise.*

■ ***The problem involves very large or very small numbers.*** *In such cases, calculations can go awry and be off by several orders of magnitude. Estimation helps keep the scale in mind and the calculations within the right math framework—the right ballpark.*

X=

Throughout the chapter, the X = sign indicates pages containing exercises and problems for classroom use and discussion.

BEST COPY AVAILABLE

ESTIMATION

Am I Close?

Our world runs on estimates—on projections, tentative conclusions, approximations. They shape our thinking and guide our decisions. Simply think of all the terms we use every day to label estimates—guess, about, around, roughly, approximately, more or less, plus or minus—and you will realize how central estimation is to our lives.

Estimation is the way to size up a challenge, take its measure and guess the likely outcomes. Because many problems crop up quickly or must be answered almost immediately, we use estimates. The world often places a higher premium on a readily available, workable answer than on slow-moving exactitude. We rarely have the luxury of collecting and processing all the necessary information, performing in-depth analyses or calculating alternative solutions. We run our homes, jobs and hobbies with estimates. Equally, the economy, unions, corporations and universities largely run on ballpark figures. Wages are negotiated based on projected rates of inflation. Research studies are budgeted and staffed based on careful estimates of the nature and duration of the work. Faculty are hired and tuition set after estimating costs and numbers of incoming students.

We rely on estimates to conserve our resources—time, money and energy. Problems are usually solved under limits or constraints. Most solutions *are not* the perfect answer, but the most useful answer within the limits set by our resources. The majority of real-world problem solving doesn't require exact answers; close is close enough and the best answer possible under many circumstances.

Why Estimate?

Estimation is perhaps the most useful skill a student can take out of math class into the "real world." As our world depends more and more on calculators and computers, estimation skills become more and more important. Machines can perform the operations and provide *an* answer. People must decide whether it's *the* answer. Estimation skills tell us if the answer's in the right ballpark, if the decimal point is where it belongs. Estimation is important in checking calculations; it's useful in checking *thinking*, too.

Students must first understand that estimation is more than just guessing: It's *educated* guessing. It doesn't take long to show them the difference. Ask them to estimate an answer to a word problem, then ask, "How did you get that answer?" If it's a wild guess, the student will respond, "I don't know." If it's an estimate, the response will be something like this: "Well, if the room's 12-feet square, that's 4 yards by 4 yards, which is 16 square yards. And since the carpet costs $15 a square yard, the room

"Of course these are only ballpark figures."

204

costs a little more than $200 to carpet. Because 10 times 15 is $150 and 20 times 15 is $300, it's in between the two." This is an estimate rather than a guess because there is conscious problem solving going on.

Teaching Estimation

Students already have a good foundation of estimation skills drawn from their lives outside school. Like all of us, in areas of interest and experience—whether estimating how much it will cost to repair a bike or how long it will take to finish a page of homework problems—they are good estimators. In math class, teachers can draw on this experience as groundwork and show students how to improve and expand their estimation skills.

Estimation is efficient. It could be called a lazy person's route to a useful answer, a shortcut. Estimation can save time, energy and personal resources—without sacrificing the usefulness or validity of the result. If you can navigate a problem successfully with an educated guess, so much the better.

Estimation avoids answers that don't fit the problem. Good estimators keep scale—the right ballpark—in mind. They don't fall into the trap of some math students, for instance, who tend to disregard decimal places, claiming 273 is "close to 2.73 because the digits are the same."

Estimation measures an answer by its usefulness. Estimation is not solely a quantitative concept, but a qualitative one as well. Good estimators know when a problem is worth a definitive answer; they take careful measure of the problem situation and its constraints to size up what's needed to arrive at a workable solution. A perfect answer may not be necessary. It may arrive too late or be too costly or too elaborate to suit the conditions of the problem.

Estimation is a skill that improves with practice and experience. An experienced car mechanic is likely to give a more accurate estimate of the car's problem—how long it will take to fix it, and what the costs will be—than a mechanic who is new to the job. Estimation is a skill students already possess to some degree; it is a skill that improves the more they use it.

Mathematics has developed techniques to yield better estimates. If a problem requires too much time or effort to solve, or appears because of its scale or nature to be unsolvable, there's probably a mathematical shortcut. In this sense, estimation covers not just the skills and techniques necessary to get better, workable solutions, but also conceptual skills to organize problems in such a way that they can be solved efficiently with available resources. ■

ON THE MOVE: Caribou cross the barren stretches of the Alaskan interior in their annual July migration. One way to estimate the population of roaming herds of caribou, elephants, timber wolves—and even sharks (see story, page 207)—is from the air. Observers fly over their habitats in a strick back-and-forth pattern, counting the animals in defined areas, called *transects*, or photographing them for a later count.

Estimation in the Classroom

Most math teachers encourage students to develop estimation skills. When a student divides 21.6 by 3 and comes up with 72, we respond: "Think again. Does that *look* like the right answer? Ignoring the 6/10ths for the moment, what do you get when you divide 21 by 3?"

"Ballparking" the answer—testing assumptions and possible limits, getting a firm grip on scale, guessing—are crucial estimation skills.* But the ESTIMATION film (and this chapter) take this aspect of problem solving a step further. Estimation also includes some mathematical techniques to derive approximations, to get "good enough" answers the quick way.

Sampling: A national census—or counting large numbers of anything—is expensive, time-consuming and prone to error. Consequently, sophisticated statistical techniques have been developed to sample various populations and derive estimates that can rival—and sometimes outdo—actual counts for accuracy.

A MATTER OF JUDGMENT: To get the ball through the wicket, a croquet player must be a good estimator—of the angle, distance, roll of the terrain, and amount of force to apply to the ball.

Error: A crucial concept in math, science and engineering is the exactness of an answer or measurement. For instance, in a scientific experiment, an engineering design or a statistical study, no measurement is perfect, but it must be exact within certain limits or tolerances, in order for the experiment (or design or study) to have any value. Equally, in carpentry, no measure is exact, but the margin of error must fall within certain limits for the piece of work to function as a table, bench, chair or whatever. The idea that even topflight scientists, mathematicians and engineers consider error an inevitable outcome of their work should be a shot in the arm to math-anxious students.

Limits: Testing limits or extreme cases are mathematical terms for "ballparking." They are useful, for instance, in solving upstream–downstream word problems, which often give students a great deal of trouble. Students can use estimation skills to check their answers by testing the problems against the extreme cases: a river with virtually no current versus a river flowing at 100 miles per hour. These give them limits within which the answer must be found.

*There are a number of excellent articles and books for teaching estimation, including *Guide to Using Estimation Skills and Strategies (GUESS)* by Barbara and Robert Reys (Dale Seymour Publications, 1983); "Instruction in Estimating Solutions of Whole Number Computations" by Harold L. Schoen et al. in *Journal for Research in Mathematics Education* (May 1981); "Estimation and Reasonableness of Results" by Bob Underhill in *The Agenda for Action: 1983 Yearbook* (National Council of Teachers of Mathematics), and "Estimate How Much" by John Van de Walle and Charles S. Thomson in *Arithmetic Teacher* (May 1985).

Scale: Part of estimation is keeping the size of the problem in mind, a kind of numerical horse-sense that most students lack but can acquire with practice. Estimation exercises which demand that students think about very large and very small numbers and do some quick thinking about their implications can steer them to a better number sense. Ask the class how many steers they think it took to make 5 billion McDonald's hamburgers. (How big is a burger? How many pounds of meat does a steer yield? It's not important to get the right answer, but to get an answer in the right ballpark.)*

*You can figure about 1/4 pound per burger and 1000 pounds of beef per steer. In the May 1982 issue of *Scientific American*, Douglas Hofstadter wrote an excellent article, "Metamagical Themas," on number sense with several examples that easily can be adapted for class use.

Probabilities: On their own time, students may already be great "bookmakers," accurately weighing the odds of a teacher throwing a pop quiz or of their favorite sport's team winning. Still, getting a feel for the mathematics of probabilities is difficult not only for students, but also for most adults. Students learn the rules and definitions in school, but they have difficulty believing that the math of probability applies outside the text book and the classroom.

Range and Averages: Estimates often communicate either a range or an average. For instance, when asked how many letters she delivers each day, a postal worker might use a range, saying between 400 and 800 pieces of mail a day. Another might offer an average, saying about 700 pieces of mail on the average. An average, or mean, is a more sophisticated estimate because it assumes an accumulation of data over a longer period of time. Students often compute averages from data without realizing their answers are reasonable estimates.

Confidence: In the ESTIMATION film, Onno Wieringa, a hazard forecaster, speaks about firing the gun at a snowfield with "good confidence" that the blast will hit the mark, after having used a gunner's quadrant to sight it. Confidence is a byproduct of good estimation skills, the kind of confidence that defeats math anxiety *and* the kind of confidence experts have in their solutions. It is not the confidence that comes with having "the right answer," but the confidence that comes with having gauged the problem and found an answer that's about the right size. ■

"Explore directions that appear plausible. Do not hesitate to embark on a new direction on the basis of partial evidence. If a route appears plausible, pursue it. It is a mistake to disregard a conclusion because it is only probable and not certain."

Moshe F. Rubinstein, *Patterns of Problem Solving*

Behind the Scenes: ESTIMATION On Film

Getting real problem solving on film is risky business. Things go wrong; people make mistakes. But if things go all too well, the challenge and the suspense evaporate, leaving the viewer wondering, "who cares?"

All four of the sequences in the ESTIMATION film were chosen to show students how useful estimation is in solving problems, and to make people "care" about the problems being addressed and the people who are addressing them.

Sharks: When filmmakers scout for stories to capture the audience, they often look for exotic locales and adventure. Samuel Gruber's airborne population survey in Bimini Lagoon fills this bill. His use of both sampling and survey techniques shows the necessity of corroborating evidence and checking one's answers. Most important, Gruber's story shows student viewers that wrong turns are an inevitable part of problem solving and often lead to better results—better estimates.

China: In the China census segment, we found a familiar story in a remote, strange locale. Here we explore not only the idea of large numbers and how to deal with them, but also the concept of error and error rates. Counting the more than one billion people in China also demonstrates the need for careful planning, feedback and management skills in solving big, multidimensional problems.

Avalanche: When is an estimate "good enough"? What happens if the estimate is wrong? The snow rangers of Alta, Utah, who are responsible for predicting and controlling avalanches, show viewers just how important expert estimation skills are. Because life and death hangs in the balance when snow rangers make their forecasts, they need back-up systems to take over when they guess wrong.

Bakery: "Bill the Baker," in the comedy segment, takes estimation into the world of the absurd. To get the pie shells under the plopper and to find his lost watch, he uses estimation skills. He devises three methods of weighing possibilities or narrowing the scope of his search: probing with a spoon; judging relative weights; and listening for the tell-tale tick-tock. With a step-by-step approach, he gets his watch, but loses his job—an estimable tale indeed! ■

GETTING STARTED: **Thinking ESTIMATION**

The following sample exercises are designed to give students some tools to hone their estimating skills *before* they view *ESTIMATION: Am I Close?*

1. Most of the numbers we come across every day on television and in magazines and newspapers are estimations or approximations. Sometimes exact figures are unnecessary or needlessly cumbersome; sometimes they are difficult or impossible to obtain. Sometimes, as in advertising, an exact figure might intimidate and an estimate may be more persuasive.

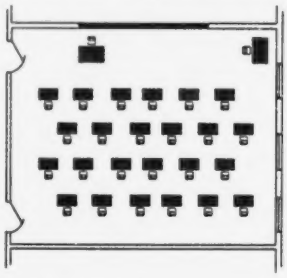
Which of the headlines below are estimates, and which are not? For each estimate, decide whether a precise number is (a) unnecessary or cumbersome; (b) too large or small or difficult to obtain; (c) more or less persuasive as an estimate.

"Unemployment Up 2%"
"Mayor Cuts Park Budget by $1 Million"
"100,000 Attend Opening Day Ceremonies"
"Fire Leaves 17 Homeless"
"Scientists Unearth 6-Million-Year-Old Skull"
"New Automobile Gets 63 MPG"
"Cut Reading Time in Half"
"Stock Market up 12 Points in Heavy Trading"
"A Family Car for Under $7,000"

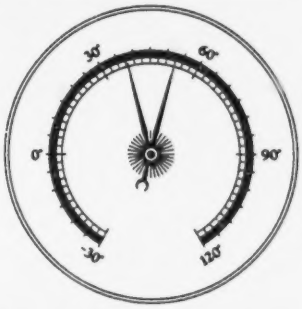
2. Estimate, then check:

- the weight of your math text

- the area of your classroom



- the temperature outside



- the number of days since you were born

- how far you can throw a baseball

3. **Estimate the answers to the following, then check:**

89 × 3112 =

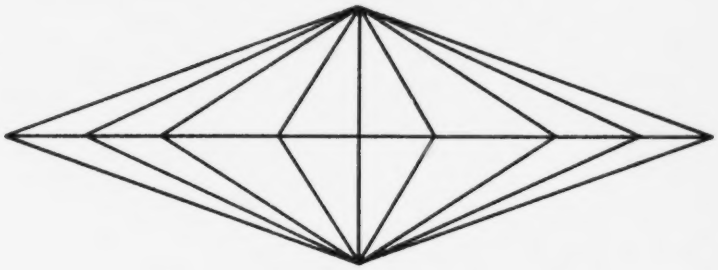
(a) 24,448
(b) 260,728
(c) 276,968*
(d) 246,788
(e) none of these

86 ÷ 3/4 =

(a) 32
(b) $47^{1}/_{2}$
(c) $114^{2}/_{3}$*
(d) $131^{3}/_{4}$
(e) none of these

*Correct answer

4. **How many triangles do you think are in the figure below? First estimate. Then count. How close were you?**



Though it may not be obvious at first glance, there are a lot of triangles in the picture—80, to be exact.

For Openers

Using the ESTIMATION film as a jumping-off point, ask your students to explore some of the following questions to give them a better "feel" for estimation:

- What do you think is meant by *estimation*?
- Why do people turn to estimation to solve problems?
- What are some examples of estimation skills that you call on every day? In school? At home? Doing errands? At play?
- What about the characters in the film segments you have just seen? *Why* and *how* did each of them draw on estimation skills to help them solve the problem(s) at hand?
  1. What is "Bill the Baker's" problem? How does estimating help him solve it?
  2. How does estimating help Samuel Gruber in his study of lemon sharks in the Bimini Lagoon? Why does he have to "settle" for less than an exact count?
  3. How did Chinese officials employ estimation skills in taking the 1982 Chinese census?
  4. What are the various ways in which avalanche forecasters draw on estimation skills to predict when and where an avalanche is likely to occur?

Stalking an **Estimation** Problem

Making Up Your Own Problems

The following is a hypothetical conversation between teacher and students. It is intended to suggest ways to encourage students to make connections between problem-solving concepts in the film, their own experiences and their knowledge of mathematics.

"

- ***Class, close your books. We're going to make up our own estimation problems.***

  How do we do that?

- ***I'll answer your question with another question. When do you guess. . .use estimation?***

  Well, I guess how long it will take me to do my homework.

- ***So you use estimation to get a rough answer. And what would it take to get an exact answer?***

  You can't get a perfectly exact answer.

- ***Why not?***

  Well, you can, but you have to finish your homework first.

- ***So why do you estimate?***

  Because it's easier.

- ***Let's think about the estimation problems in the film.***

  Well, the shark story is like my homework problem. Counting sharks is too hard, so he tries to figure out a way to count some of them and get an estimate of the total number.

- ***He uses a sample. Right? He says, there's not enough time or money to count all the sharks. . .so you might call this a "scarcity" estimation problem. What about the avalanche story? Why is that an estimation problem?***

  Well, it says in the film you don't know exactly where or when an avalanche may happen, so you have to guess when the conditions may be right.

- ***They estimate probabilities. When the chances are high, they have to take action. Can you think of examples like this from your own life? No? What about predicting whether it will rain today? How do you decide whether to take an umbrella?***

  I listen to the weather report and then look outside.

- ***And make an estimate. Why?***

  Well, it's the only answer available. Unless it's already raining.

- ***Like the avalanche. Your estimate is the best answer available when you need it, right?***

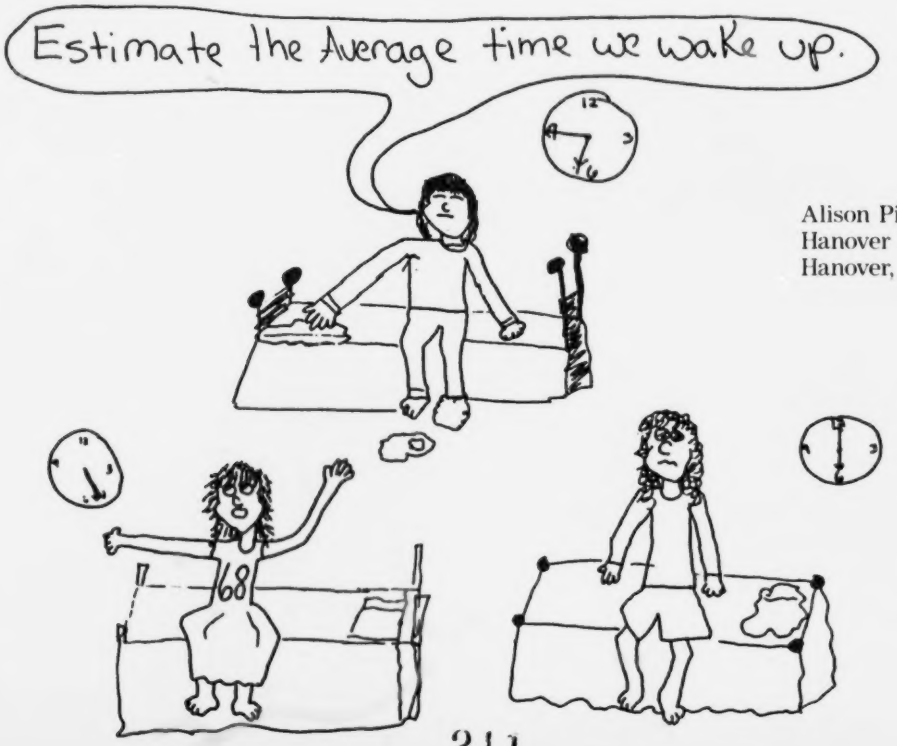
  You need an answer and an estimate would be better than no answer at all. At least I'd be prepared for rain if I thought there was a good chance of rain in the forecast.

- ***Any other kinds of estimation problems?. . .Well, I call an estimation problem a "good enough" problem. Sometimes you need a more precise answer, and sometimes a good guess is good enough. Can you think of an example?***

  Well, if I went to the grocery store and I wasn't sure that I had enough money to pay for everything, I'd do a rough estimate—to the nearest dollar—of how much everything in my shopping cart would cost.

- ***We can do an exercise to test our estimation skills at the supermarket. I call it "Keeping Tabs on the Tab." (See page 225.) Let's try it.***

"



Alison Pierce
Hanover High School
Hanover, N.H.

Following Through: Problem Ideas from the Film

The ESTIMATION film provides plenty of raw material for making up your own problems. Here are some examples, ranging from sample exercises to open-ended problems.

Sharks: The two techniques that Samuel Gruber uses to estimate the shark population in the Bimini Lagoon are basically the same techniques used by naturalists around the world in estimating the size of various animal populations.

- The tag-and-recapture method of sampling is based on probabilities. What if you took a bottle of 300 aspirin and replaced 10 of the aspirin tablets with "sugar" pills of the same color and size? What is the probability that you would choose one of the sugar pills the first time you went for an aspirin? The twentieth time?

- How would you count the dogs in your school district? Devise two methods to survey or sample the dog population. (*Suggestion:* Survey 10 blocks in various neighborhoods in the school district, counting the number of dogs seen in a given period of time; then, based on the numbers that you get, make an estimate for the total number of dogs in the district.)

China: In 1982, the Chinese conducted the largest, and perhaps the most accurate, census in history. The total: 1,008,175,288, plus or minus .015%.

- At one number every second, around the clock, it would take approximately 31 years to count to 1 billion. How long would it take to count to 1 million? One thousand?

- How far is it from Chicago to Tokyo? Using the scale on a map of the world, approximate the distance. Check your estimate against the figure in an almanac or atlas. How different are the two figures? What's the margin of error? Why are they different?

Avalanche: Onno Wieringa and Binx Sandahl are responsible for estimating the probability of avalanches in Alta, Utah.

- To keep track of snow buildup and changing conditions, Sandahl logs in weather data every day. Keep your own weather log of temperature and precipitation for two to four weeks. Graph the figures. Do you see a pattern or trend emerging?

- Insurance rates are often based on the results obtained from detailed probability studies, eventually published as actuarial tables. An *actuarial table* establishes the premium to be charged by the insurance company to cover a given contingency. Although life insurance is the most common type of policy, you can get coverage for almost any contingency if you're willing to pay the premium.

  Make an actuarial table for the probability of rain interrupting your plans for a class outing. Keep track of the weather for a 30-day period. What is the probability of being rained out? Think of some other contingencies; plan and execute how you might make an actuarial table to study the probability of that "something" occurring.

"Guess and check. . . the problem solver makes an educated guess and then checks *it against the problem conditions to determine how to improve his guess. He repeats this process until he has an acceptable answer."*

The Ohio Department of Education, *Problem Solving . . . A Basic Mathematics Goal*

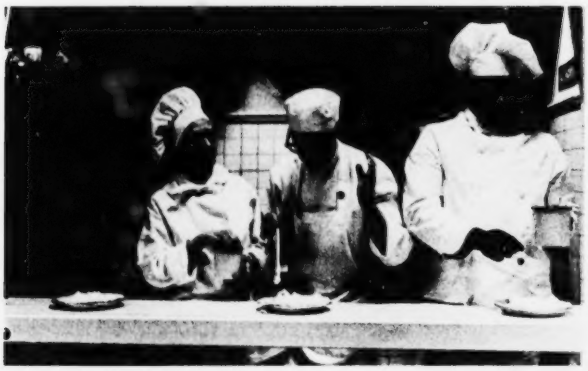
On the Lighter Side

Math can be fun, but many students don't believe it. A lot has been written lately about math anxiety: Both the pressure to get the right answer and the fear of failing make many students hate math.

ESTIMATION has an antidote: "Bill the Baker." He makes mistakes and lives to tell the tale. He even triumphs in the end.

The light-hearted saga of how Bill loses and finds his watch provides an opportunity for students to enjoy a laugh and learn from a negative example. They can develop their critical powers as problem solvers by exploring where Bill went wrong. And, they can learn a little math as well.

A Comedy of Errors: The Bakery



"My watch had disappeared! It must have been sucked into the holding tank. I knew there was roughly an hour's worth of custard in the holding tank. The plopper discharged every 10 seconds, so my watch had to be in one of 360 pies. 'Excuse me. . . have you seen my watch?'"

"My biggest fear was that one pie would go into the oven unchecked. I tried not to let the situation get the better of me. But never in my six years at the bakery had I ever had to solve a problem of such complexity."

"Half-Baked": Some Comic Word Problems

"Bill the Baker" estimates that there's an hour's worth of custard in the holding tank. The custard plopper discharges every 10 seconds, so his watch is in one of 360 pies.

- If it takes Bill 4 seconds to check every pie, how long will it take him to check all the pies?
- If he finds his watch after checking for 20 minutes, how many pies will he have checked? How many pies will have been filled?
- If Bill can enlist two co-workers to help him (and they can work at the same rate as Bill), how long will it take them to check all the pies? If Mother Hudson stops them after 15 minutes, how many pies would the three bakers have checked? How many filled pies would not yet be checked?
- Ask students to use their imagination to make up their own word problems.

"Then, inspiration struck. An AUDIO ESTIMATOR! All I had to do was find the pie that was ticking. My methods had paid off. Unfortunately, Mrs. Hudson had returned.

"Six years of fearless service and I was fired. Like that! I didn't have a job, but I had my self-respect. And I maintained it, even in the face of indignity."

Pieces of Pie

Use the idea of pies in the comedy segment to devise some estimation exercises, translating fractions to decimals and percents with pie charts.

For instance: **How big is the shaded piece of pie?**

a. About 60%
b. About 1/2
c. About .85

a. About 45%
b. About 2/3
c. About .65

Estimation With Class

Making Up Your Own Problems

This hypothetical dialogue between teacher and students provides examples of how to draw students into the process of devising and solving their own estimation problems.

"

♦ ***For the rest of the period, we're going to make up our own problems.***

How do we do that?

♦ ***You have problems in your own life that you have to solve. We'll use those. But remember this is math class, so let's work on problems with some numbers in them.***

I don't get it.

♦ ***Did you ever plan a vacation trip? Or save up for a bicycle? Or figure out how to get your parents to buy you a computer? Or build a treehouse? Or figure out how much paint you need to repaint your room? Any of this sound familiar?***

Does this count? When I had a slumber party, I had to order all the stuff.

♦ ***That's a great example. But you did more than just "order the stuff." You made up your own problem. You wanted to have a slumber party. You got your parents' permission. You planned how many people to invite, what to do and what to eat. Can you tell us about the whole process?***

Well, I invited about 15 girls over for a slumber party, but only 12 could come.

♦ ***Let's start farther back toward the beginning. Did you start out wanting a slumber party or did you have a couple of ideas?***

I knew I wanted either to have a slumber party or a picnic in the park, but I knew that if I had a picnic then my mom and another mom would have to drive and that's always a hassle. So, I decided it would be easier to stay at home.

♦ ***Okay, you narrowed down your options, based on an estimate of what resources were readily available.***

I did?

♦ ***What came next?***

I invited all the kids and. . .

♦ ***Wait a minute. How did you know how many kids to invite?***

My mom and I had a fight about that. I wanted to invite about 20 kids and she said no more than 15. So I invited 15.

♦ ***So 15 was the limit your mother set. How did you figure out how much food to order?***

The last slumber party I went to I remember I had about 3 slices of pizza and a bunch of cookies and 2 cokes and a glass of milk. So I multiplied those quantities by 12, the number of people who were coming.

♦ ***Here's the math part! Let's do this together. I'll put it on the board.***

3 slices × 12 = 36 slices
2 cokes × 12 = 24 cokes
1 glass milk × 12 = 12 glasses of milk

♦ ***Now we have these figures. How many pizzas did you order?***

Five.

♦ ***Why?***

There are 8 slices in a pizza and 8 into 36 is 4 pizzas with 4 left over, which means 5 pizzas to be on the safe side.

♦ ***And the cokes?***

That came out even. Twenty-four cokes is 4 six-packs. But the milk was harder. I think there's about 3 glasses in a quart, so that's 4 quarts or a gallon. The cookies I just guessed. I thought three boxes would be alright, and it came out okay.

♦ ***How did your other estimates come out? Did you have food left over?***

I overestimated on pizza. There was a whole one left over. I guess I forgot that I like pizza a lot and not everybody else does. The cokes were okay, but we only drank about 6 glasses of milk, so half of it was left over.

♦ ***And how was the party?***

Pretty good. It's better to have too much food than not enough.

♦ ***And I think you did a good job making up and solving your own estimation problem. And next time you'll be even better because you learned that not everybody likes milk or pizza the way you do and you'll order less next time. . . .***

Who's next with an estimation problem?

"

If it takes 15 minutes to skate around the very outside of the lake, how long would it take to skate straight through the middle?

Kathy Gee
Walled Lake Junior High School
Walled Lake, Michigan

One If by Air... Two If by Sea

How many sharks "make their living" in the waters around the 100-square-kilometer area of the Bimini Lagoon? The question is much easier to ask than to answer.

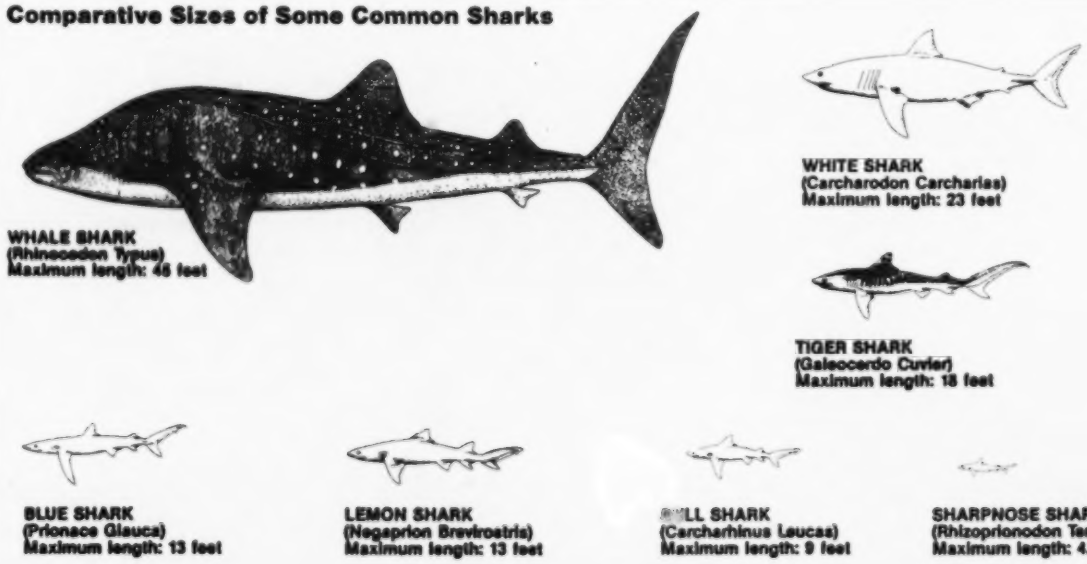
Despite the fact that sharks have received a good deal of notoriety—especially at the hands of filmmakers who have cast them in particularly evil roles in past box office hits—little is really known about them. What scientists do know is that sharks are fragile creatures. Few species can survive in captivity long enough to be thoroughly studied. In fact, even "tagging" a shark—one of the means by which scientists can count their numbers and monitor their movements—can cause injury and even death.

Marine biologist Samuel Gruber is one of those who is extending our knowledge of sharks. His interest has something to do with a chance encounter with a hammerhead shark while scuba diving more than 25 years ago. He won't say exactly what happened, but it left him determined to find out everything he could about these marine predators. Like humans, sharks are *apex predators*, that is, they live at the top of the food chain for their species, eating those below them in the food chain, but not being eaten themselves.

Gruber's studies have focused on the lemon shark (see accompanying box). In his laboratory at the University of Miami, he analyzes their energy budgets—how they use energy as they move; how much food they eat and how efficiently they metabolize it; and how fast they grow, mature and reproduce.* But, as Gruber points out, "Whatever goes on in the lab is, by definition, artificial. We can grow sharks ten times faster in the lab than they grow in the field."

*Gruber's findings have shown that lemon sharks take 20 years to mature, not 2 to 6 years as previously thought. This information is necessary for setting quotas for catching and killing sharks. Slow-moving, slow-reproducing animals such as whales or sharks can rapidly be hunted to the brink of extinction. Five species of shark have already suffered this fate.

Comparative Sizes of Some Common Sharks



Gruber Gets His Feet Wet

So, in the waters of Bimini, an island in the Straits of Florida between Florida and Cuba, Gruber continues his work on lemon sharks. Here he tests his laboratory findings to see how they apply to sharks "in the field"—or in this case, in the sea. He monitors the marine ecosystem, noting the temperature and salinity of the water, for instance, and relates his findings on individual sharks to the shark population as a whole.

How many sharks "make their living" in the waters around the 100-kilometer-square area of the Bimini Lagoon? The question is much easier to ask than to answer. First, Gruber sampled the population using a method known as tag-and-recapture (see accompanying box).

It was here that Gruber's plan ran into trouble, however, and his study was criticized for being biased as a result. In order to sample properly, sharks must be caught and tagged on a random basis. But the sharks in Gruber's sample failed to cooperate. Sharks are social animals and "site-attached," meaning that they travel in groups and tend to visit the same area repeatedly. Because Gruber and his team caught and tagged sharks only at spots in the lagoon where Gruber knew them to congregate, the sample was judged inaccurate. In the same way, if Gruber were to tag his population of sharks in an area where they were known *not* to congregate, the sample would be skewed the other way. Gruber had to devise another method to test the validity of his tag-and-recapture results.

In Case Someone Asks: A Fact Sheet on Lemon Sharks

Average length
8½ feet (at maturity)

Average length at birth
22 inches

Maximum weight
400 pounds

Life span
75–100 years

Habitat
Tropical Atlantic Ocean from New Jersey to Brazil; adults live at depths as great as 300 feet; young in shallows only

Distinctive markings
None, but broad pectoral fins, unserrated teeth, yellowish-gray color, dorsal fins about the same length; "very beautiful."

Average size of schools
6–8 sharks

Swimming speed
1–2 mph; up to 20 mph at top speed (can rest on bottom)

Average number of offspring
8–12

Population (worldwide)
Unknown, but not endangered

Eating habits
Broad range of prey (including fish, lobsters, octopus, shrimp, stingrays, sharks, even lemon sharks); eat about every 4 days.

Comments
Lemon sharks are known to be very dangerous when handled; a few unprovoked attacks have been reported, but basically shy. One of the few species of shark that can survive in captivity.

YOU'RE IT: Samuel Gruber tags a lemon shark in the first phase of his census (above). Since the sharks' social habits are to congregate, leaving large stretches of the lagoon uninhabited at any given time, Gruber takes to the skies (below) to reinforce his sampling with a bird's-eye view of the whole lagoon.

Flying Field Work

His answer was to take to the air to conduct an aerial survey of the shark population. A single-seated, open aircraft called an *ultralight* carried him aloft. Using a navigational map, Gruber divided the lagoon into a series of rectangular strips, called *transects*, whose outer bounds were defined by compass readings and the two pontoons of the ultralight. (See box on transect sampling.) Gruber would then sweep back and forth over the lagoon, like a farmer plowing a field. He would count only those sharks that he actually saw within the boundaries of the transect over which he was flying. Sharks on their way in or out of the transect before he reached them didn't count.

Gruber describes his first ultralight flight: "Then, unexpectedly, there they were. In a space of a few hundred square feet, 20 or more lemon sharks were milling slowly around in a tight circle—almost nose to tail. . . . The glider spooked the sharks; they took off in every direction, raising puffs of white sediment as they sped away."

The aerial survey also allowed Gruber to chart currents, bottom topography and macrofauna and to observe shark behaviors never before described.

The exact shark population of Bimini Lagoon will probably never be known. But by comparing two methods of estimating the population, Gruber may be able to move from "close" to "close enough" to enable research and investigation to go forward. ■

The Art of Tag and Recapture

In order to estimate the number of lemon sharks in Bimini Lagoon, Samuel Gruber uses a technique called *tag and recapture*, which is often used to count large populations of animals in the wild. Gruber explains the technique: Imagine being given a bag full of Ping-Pong balls and being asked to estimate how many balls are in the bag. A good way to guess would be to paint 10 of the balls red, put them back in the bag, shake up the bag and then draw out 10 balls at random. If one of the balls in your second draw is red, you could estimate that the bag contained about 100 balls altogether.

Applying this technique to lemon sharks in Bimini Lagoon, Gruber and his team catch, tag and release as large a number of sharks as possible. After waiting for the sharks to swim around the lagoon—presumably distributing themselves randomly—they then catch a smaller sample of sharks and record the number of tagged sharks in the "recapture" sample.

The relationship between tagged and untagged fish and subtotal and total populations in the Bimini Lagoon can be expressed in the formula:

$\frac{T}{N} = \frac{t}{n}$, where T = number of sharks tagged;
N = total number of sharks in lagoon;
t = number of tagged sharks recaptured in second sample;
n = total number of sharks captured.

Imagine that Gruber tags a total of 400 sharks. If he recaptures 100 sharks and finds that half of them are tagged, the equation would look like this:

$$\frac{400}{N} = \frac{50}{100}$$

By solving for N, Gruber can estimate that N (the total number of sharks in the lagoon) is 800.

LANDING A SMALL ONE: Samuel Gruber kneels in the shallows of the Bimini Lagoon preparing to tag a lemon shark as part of his sample. Using the information from this sampling technique and his transect sampling technique, Gruber can get a fix on the size of the overall population before beginning his studies into the habits of the lemon shark.

Checking the Answers

Check this person's work. Find the 9 errors in the answers given.

Don't actually *do* the calculations that are indicated. Instead, *estimate* what the correct answer should be and check against the answers given.

Add:

1) 68,342 + 275 + 9,476

2) 458 + 174 + 649 + 362

3) 74 + 5,492 + 67,789 + 446 + 52,473

Answers:
1) 78,093 2) 1,643 3) 12,627

Subtract:

4) 8,749 − 2,345

5) 83,597 − 61,428

6) 8,294,521 − 27,512

7) 58,000 − 32,046 =
8) 33,747 − 912 =
9) 842,000 − 16,777 =

Answers:
4) 11,094 5) 52,169 6) 8,267,009
7) 25,954 8) 34,835 9) 825,223

Multiply:

10) 1,086 × 5

11) 15,795 × 3

12) 987 × 99

13) 21 × 79 =
14) 2 × 7 × 9 =
15) 200 × 53 =

Answers:
10) 9,030 11) 74,385 12) 87,843
13) 1,659 14) 126 15) 1,060

Divide:

16) 6)846 17) 5)6,995 18) 4)29,784

19) 23)97 20) 24)984 21) 48)74,976

22) 16,272 − 36 = 23) 363,561 − 23 =

Answers:
16) 141 17) 139 18) 7,446
19) 4 R5 20) 41 21) 1,562
22) 45 R2 23) 15,807

Packing It In

Suppose you and two friends were going on an eight-day camping trip. You are in charge of the food.

How much food do you need? You don't want to run out, but you don't want to carry food you won't eat. How would you estimate your needs? Here's a suggestion: Write down what you'd like to have for breakfast, lunch and dinner tomorrow. Have your two friends do the same. Throw in a few snacks. Then combine your menus and you have the amount of food you'd need for a day. Based on that, project how much food you'll need for eight days.

Exercises:

1. Write a ratio to represent each of the following statements:

a. 5 of 30 lemon sharks in a sample were tagged;
b. 1 of 8 lemon sharks in a sample was tagged;
c. every other shark in a sample of 120 lemon sharks was tagged.

2. Assuming that Gruber tags 500 sharks in his first sample, what would be your estimate of the total number of sharks in the lagoon if he recaptured 50 sharks in the second sample and 5 of them were tagged?

3. What if he recaptured 300 and 60 were tagged?

4. What if he recaptured 175 and 70 were tagged?

5a. Complete the following table (the first entry has been done for you):

T = 500 = number of tagged sharks in the first sample.
N = Total number of sharks in the lagoon.
t = Number of tagged sharks in second (recaptured) sample.
n = Total number of sharks in the second (recaptured) sample.

T	500	500	500	500	500	500
t	5	60	70	4	2	1
n	50	300	175	100	100	100
N	5000					

5b. Looking at the last 3 entries in this table, what can you say about the total number of sharks in the lagoon if a recaptured sample of 100 sharks did not have any tags?

6. A fish warden stocked a fresh-water lake with 300 bass. If 3 out of every 20 bass were tagged, how many were tagged in all?

7. Compare Exercises 2, 3 and 4. In which of the three cases are you most confident that your estimate is close to the true total?

Answers:

1. $\frac{5}{30}; \frac{1}{8}; \frac{60}{120}$
2. 5000
3. 2500
4. 1250
5a.

T	500	500	500	500	500	500
t	5	60	70	4	2	1
n	50	300	175	100	100	100
N	5000	2500	1250	12500	25000	50000

b. There are not enough sharks tagged to make a meaningful estimate.
6. 45
7. #3, because of the larger sample size.

"Many a guess has turned out to be wrong but nevertheless useful in leading to a better one."

George Polya, *How to Solve It*

Tag and Recapture in the Classroom

How accurate is the tag-and-recapture method? You can set up an experiment with your class to find out. You'll need the following:

several bags of "goldfish" crackers*
one goldfish bowl*
large scoop (ice cream or flour scoop or equivalent)

Put the "goldfish" crackers into the fishbowl. Have each student guess how many crackers are in the bowl, and record all the guesses.

Take out a scoopful of goldfish crackers. Count them. These will be your total tagged sample. "Tag" them by marking them in some distinguishing way; then "throw" them back in and shake up the bowl.

Now take out another scoopful. Count them. How many are tagged? Set up a table like this one (the numbers below are examples; your findings will vary):

Sample	1	2	3	4	5	6
t	2					
n	18					
T	24					
N	?					

Fig. 1

From this table you will eventually set up a proportion to solve for the total of goldfish in the bowl:

$$\frac{T}{N} = \frac{t}{n}$$

You could make an estimate of the number of goldfish in the bowl from this one try, but the odds are it wouldn't be accurate. In order to get a fair sample allowing for their random distribution, you must average the results of several attempts, say five in all. Record each count. Now your table might look something like this:

Sample	1	2	3	4	5
t	2	4	2	3	0
n	18	21	15	18	16
T	24	24	24	24	24
N	216	126	180	144	—

Fig. 2

There are several ways we can use this data.

1. The class can find the average of the calculated Ns and use this as an estimate.

$$216 + 126 + 180 + 144 = 666$$
$$666 \div 4 \approx 167$$

2. We could treat the samples as one large sample and calculate N.

$$t = 2 + 4 + 2 + 3 + 0 = 11$$
$$n = 18 + 21 + 15 + 18 + 16 = 88$$
$$T = 5 \times 24 = 120$$

or, $$\frac{T}{N} = \frac{t}{n} \rightarrow \frac{120}{5N} = \frac{11}{88} = 192$$

Are both of the answers (167 and 192) reasonable?

Students can *check the result* by counting all the goldfish. How close was the sampled estimate? Which student was closest?

Try variations with different numbers of samples. What effect does it have on the result if the class takes 10 samples? 15? 2?

Can the students devise a rule of thumb for a minimum number of samples that is likely to yield an accurate (that is, reasonably close) estimate? How close is reasonably close?

*You can also use jellybeans, marbles or toothpicks; a shoebox makes a good substitute for a goldfish bowl.

Transect Sampling: A Bird's-Eye View

Gruber checks his tag-and-recapture estimates by dividing the Bimini Lagoon into rectangular strips (transects) and counting the sharks in each transect from the air.

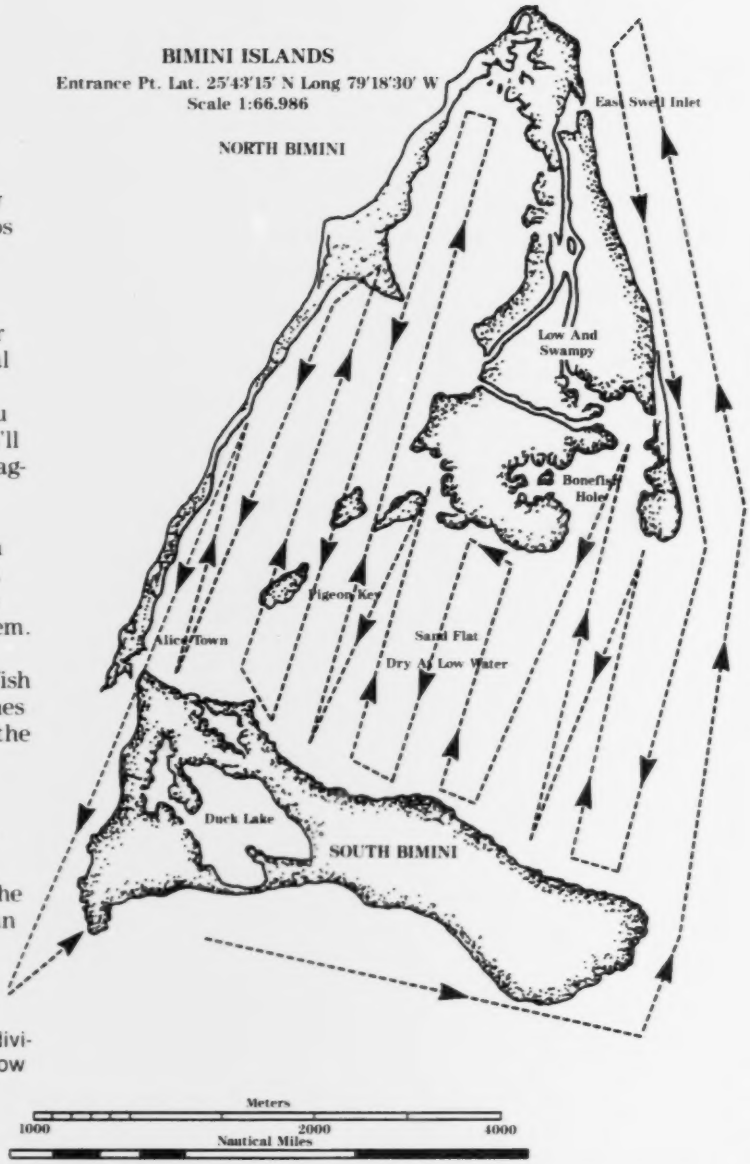
Here's a similar experiment you can do with your class. You'll need a floor that is divided into equal rectangular strips: wooden beams or linoleum squares will serve as readymade transects, or you can use masking tape to mark off your own. You'll also need the goldfish crackers you used in the tag-and-recapture experiment.

Define the area you'll use for this experiment (in this example, the area is 20 transects). This area represents the lagoon. Throw the crackers on the floor. If any land outside the "lagoon," ignore them.

Take 5 of the 20 "transects" and count the goldfish in each transect. Any goldfish that are on the lines between transects should be counted as part of the transects they are "swimming" into.

Estimate the total number of goldfish in the "lagoon." Now count them. How close were you?

Now repeat the experiment, this time counting the fish in 10 transects. Was your estimate closer than before?



The dotted lines in the map at right show Gruber's division of Bimini Lagoon into transects. The arrows show his flight path.

Further Readings

Chapman, D.G. "The Plight of the Whales." *Statistics: A Guide to the Unknown*, edited by Judith M. Tanur et al., San Francisco: Holden-Day, 1978.

This essay describes several different methods of estimating the size of a whale population. Since the methods used are similar to those involving sharks, this should be helpful to students.

Clark, Eugenie. "Sharks: Magnificent and Misunderstood." *National Geographic*, August 1981.

A fascinating, well-illustrated article by a biologist and zoologist who has studied sharks for over 25 years. A general introduction to shark behavior and research.

Gruber, Samuel H. "Shark Research by Ultralight: Parts 1 and 2." Rosenstiel School of Marine and Atmospheric Science, University of Miami, 1982.

This paper describes Gruber's experiment with counting sharks from an ultralight airplane. Accessible to high school students.

McKean, Kevin. "The Fine Art of Reading Voters' Minds." *Discover*, May 1984.

This article provides a well-informed discussion of polling techniques, and describes the steps pollsters take to ensure that their samples are as accurate and "random" as possible.

Everyone Counts

How do you count 1,000,000,000 people, more or less? The Chinese Census of 1982 was an enterprise of staggering scale and great importance in determining the country's ability to provide work and food and care for its people.

To get the 1982 China Census done required about 2,000 computers, 3,000 keypunch operators, 200,000 data coders and an army of about 5.1 million head counters—more counters than Ireland has heads to count.

With a census of any size, accuracy is a primary concern, but *complete* accuracy is impossible. Even the best census is an approximation—an *estimate*. Some people and some facts are inevitably altered or lost. A car mechanic may list his occupation as a steelworker because he is to start work at a foundry in a week's time. A college student may be counted twice—with her family in her hometown and again as a resident of the town where her college is located. A slip of the pencil by an enumerator can change age, income or sex of an individual. A keypunch operator's momentary lapse can erase a family member.

The larger the population and the larger the country over which that population is dispersed, the more complex the census gathering and analyzing apparatus—and, certainly, the greater the chance for error.

BEST COPY AVAILABLE

To Err Is Inevitable

Because a perfect census is impossible, an important part of planning the 1982 China census was deciding on an "acceptable" margin of error. "If the error rate is below 1%," says Yeun-chung Yu, a United Nations statistician who helped the Chinese, "we consider the census very, very good." Not good enough for China, however. Pre-census estimates had put China's population at 1 billion. Thus, China aimed at an error rate of 0.2%, even smaller than the error rate of 0.4% achieved by the 1980 U.S. Census.

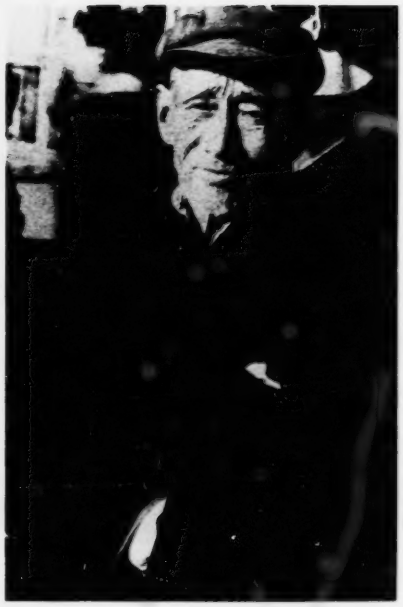
The planning stage took three years. Personnel were trained and "pilot" censuses were conducted. These small-scale enumerations are designed to refine procedures and identify potential trouble spots, including the tricky problem of language. There are hundreds of languages and dialects used in China. Census forms had to be printed in many languages and dialects to reach the total population.

Once all the census-taking machinery was ready, the public had to be brought into the act before it was brought into the picture. The flaws of the earlier censuses stemmed in part from an inadequately informed public.* June was declared "census publicity month," and the public was inundated with information from billboards, radio and TV.

*The 1982 Census was only the third in China's history and the first to use computers. Previous censuses in 1953 and 1964 were conducted with abacuses and hand tabulation. The earlier counts are regarded as inaccurate.

COUNTING HEADS BY COMPUTER: Above, Chinese tabulators use a specially installed computer system to keep track of census data. Below, a sample census form. Answers to 19 questions on millions of forms like this one provided raw census data. The form is divided into two parts: long, narrow box at top asks for information about the household, while deeper, narrow box at bottom requests information on each individual.

第三次全国人口普查表



THE OLD-FASHIONED WAY: Side by side with their high-tech colleagues, some tabulators exercise their skills on that ancient, technology-proven computing machine, the abacus.

Finally, the counting began. Enumerators visited some families in their homes; in other areas, one family member was required to report to an official census station. Questions were asked and answered; tens of millions of census forms shipped and filled out; millions upon millions of figures entered and tallied. Yard after yard of printouts peppered with numbers spewed out of the computers. Even then, checking and cross-checking procedures continued for days to guarantee an estimate accurate within 0.2%.

The Data Game

The census data will be crucial to planners and help government officials in China in ensuring the country's ability to provide work and to care for its people. The numbers provide answers—they also pose questions. Chinese leaders have set a population target of 1.2 billion by the year 2000. The 1982 census showed the Chinese they are already close to that figure; it also showed that, in the 20 years since the last census, the population has increased by nearly one-third.

With its 1982 census now complete, China has the figures necessary for more accurate planning. At zero hour —midnight, July 1, 1982—the census recorded 1,008,175,288 Chinese—with a .015% error factor representing a mere (or relatively mere), 150,000 people—more or less.

It remains the biggest group portrait in history. ■

How Much Error?

Despite careful planning, every national census, including the Chinese, includes some error. With a large population, it can be quite substantial.

Knowing that error can be limited but not eliminated, governments have developed techniques to limit error and to define it. It's not quite as good as having precise figures to begin with, but it's the best that can be achieved under the circumstances. And with careful control, a government can come remarkably close to an exact population count by knowing what the margin of error is.

In order to calculate error rates, census takers recount random districts, recheck the recording of data in random cases and recheck computations. Then, if the original and rechecked figures disagree by, for instance, 2 percent, you could assume a 2 percent error rate for the census as a whole.

What Happens to the Error?

What happens to errors when we do computations with them? It depends on what the computations are.

When you add or subtract errors, they are, at worst, additive. (They may cancel each other out.) But when you multiply numbers that have error in them, the resultant error can be far greater than you might expect.

Here's an exercise you can take your class through to show how errors can affect computations.

Perimeter: The Outside Limits

Suppose you have a rectangle whose sides are measured at 4 cm and 3 cm each. Since neither your eye nor your measuring device is perfect, assume that the error could be as much as 0.1 cm per side. So the greatest possible error you might have in calculating the perimeter (the sum of the four sides) is 0.4 cm.

The percent error is calculated by dividing the error by the true value (or the approximation of it). That is, to find the percent error in the perimeter, we divide as follows:

$$\frac{\text{greatest possible error in perimeter}}{\text{perimeter}} = \frac{.4}{14} = 0.02857 = 2.9\%$$

Likewise, we can find the percent error in the length and the width.

$$\frac{\text{greatest possible error in width}}{\text{length}} = \frac{.1}{4} = 0.025 = 2.5\%$$

$$\frac{\text{greatest possible error in width}}{\text{width}} = \frac{.1}{3} = 0.0333 \approx 3.3\%$$

Thus, the percent error for the perimeter is about the same as for the sides.

Further Readings

Atwood, Albert W. and Aikman, Lonelle. "Census 1960: Profile of the Nation." *National Geographic*, November 1959.

A description of the 1960 U.S. Census—its planning and execution. Out of date, but a good, thorough introduction to the planning necessary to get an accurate count.

Passel, J.S.; Siegel, J.S.; and Robinson, J.G. "Coverage of the National Population in the 1980 Census, by Age, Sex and Race: Preliminary Estimates by Demographic Analysis." Bureau of the Census, February 1982. U.S. Government Printing Office, Washington DC.

A preliminary overview of the results of the 1980 U.S. Census, describing the basic numerical results and showing the general trends that can be deduced from those numbers. A good source for making up census problems.

Keyfitz, Nathan. "The Population of China." *Scientific American*, February 1984.

A description of the demographic trends uncovered in the 1982 China census, this article describes the different problems and possible solutions that a census can uncover. Good, clear graphs; well-written.

Qing, Xiao. "China's Census: Accurate to 0.2 Percent." *China Reconstructs*, Oct. 1982. (Available from China Books and Periodicals, Inc., 2929 24th Street, San Francisco CA 94110.)

This article provides a concise overview of the China census—how it was planned and carried out, why it was important, how the planners and takers insured its accuracy. Accessible to junior high students.

Tanur, Judith M., ed. *Statistics: A Guide to the Unknown*. San Francisco: Holden-Day, 1978.

This excellent collection of essays covers a variety of statistical problems and solutions. "How to Count Better: Using Statistics to Improve the Census," by M. H. Hansen, is a concise description of the basic principles of planning and carrying out a population census. "Information for the Nation from a Sample Survey," by C. Theber, describes the type of information that can be gained from the Current Population Survey (CPS) that the U.S. Census Bureau conducts each month. In addition, this book contains a number of other essays that are accessible to high school students.

Area: The Inside Story

In computing the area (the product of the length and width), the possible error is more substantial.

First, we find the smallest possible length and width:

length = 4 cm − .1 cm = 3.9 cm
width = 3 cm − .1 cm = 2.9 cm

and the longest possible length and width:

length = 4 cm + .1 cm = 4.1 cm
width = 3 cm + .1 cm = 3.1 cm

The lower bound is found by multiplying the smaller values:

$3.9 \times 2.9 = 11.31$

The upper bound is found by multiplying the larger values:

$4.1 \times 3.1 = 12.71$

So the true area is between 11.31 sq cm and 12.71 sq cm.

The possible *positive* error is found by subtracting the calculated (12) from the upper bound (12.71), i.e., $12.71 - 12 = .71$. The positive percent error is $.71 - 12 \approx 5.9\%$. This is more than twice the percent error of the perimeter.

The possible *negative* error is found by subtracting the lower bound (11.31) from the calculated area (12), i.e., $12 - 11.31 = .69$, or 5.6%.

We also note that the possible negative error .69 is less than the possible positive error. Figure 1 shows this. The shaded area shows the possible negative error, the dotted area shows the possible positive error.

Fig. 1

Exercises:

1. Percent error is calculated by dividing the error by the true value or estimated true value. Find the possible percent error for the following:

error	measurement
1 cm	35 cm
1 mm	7 cm
1 mm	2 m

2. If the length and width of a rectangle are measured to be 7 cm and 8 cm, respectively, with possible errors of 3 mm and 4 mm, respectively, what are the bounds for the perimeter? What is the possible percent of error? What are the bounds and percent error for the area?

3. Pat drove about 200 miles in approximately 4 hours. How many miles per hour is that? If the error in the distance could have been as much as 2 miles and the error in the time could have been as much as four minutes, what are the upper and lower bounds on the rate? What is the possible percent of error for the rate?

4. Estimate, *don't* calculate: In the China census, 5 million enumerators counted 1,008,175,288 people. About how many people did the average enumerator count?

5. A census indicated that in a small town in Arkansas there were 4,953 people. The percent error was estimated to be 2.75%. What are the estimated upper and lower bounds on the population?

Answers:

1. 2.9%
   1.4%
   5.0%

2. Perimeter: lower bound : 28.6 cm
   upper bound : 31.4 cm
   error : 1.4 cm (4.7%)

   Area: lower bound : 50.92
   upper bound : 61.32
   error : 5.32 (9.5%)

3. 50 mph
   Lower bound : 48.68 mph
   Upper bound : 51.36 mph
   Error : 2.72%

4. 200

5. Lower bounds : 4,820
   Upper bounds : 5,093

Counting Noses: Planning a School Census

Stage I Here is a classroom exercise which will take your students through the *steps of planning* a school census.

Stage II If time and the conditions at your school allow, have them *implement* the census and check some of their findings against school records.

1. *Start with a rough estimate.* What's the average number of students per class? About how many classes are there in the school?

2. *Determine what information you want to obtain.* A census isn't just a head count. It's also a means of gathering information about the people in a society. Here are some suggestions:

- the ratio of boys to girls in the student body;
- the number of students who plan to go to college;
- the number of students who were born in your town;
- student opinions on public or campus issues;
- the number of students who will have summer jobs or internships.

In what ways would this kind of information be useful?

3. *Invent a procedure.* Your class will need enumerators. How many? How would they let the other students in the school know that the census is taking place? What day or days should they choose for the census—are there good times *not* to have it?

4. *Present your findings.* What would be the best way to organize and present the information gathered in the census? It might be helpful to study data tables based on the U.S. census in an almanac.

Notes on Implementing a Census

If your class goes beyond planning to actually taking a student census, have them try a smaller practice run first. Then discuss the results. What kinds of difficulties did they encounter? How might they be avoided?

After they've taken the census, have your students take a selected part of it again. Compare results with the "official census." Can they calculate an error factor?

Finally, compare results with official school records. Do they agree? If not, by how much do they disagree? How might your students account for any discrepancies? Are they within the error factor?

Snow Job

To prevent huge, life-threatening slides, avalanche controllers regularly start small "controlled" slides. The critical part of their job is to estimate when, where and how a small avalanche should be started.

An average annual snowfall of 40 feet makes Little Cottonwood Canyon in Utah's Wasatch Mountains the scene of some of the best skiiing in the United States—*and* some of the worst avalanches as well. Route 210 brings thousands of skiers into the resort towns of Alta and Snowbird every year passing through 19 major slide paths—spillways down which tons of snow can roar, burying the road and even pouring into the villages themselves. Some call the place "Death Canyon." It is the job of men such as hazard forecaster Bengt (Binx) Sandahl and Alta's snow safety director Onno Wieringa to see that it remains Death Canyon in name only.

To prevent the huge slides, Sandahl and Wieringa regularly start small "controlled" avalanches.

A full-blown avalanche can develop a mass of 1 million tons, and reach a speed of 200 miles per hour. It can snap mature trees and flatten trucks into steel pancakes only a foot thick, kill people outright or entomb them alive. Snow isn't the only killer: An avalanche can drive before it a blast of compressed air that can level buildings the snow never reaches. Behind the avalanche, suction is created as air races in to fill the vacuum left by the advancing mass of snow.

NATURE'S AWESOME POWER UNLEASHED: With a mass of up to 1 million tons and speed of up to 200 mph, an avalanche packs a destructive wallop that can turn a peaceful landscape into a scene of devastation in minutes. Avalanche forecasters rely on experience and intuition to decide when disaster is likely.

Hazards of Forecasting

As hazard forecaster, Sandahl's responsibility is to protect the lives and livelihoods of those in the Little Cottonwood Canyon area. Using data from the National Weather Service, ongoing evaluation of the snowpack, and a sixth sense built on experience, he decides when and where roads and resorts should be closed. Sandahl walks a thin line: If he's too cautious and closes ski areas too often, he could put them out of business; yet he has to protect the lives of the skiers.

In order to evaluate the snowpack, snow rangers climb mountains into the path of a potential avalanche. There they dig snowpits all the way to ground level, which allows them to examine the snow layer by layer, noting variations in 10-centimeter intervals. They even examine samples from each layer under a lens to identify the crystal type. Sandahl uses this information to make his own prediction—an estimate of the situation at hand: How thick are the layers, and how hard? Are any deteriorating? Where and when is the snowpack liable to break off? Depending upon the kinds of layers making up the snowpack and the expected weather conditions, the snow may or may not slide. Sandahl must decide when it is time for action.

When snow rangers decide to start a small avalanche to avoid a big one, they put on quite a show. They "call in the artillery"—75-millimeter-pack howitzers, with sights trained on precise points in the snowpack. Estimation skills help judge how much firepower to use and how and where to aim the guns to effect a "controlled slide." If the controllers have done their job of estimating right, the unstable snow will give way and move harmlessly down the mountainside. The potential for a major disaster will be avoided—this time around. ■

AIMING FOR AN AVALANCHE: One way to prevent a big slide is to start a small one. At top right, snow rangers fire a 75mm pack-Howitzer into a potentially dangerous area. Above, a snow ranger starts a controlled slide—not recommended for the novice.

Mathematical Likelihoods

Estimating when and where an avalanche will occur uses the mathematical idea of *probability*.

The probability that an event will occur is a number between 0 and 1 ("0" represents an impossibility, and "1" a certainty), which can be represented as a fraction, decimal number or percent. For example, there might be 1 chance in 100 that a particular slope is unstable enough to be released by a skier—a probability that could be written 1/100, .01 or 1%.

If an outcome depends on several independent actions, the probability is equal to the product of each individual probability. So if there is a 1/3 probability that a skier will be on the unstable slope, the probability of the outcome—an avalanche caused by the skier on the slope—is $1/3 \times 1/100$ or 1/300 (one chance in 300).

Exercises:

1. What is the probability that:

a. a wild kangaroo will start an avalanche in northern Utah?
b. any day of the week has the letter "s" in it?
c. any month of the year has the letter "r" in it? What is more likely, an "s" day or an "r" month?
d. any day will be an "s" day in an "r" month?
e. the sun will rise tomorrow?

2. Suppose the probability that a given slope will trigger naturally in the next 12 hours is 1/50, but if a skier should be on it, the probability is 1/10. If the slope releases, there is a road below with a 1/25 probability of having traffic on it.

a. What is the probability of a traffic accident, if the slope is triggered naturally?
b. What is the probability of a traffic accident if a skier is on the slope?
c. What is the probability of a traffic accident if there is one chance out of two that a skier will be on the slope?

3.a. What is the probability that a wild kangaroo will *not* start an avalanche in northern Utah?

b. If the probability is 1/4 that a skier will be on a slope, what is the probability that a skier will *not* be on a slope?

In question #2, what is the probability of:

c. a given slope *not* triggering naturally within 12 hours?

d. there *not* being traffic on the road?

e. there *not* being a traffic accident resulting from a natural triggering of the slope?

4. Suppose that it is known that an avalanche returns to a particular hazardous area every three years. The probability of a hit in the first year of the cycle is 1/3.

What is the probability that:

a. the avalanche did not hit one year?

b. the avalanche did not hit one year, but hit the second year?

c. the avalanche did not hit the first or second years, but hit the third year?

d. the avalanche will hit one year and the next year?

e. the avalanche will hit one year or the next year?

CHALLENGE Exercise:

5. If the avalanche return interval is every 10 years, determine the probability of an avalanche occurring in any one of the first 5 years. Express your answer as a percent.

Answers:

1. a. 0; b. 5/7; c. 8/12, "s" day; d. 10/21; e. 1
2. a. 1/1250; b. 1/250; c. 1/500
3. a. 1; b. 3/4; c. 49/50; d. 24/25; e. 1249/1250
4. a. 2/3; b. 2/9; c. 4/27; d. 1/9; e. 5/9
5. 40,951/100,000 (about 41%)

Further Readings

Cupp, David. "Avalanche!" *National Geographic*, September 1982.

The story of a successful avalanche rescue effort at a Rocky Mountain ski area and a description of avalanche control and prevention methods used around the world—from Colorado to Switzerland. Excellent photos and illustrations.

Dept. of Agriculture Handbook No. 489. *Avalanche Handbook*. U.S. Government Printing Office, Washington DC.

This official handbook tells you everything you could want to know about avalanches, and more. It is accessible to high school students, and contains a section of relevant word problems.

LaChapelle, Edward R. *The ABC of Avalanche Safety*. Seattle: The Mountaineers, 1981.

This compact (55-page) booklet contains everything a student could want to know about the causes, formation, dangers and prevention of avalanches. It specifically focuses on avalanche safety and rescue, but the scientific information on the causes of avalanches is thorough and accessible at the junior high school level. Available from The Mountaineers, 719 Pike Street, Seattle WA 98101.

LaChapelle, Edward R. *Field Guide to Snow Crystals*. Seattle: University of Washington Press, 1969.

This book explains the nature of snow crystals—their formation in the atmosphere and their metamorphosis inside a snowpack—and how they influence avalanche formation. Lots of photographs and simple charts and diagrams.

Scientific American. *Atmospheric Phenomena*. San Francisco: W. H. Freeman, 1980.

This book contains two reprinted articles—"Water, Ice, Snow Crystals" and "The Growth of Snow Crystals"—which provide a good introduction to the molecular structure of water and ice, as well as the formation of snow crystals.

Guess What?

Estimate the following:

1. The width of a cafeteria table.
2. The height of the tallest person in your class.
3. The diameter of a soda can.
4. The height of your classroom door.
5. The width of your chair.
6. The length of the hall outside your classroom.

Compare your answers with your classmates'. Take an average of the class's answers to each question. Now check. How close was the class average to the exact number?

Now, decide whether the estimate below is more or less than the actual quantity. (Supply your own examples if they would be more relevant.)

7. Pages in the telephone book: 650 MORE LESS
8. Trays in the cafeteria: 700 MORE LESS
9. Rainy days in the last year: 75 MORE LESS
10. Area of your classroom: 200 sq ft MORE LESS

Estimate:

11. How many ads are in today's paper?
12. How many words are on page one of today's paper?
13. How many commercials are there in a one-hour TV show?
14. How many minutes do the commercials take up?

Keeping Tabs on the Tab

One place where estimation comes in handy is at a store. In order to keep within a budget, or to avoid embarrassment at the cash register, it helps to keep an approximate running total while shopping.

Take your class on an imaginary shopping spree. Read them the price of each item and have them keep an estimated running total in their heads—no pencil and paper allowed. The object is to come as close as possible to the actual total.

	Grocery Store	5¢ & 10¢	Sporting Goods	Book Store	Clothing Store
Item #1	$1.99	$0.59	$15.95	$2.99	$35.00
Item #2	$3.98	$0.29	$29.95	$1.79	$50.00
Item #3	$4.29	$0.15	$12.88	$3.29	$25.00
Item #4	$0.79	$0.69	$ 9.45	$1.25	$ 5.00
Item #5	$0.79	$0.25	$25.50	$5.50	$75.00
Item #6	$0.89	$0.99	$ 6.98	$3.00	$69.00
Item #7	$5.95	$0.99	$39.95	$9.00	$18.00
Item #8	$0.69	$0.89	$59.95	$2.50	$22.50
Item #9	$3.95	$1.29	$19.99	$5.00	$38.25
Item #10	$0.29	$0.65	$ 3.29	$1.69	$19.99
Total	$23.61	$6.78	$223.89	$36.01	$357.74

For more practice, let your students make up their own shopping sprees, perhaps using receipts from actual trips to supermarkets or variety stores, and challenge the class to estimate the total purchase price.

Teacher's Notes

(see opposite page)

The Queen's Larder

On a much grander scale, the chief steward has an estimation problem that any family solves each week. Your students might get some help from their parents here. It might be useful to do the class estimate as a class.

Basic Catering Requirements for a Normal 8-Day Transatlantic Voyage Aboard Queen Elizabeth 2

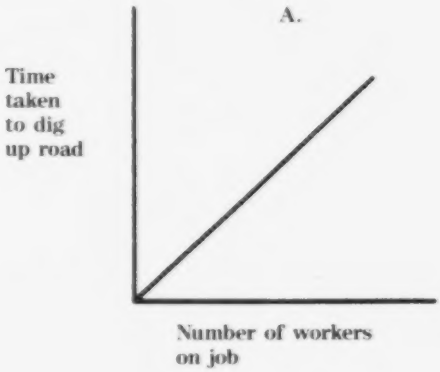
Queen Elizabeth 2 must be capable of meeting the demands of an international clientele. Her basic catering requirements—detailed below—for a normal Transatlantic run include such varied items as 25,000 lb beef, 22,000 lb of fresh fruit, 150 lb caviar, 600 jars of baby food and 50 lb of "Super Crunchies for Hungry Doggies." She carries 41 brands of whiskey and 43 brands of cigarettes. On a normal Transatlantic round trip, the ship would carry, on average, the following food supplies:

Biscuits	2,000 lb	Eggs	220 cases (360 case)
Flour	3,000 lb		
Cereals	800 lb	Cream	3,000 qt
Rice, etc.	3,000 lb	Milk	2,500 gal
Tinned fish	1,500 tins	Fish	1,400 lb
Tinned fruit	1,500 gal	Lobsters	1,500 lb
Herbs, spices	50 lb	Crabs	800 lb
Jam, marmalade	700 lb	Fresh fruit	22,000 lb
Juices	3,000 gal	Frozen fruit	2,500 lb
Tea bags	50,000	Ice cream	5,000 lb
Tea (loose)	500 lb	Kosher food	600 lb
Coffee	2,000 lb	Beef	25,000 lb
Sugar	5,000 lb	Lamb	6,500 lb
Baby food	600 jars	Pork	4,000 lb
Dog biscuits	50 lb	Veal	3,000 lb
Caviar	150 lb	Sausages	2,000 lb
Foie gras	100 lb	Chicken	5,000 lb
Butter	3,500 lb	Ducks	3,000 lb
Bacon	2,500 lb	Turkey	5,000 lb
Ham	1,200 lb	Fresh Vegetables	27,000 lb
Cheeses	3,000 lb	Potatoes	300 cwt
		Pickles, sauces, etc.	2,000 bottles

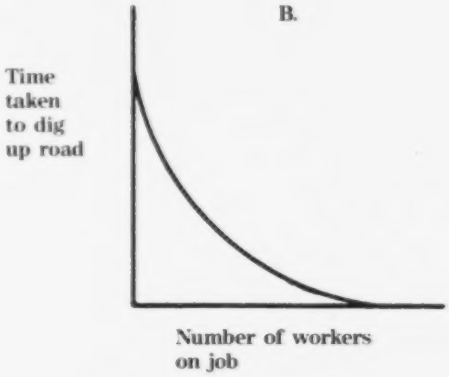
Digging Up the Road

These three graphs tell three different stories. Two are clearly impossible. Graph A indicates the more workers hired, the longer the job will take (and the graph says that no workers would finish the job in no time at all). Graph C shows that the job will get done eventually even if no one is doing it.

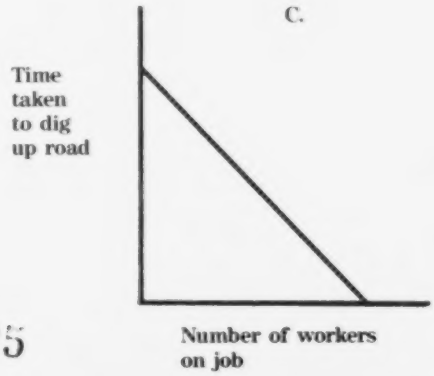
A.



B.

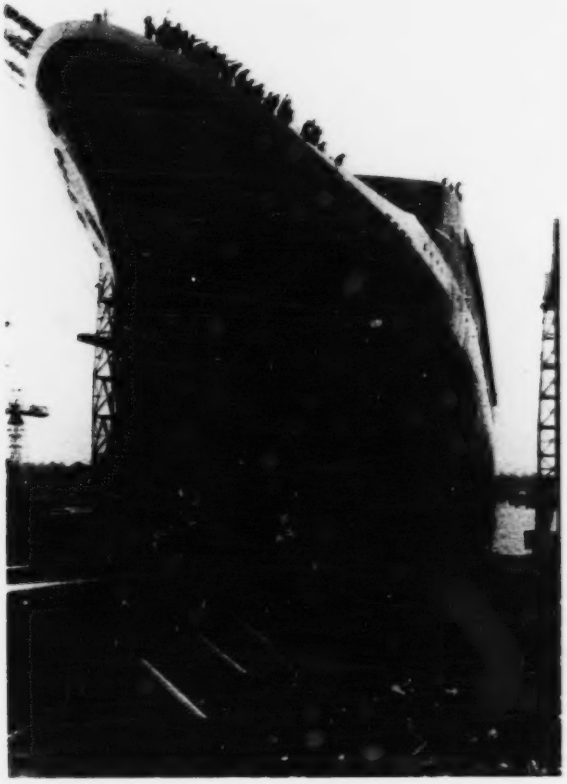


C.



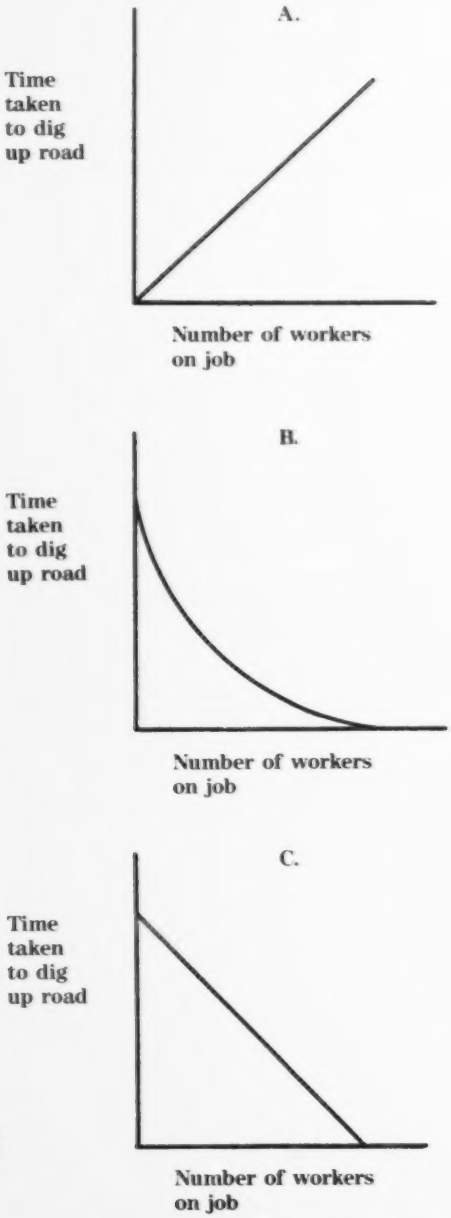
The Queen's Larder

Now, suppose you were in charge of buying food for a large ocean liner like the Queen Elizabeth 2, carrying 1,800 passengers and 1,000 crew members on an 8-day cruise. That's a lot of people and they need a lot of food. Where would you start?



Digging Up the Road

How many people does it take to dig up a two-mile stretch of road in a given period of time? These graphs can't give you an exact answer. Which graph or graphs can supply a reasonable estimate?



The Sole of a Nation

How many shoes do you suppose are being worn on American feet right at this moment? What do you need to know in order to find out?

This may be a frivolous question, but your answer will only be as good as your estimation skills. You can't just take the population and multiply by two. You have to identify and estimate the numbers of people *not* wearing shoes:

- in the population—above and beyond the last census
- asleep, thus not wearing shoes
- at the beach
- in the bathtub or shower
- in the hospital, etc.

The more you can narrow it down, the more accurate your answer is liable to be.

Manufacturers use similar estimates to decide how many products they should be prepared to make each year. To plan production, an automobile manufacturer must estimate the number of people who will be buying a car during the year, and the number of buyers who will choose the manufacturer's brand over other cars. Accuracy is at a premium: overestimate, and you have warehouses full of unwanted goods; underestimate, and your competitors will get the orders you can't fill.

Budgeting a Banquet

Plan and budget a class banquet. Your budget will be an estimate of how much money you are likely to spend. Sometimes you base your estimate on similar past projects; sometimes you have to start from scratch and base your budget on an educated guess of how much each aspect of the project will cost.

In preparing a budget, you must consider two things:

1. How much money is available for this project? (Will you get money from a car wash, sell tickets to the event, get money from a school fund?)

2. What are the individual pieces of the project, and how much is each likely to cost? (What costs are fixed? If you have a band, the cost will be the same if 20 or 200 people attend. What costs depend on the number of people attending?)

Here's a sample budget for a class banquet to use as a guide:

BANQUET BUDGET*

	Cost per person	Total cost
Pizza	$2.25	$ 337.50
Soda	.75	112.50
Facilities	2.00	300.00
Entertainment	3.00	450.00
Subtotal	$8.00	$1,200.00
Sales tax @ 6.5%	.52	78.00
TOTAL	$8.52	$1,278.00

*Number of students = 150.

This budget does not cover everything; don't forget to consider decorations, transportation, staffing (if necessary) and any other expenses you might incur. Double check your answer. What happens if the budget is wrong?

Jessamin Swearingen
Calhoun School
New York, N.Y.

Making Sense of the Census

The census, the government's official estimate of the size of the population, is society's most important planning tool. Will the country need more hospitals to accommodate an increasingly large elderly population? Should schools be closed as the number of school-age children declines? Will there be enough jobs for the emerging work force? Should farm production be increased to feed a growing total population? Census results allow society's planners to project trends several years into the future and make their plans accordingly.

The table in Figure 1 shows the United States population as it was at the time of the 1980 census. If you were planning for a business, a community or the nation, what kinds of plans would you make based on this table? Normally, you'd have to allow for deaths to decrease the population of all age groups, and for newly naturalized immigrants to increase them. But for the purpose of this exercise, assume that they cancel each other out.

Interpreting data tables is a necessary skill for projecting trends. Here are some suggestions for practice:

1. Draw a graph showing the number of 12 year olds from 1970 to 1980.

2. When will 1980's 2 year olds be high school seniors?

3. Draw a bar chart comparing the total number of people in the following age groups:

1—12
13—18
19—24
25—34
35—49

4. What is the ratio of 25 year olds to 40 year olds as of 1980? Why might this ratio be important?

5. As of 1980, what year had the most births? What year had the fewest? What kinds of predictions could you make based on that knowledge?

U.S. Population for Census Period Ending April 1, 1980

Total Population Including Armed Forces Overseas

All Ages 227,061,000

Under 1 year	3,534,000		
1 year	3,270,000	41 years	2,380,000
2 years	3,224,000	42 years	2,329,000
3 years	3,179,000	43 years	2,240,000
4 years	3,142,000	44 years	2,265,000
5 years	3,163,000	45 years	2,244,000
6 years	3,109,000	46 years	2,141,000
7 years	3,273,000	47 years	2,224,000
8 years	3,395,000	48 years	2,165,000
9 years	3,760,000	49 years	2,322,000
10 years	3,717,000	50 years	2,348,000
11 years	3,581,000	51 years	2,295,000
12 years	3,519,000	52 years	2,363,000
13 years	3,643,000	53 years	2,337,000
14 years	3,783,000	54 years	2,368,000
15 years	4,060,000	55 years	2,390,000
16 years	4,181,000	56 years	2,330,000
17 years	4,225,000	57 years	2,313,000
18 years	4,268,000	58 years	2,330,000
19 years	4,492,000	59 years	2,252,000
20 years	4,441,000	60 years	2,161,000
21 years	4,337,000	61 years	2,074,000
22 years	4,326,000	62 years	2,008,000
23 years	4,233,000	63 years	1,931,000
24 years	4,192,000	64 years	1,913,000
25 years	4,143,000	65 years	1,905,000
26 years	4,002,000	66 years	1,814,000
27 years	3,953,000	67 years	1,764,000
28 years	3,728,000	68 years	1,679,000
29 years	3,803,000	69 years	1,621,000
30 years	3,742,000	70 years	1,517,000
31 years	3,622,000	71 years	1,440,000
32 years	3,727,000	72 years	1,371,000
33 years	3,668,000	73 years	1,262,000
34 years	2,871,000	74 years	1,208,000
35 years	2,912,000	75 years	1,117,000
36 years	2,939,000	76 years	1,034,000
37 years	2,992,000	77 years	955,000
38 years	2,607,000	78 years	880,000
39 years	2,559,000	79 years	808,000
40 years	2,474,000	80 years	723,000

Fig. 1

Exceeding All Estimates

"Counting" crowds is important to police officers and news reporters, among others. Should extra police be brought in to protect the crowd at an outdoor concert or parade? Was a rally attended by enough people to justify a front-page story? These judgments depend on knowing how many people are in the crowd.

Of course, you can't count every person in a large, moving crowd. You have to estimate the number of people. Police officers, in particular, are skillful enough "guessers" that their estimates are usually accepted as official crowd counts.

Develop a system for estimating the number of people in this picture. One suggestion: Take a one-square-inch section of the photograph and count the people in it. Then multiply that number by the total area of the picture.

Tower of Burgers

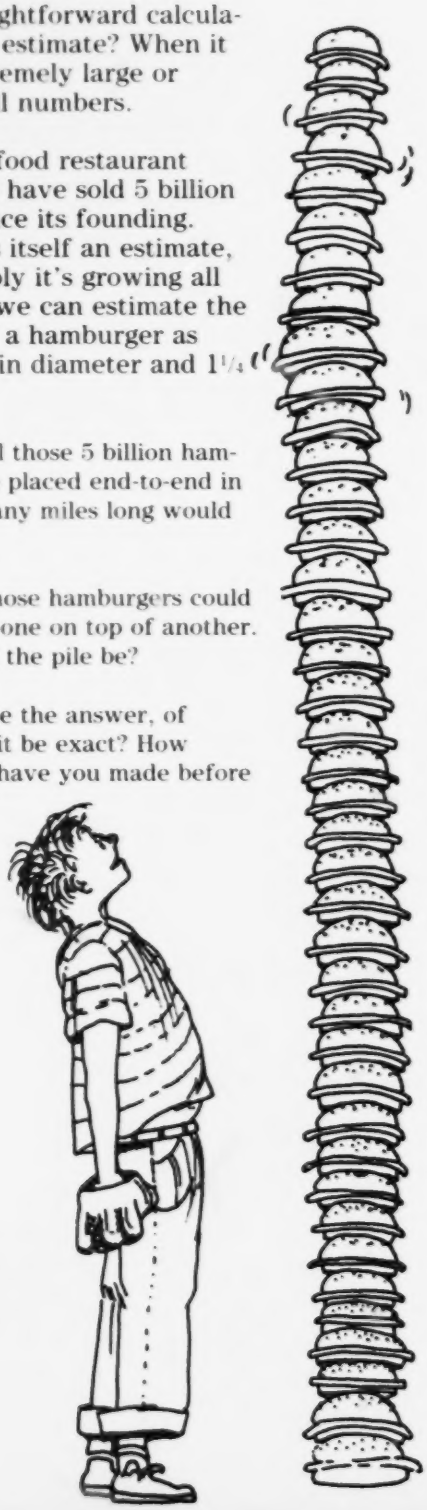
When is a straightforward calculation merely an estimate? When it deals with extremely large or extremely small numbers.

A famous fast-food restaurant chain claims to have sold 5 billion hamburgers since its founding. That number is itself an estimate, since presumably it's growing all the time. And we can estimate the average size of a hamburger as being 4 inches in diameter and $1\frac{1}{4}$ inches high.

Now, suppose all those 5 billion hamburgers could be placed end-to-end in a chain. How many miles long would the chain be?

Or suppose all those hamburgers could be put in a pile, one on top of another. How high would the pile be?

You can calculate the answer, of course, but will it be exact? How many estimates have you made before the calculation?



ARGUMENT

Am I Right?

ARGUMENT: *Am I Right?*

Running Time: 21:10 minutes

Documentary Segments (4):

1. ***BOOMERANGS:*** Making a boomerang demands fine tuning. **Even when it's working, you ask yourself, "Could it work better?"** . . 246
2. ***SUN DAGGERS:*** There's a good case that ancient rock carvings in the American Southwest functioned as solar calendars. **But is the argument conclusive?** . . 250
3. ***BLIMP:*** The LTA 20-1, a freight-lifting blimp, is the airship of tomorrow. **Or is it? Only time and tests with models and prototypes will tell.** . . 254
4. ***STONEHENGE:*** **Is Stonehenge a temple of sun worship, an observatory, perhaps an ancient astronomical calculator?** We'll never know *why* Stonehenge was built, but that doesn't stop us from asking questions. . . 260

Comedy Segment:

5. ***PIANO MOVING:*** "Bill and Phil, the Piano Movers," have different ideas about where they're going and how to get there. **Can argument help two wrongs make a right?** . . 244

Math Concepts:

BOOMERANG: Measurement (distance and angles), lift and drag.

SUNDAGGERS: Circle geometry, angles, statistics.

BLIMP: Air temperature and density, properties of spheres, lift and drag.

STONEHENGE: Force, levers, angles and measurement (surveying).

Problem-Solving Concepts: Checking result and method; quality of solution; efficiency of solution strategies.

HOW TO SPOT AN ARGUMENT PROBLEM

You can tell it's an argument problem if . . .

■ ***You think you've already solved it.*** *Any problem worth solving is worth checking. Is this the right solution? The right procedure? The right problem?*

■ ***It's a complex problem with no clear-cut or obvious answer.*** *Then, the question is, which is the best or most acceptable of all possible answers.*

■ ***"Everybody knows" the solution.*** *Put answers and accepted solutions are waiting for an original and critical mind to argue with them. Just because everybody solves a problem one way, doesn't mean you can't find a better solution.*

■ ***It seems unsolvable, but it's worth solving.*** *The problem solver who keeps challenging a problem may just crack it.*

■ ***By changing your perspective, the problem appears in a new light.*** *You may even get a new solution. Then the argument is, which solution is better?*

■ ***The solution process is long and elaborate.*** You probably can find a better way to solve the problem.

Throughout the chapter, the $X =$ sign indicates pages containing exercises and problems for classroom use and discussion.

"AND I SAY HE'S SAFE!": In problem solving, unlike baseball, "argument" means a willingness to see more than one side of a question. Manager Billy Martin is following a venerable baseball tradition by arguing with the umpire—but it probably won't lead him to a satisfactory solution.

ARGUMENT

Am I Right?

Problem solving doesn't end with an answer; it ends with an argument: Am I right? Is this answer right? Is it the best answer possible? Every problem solver must listen to the voice of the critic challenging him to double-check, to prove and persuade. This dialogue can happen in an open forum, as proponents of various solutions challenge one another, or inside the problem solver's head.

Math students may resist double-checking a page of homework exercises. Outside the classroom, where the stakes are higher, trial by argument is an integral part of the problem-solving process. An accountant checks and rechecks his figures and the tax code before informing a client that she owes $6,500 in federal taxes; a jury reevaluates all the evidence before handing down a verdict; a scientist who has discovered a cure for the common cold is going to review all the data and methods, check results and repeat the experiment before publishing his paper.

The world submits its toughest problems to the most stringent argument. Issues like economic policy, criminal justice, drug safety or toxic waste are tested in public debate. The data is reviewed and challenged, the methodologies or solution processes submitted to close scrutiny, alternative solutions considered and unacceptable options rejected. Through argument, the world struggles toward understanding and consensus.

Teaching **Argument**

Important, complex problems deserve to be argued. A routine or unimportant problem can be solved and passed on. But when the President sets foreign policy, a city council raises taxes, a school revises its curriculum, or a state raises the drinking age, then the problem must be argued. Complex problems breed a number of possible solutions; the only fair way to decide among them is to challenge each in turn, to test them by debate or argument.

Argument reviews the merits of the solution—and the problem, too. Argument reviews the evidence, point by point, and then attacks the chain of reasoning, trying to find a weak link.

Argument makes progress possible. By taking a hard look at pat answers and conventional wisdom, argument makes the future happen. Argument may produce a better mousetrap or a better space shuttle by rethinking old solutions and then restructuring the problem.

Argument is not just great leaps forward; it's also fine tuning an already good solution until it's a better one. Designers, architects, engineers and tinkerers are always looking at what is, and asking what it could be. . . if only.

Argument sparks learning. Argument ferrets out mistakes and pinpoints errors and inefficiencies. Its persistent voice may be as irritating and unwelcome as the voices that cried, "The emperor has no clothes." But questioning can turn failure into learning and make success that much sweeter.

Some arguments never end—nor should they. Where there's room for improvement, there's reason to continue the debate.

REACHING A VERDICT: When the jury's out, it's out to reach a verdict based on the arguments that have been presented in the trial. Is the defendant innocent or guilty? Each side uses the evidence to support its claims. Now it's up to the jury—in this case "Twelve Angry Men" from the 1957 film of the same title.

Why **Argument?**

"♦ There it is, the answer. I'm done with this problem.

No you're not.

♦ Yes, I am.

No, you're not. Have you checked your work?

♦ Okay, okay. 6 is to 4 as 1 is to x. $6x$ equals 4. 4 divided by 6 is 2/3rds. Yup, that's the answer.

You're not finished yet.

♦ YES, I AM.

No, you're not. You've only checked your calculations. Now read the problem again . . . and think. Is your answer the right answer?

♦ (Groan.) 'If a 6-story building is served by one elevator, how many elevators will a 4-story building need?'

Two-thirds of an elevator?"

Reminding students to check their work is a first step in teaching argument, but only a first step. Teaching argument involves helping students develop critical skills necessary to analyze their own ideas and discoveries as well as those of others, to evaluate facts and evidence, to find flaws and inconsistencies in logic and to be able to identify assumptions.

Test data. The news and advertising media daily bombard their audiences with data: "Four out of five doctors recommend Brand X"; "More cats prefer our cat food"; "In a reader poll, 23% thought the President was overpaid"; "Of 125 utilities built in the last decade, 78% had cost overruns in excess of $1,000,000." What do the numbers mean? Students should be able to review data c[illegible]cally, identify implied claims and evaluate whether the data support the claims, either stated or implied.

Use Logic. Many students enjoy logic puzzles; many others are frustrated by their abstract quality. Both groups can expand and improve their understanding of logic by using it as a critical tool in examining examples from their own experience, picking out and criticizing assumptions and inconsistencies as well as logical flaws. For example, a survey shows that students with home computers earn better grades than those who don't have computers at home. Ask students if, on the basis of this survey, they would recommend that parents buy computers for school-age children. Why—or why not? ■

"FOUR OUT OF FIVE DOCTORS RECOMMEND. . ."

"The critical question in evaluation is this: 'Does the answer I propose meet all of the goals and conditions set by the problem?' Thus, after the effort of finding a solution, we must turn our attention back to the problem statement and check carefully to be sure our solution satisfies it."

John R. Hayes, *The Complete Problem Solver*

Argument in the Classroom

One of the goals of studying math is developing the ability to think carefully and critically, as well as quantitatively. This, in a general way, is what ARGUMENT is all about.

Geometry: Geometric theorems and proofs are the place in the curriculum where most math students get an introduction to reasoning, or argument. Constructing a proof, the pattern of premise, statement and conclusion, carefully linked together in deductive reasoning, introduces students to mathematical logic. As they learn to examine their own thinking and to devise the simplest and most direct chain of reasoning, they appreciate one of the facets of argument—the elegance and efficiency of the solution process. It is not enough to have an answer; good problem solvers must also convince themselves that they have found the best and most direct path to the answer.

Statistics: The standards of a statistical proof are quite different than those of a geometric proof, but equally important for students to comprehend. Nowadays, it seems that everything from public policy to consumer purchases is decided on the basis of statistical evidence. Is saccharine safe? Do seatbelts save lives? How many humpback whales are there? These questions and many others are answered statistically. But there's room for argument in deciding how convincing the statistical case is. Being able to review data critically and to analyze how it is used to persuade are crucial skills for students to develop in order that they may be effective consumers and citizens.

Solid Geometry/Spheres: The LTA 20-1 is a giant helium-filled sphere; the geometric properties of a sphere are an important part of its design strengths. A sphere encloses the largest volume with the least surface area. Using the blimp as an example, have students compute surface and volume of a *sphere* with a diameter of 4 meters, and compare it to the surface and volume of a *cube* with 4 meters on a side.

Surveying: In the ARGUMENT film, Richard Atkinson is surveying several of the monoliths of Stonehenge, gathering data to test a theory. The principles of surveying were known to the Egyptians and Sumerians and are demonstrated by Euclid. The area of any figure or piece of land can be found (provided that the edges are straight), by breaking the figure up into triangles. Students can use this method to find the area of straight-edge shapes on paper or use the same method to draw a map and find the area of a marked piece of land. ■

OLD AND NEW: Modern-day surveyors have devices like this *theodolite* to measure horizontal and vertical angles, but the basic principles of surveying go all the way back to the ancient Egyptians.

Behind the Scenes: ARGUMENT On Film

Our main objective in filming ARGUMENT was to find stories that would put student viewers in the middle of a debate. We needed stories where an interesting problem was being solved and the subject was putting the solution to the test. We wanted viewers to conduct their own argument process along with the subject, intrigued by the promise of the proposed solution, yet eager to detect any flaws.

Boomerang: Boomerangers call their sport "the thinking-person's frisbee," perhaps because the exotic flight path of a boomerang gives the onlooker pause to think, "how can this be?" This short segment focuses on some important aspects of problem solving. Many problems have more than one answer, so the quality of a solution must be tested—and retested—before the problem can be considered finished.

Sun Daggers: When Ann and Bob Preston hypothesized that some of the Anasazi rock carvings functioned as seasonal markers, they thought they'd take a few measurements and quickly *disprove* their theory. But the first measurements checked out, and so did measurements for 60 petroglyphs at 19 other sites. Their investigation, involving measuring and charting the sun's path, is a good introduction to the measurement of time and calendars. Their argument—that these carvings function as solar calendars—involves the concept of statistical proof.

Blimp: One glance at the LTA 20-1 and we were hooked. A blimp in this day and age, and a spherical one at that! Designer Frederick Ferguson challenged the accepted wisdom of the past that blimps were a dead issue. By focusing on their assets and trying to eliminate their weaknesses, he found an answer he *thinks* will work, and he's putting his ideas to the test in the scaled-down prototype we filmed. This segment focuses on the solid geometry of a sphere, the physics of flight, computer and mechanical modeling, scaling in three dimensions and the Magnus effect.

Stonehenge: Hundreds of scientists, scholars, mystics and madmen have tried to unravel the mysteries of Stonehenge. Some have been solved; others remain questions. For 30 years, archaeologist Richard Atkinson has worked on the who, what, when and how of Stonehenge. This segment focuses on both measuring and surveying, as well as standards of proof in proposing solutions to the age-old mysteries of Stonehenge.

Piano Moving: We couldn't resist trying our own version of the classic comedy routine of piano moving. There are two kinds of argument in this segment: Bill and Phil don't "argue" with their answers; they just accept them untested. Consequently, they end up arguing with each other. They don't bother to stop and think, so they produce one wrong answer and then another. When it looks as though they'll have to move the piano a *third* time, they finally do what they should have done at the beginning: They test their assumptions and decide to let the sound of music travel and leave the piano where it is! ■

GETTING STARTED: **Thinking ARGUMENT**

The exercises in this chapter are designed to show three important functions of argument in the problem-solving process. Some exercises stress *verification*, or checking and testing a solution; others emphasize *optimization*, the process of determining the best solution from a number of possibilities; and still others demonstrate the role of *persuasion*, how statistics and graphs can be used to persuade as well as to inform.

Before showing your students the ARGUMENT film, you might have them do the following exercises to give them a "feel" for argument.

1. Lawyers defend their clients with arguments; legislators develop arguments for and against resolutions; scientists perform experiments to argue in support of their hypotheses; advertisers continually pit their products against others to argue that their products "work better," "taste better," "last longer," etc.

Discuss the role argument plays in clarifying issues and testing solutions. **Debate one of the following issues (or another issue of your own choosing). In addition to pros and cons, offer some factual support for your argument.**

- holding the Olympics in the same location every four years rather than moving the site from country to country each time
- establishing a 4-day school week
- establishing a 12-month school year
- banning violence on television
- raising/lowering the driving age

2. Using the clues in Table 1, supply the missing information. Then fill in the algebraic expression that describes the last column.

Explain how you deduced the values of *x* and *y*. Does your expression for the last column fit every row? If not, be prepared to go back and start over again!

	x	y	x + y	xy	x − y	x/y	2x + y	4x − 3y	$x^2 - y^2$	?
1.	6	3								−3
2.	4		6							−2
3.				12	−1					−9
4.			11	24						−1
5.		4							9	−7
6.	8						18			2
7.					9	4				3
8.		−5						15		15

Table 1

Answers:

	x	y	x + y	xy	x − y	x/y	2x + y	4x − 3y	$x^2 - y^2$	x − 3y
1.	6	3	9	18	3	2	15	15	27	−3
2.	4	2	6	8	2	2	10	10	12	−2
3.	3	4	7	12	−1	$^3/_4$	10	0	−7	−9
4.	8	3	11	24	5	$2^2/_3$	19	23	55	−1
5.	5	4	9	20	1	$1^1/_4$	14	8	9	−7
6.	8	2	10	16	6	4	18	26	60	2
7.	12	3	15	36	9	4	27	39	135	3
8.	0	−5	−5	0	5	0	−5	15	−25	15

X=

247

3. Some arguments are meant to persuade. Advertisements may manipulate statistics or dramatize figures to convince us of their point. **What is misleading about the illustration in Figure 1? How might this same information be more accurately presented?**

THE CRESCIVE COW

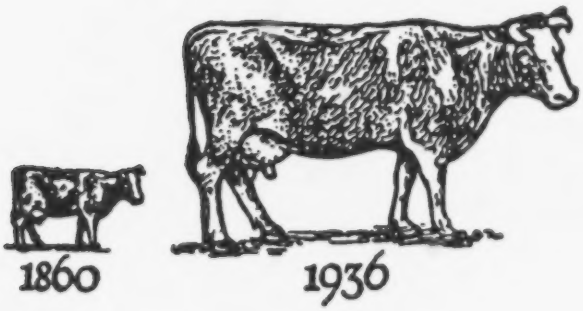


Fig. 1

In an attempt to demonstrate the increase in the number of milking cows in the United States between 1860 and 1936 (an increase from 8 million to 25 million), this illustration not only sensationalizes the increase but also gives the impression that the size of cows themselves has increased. The overall effect is misleading.

For Openers

After your students have seen the ARGUMENT film, you might ask them the following questions:

- What is a good argument? How do you recognize one?
- Is it important to understand all sides of an argument? Why/why not?
- Argument is very important in courtrooms, newspaper editorials, medical and scientific journals. Name other places where argument is important, and explain why.
- On what occasions have you had to make a case for something you believed to be true? Was your case convincing? How might it have been better?
- How do the characters in the film segments in ARGUMENT make use of argument to prove or persuade others about their point of view?
  1. What's the importance of argument in making a boomerang?
  2. Ann and Robert Preston argue that the stone carvings made hundreds of years ago by the Anasazi Indians were used as calendars. What is the basis of their argument? Can they prove it?
  3. What does the film mean when it calls the LTA 20-1 "an argument for a new kind of flying machine"?
  4. What are the difficulties in making a convincing argument about Stonehenge?
  5. How do Bill and Phil, the piano movers in the comedy segment, argue their case for a rather unusual solution to an awkward moving problem?

Stalking an **Argument** Problem

Making Up Your Own Problems

The following is a hypothetical conversation between teacher and students. It is intended to suggest ways to encourage students to make connections between problem-solving concepts in the film, their own experience and their knowledge of mathematics.

"

♦ ***I hope you're in a feisty mood today, because we're going to tackle argument problems. Who can tell me what argument means and what an argument problem is?***

It means checking your work.

♦ ***It's that and more. What about the Prestons and the sun daggers? What does argument mean there?***

It means proving their theory.

♦ ***Have they proved it?***

Well, not exactly proved, but they've made a strong case. Every time they find a new rock carving that works with the sun as a marker, they have a better case.

♦ ***So, for their argument, there's strength in numbers. The more cases they can demonstrate, the stronger their argument is. But what about all the rock carvings that don't give evidence of sun daggers? What if they found a rock carving with a sun dagger passing through it that did not seem to mark any special time on the solar calendar? What would you say that would do to their argument?***

It wouldn't make it any better, that's for sure. But it wouldn't *disprove* it because so many of them do work.

♦ ***What would disprove it?***

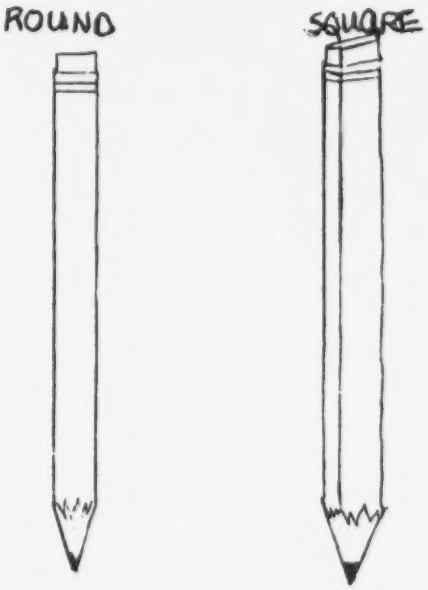
If none of them worked. Or, if they found some writing by the Anasazi who made them which said, "we carved them because they were pretty, not for any other reason." But they didn't leave any writing behind.

♦ ***It looks like the kind of solution that's hard to prove and hard to disprove, too. It's accepted for the moment, but we don't count the problem solved. Does that seem fair? And what about Stonehenge? Isn't that the same kind of thing?***

I think it's the same. In both stories, the people make careful measurements and try to eliminate coincidence, and they don't claim too much for their theories.

Problem 8

Which pencil would be the most efficient and easiest to manufacture?



Ashley Harvey
Howard Bishop Middle School
Gainesville, Florida

X=

♦ ***I think they're both special kinds of argument problems because the people are trying to figure out what happened a long time ago. There's no one they can test their theories on. No one can say, "Yes, it was like that," or "No, it wasn't." They have to be especially critical of their work. Let's switch to a more modern-day problem—the blimp. What kind of argument is going on there?***

Two kinds of argument, I think. First, the man is challenging old assumptions that blimps are no good. I also think he's testing his own design with models. That's argument, right?

♦ ***Yes. I think the phrase "everybody knows" should be a red flag to anyone with an inquiring mind. Frederick Ferguson must have heard lots of times that "everybody knows" blimps are dangerous and unmanageable, but he didn't accept it. He argued with it. What about the boomerang?***

That's a kind of trial and error, but I don't really understand why it's argument.

♦ ***Anybody have an idea?***

I think it's because when they're making the boomerang, they have to keep improving it. They get one answer and they work on it and get a better one. That's a kind of argument, almost as if you were arguing with yourself.

♦ ***That's good. Now we have all these examples from the film. Who's going to give me an example from their own life?***

Well, my family is going to buy a camera, but my parents want one type and my brother and I want another.

♦ ***That raises a lot of issues, doesn't it? Like costs and different models, and how much and for what purpose you and your parents intend to use the camera. How might we address these issues?***

We could make a list of pros and cons from each person's perspective. We could research the advertising claims that the different manufacturers make for various cameras. I guess there's a lot we could do before one of us would have to give way to the other.

♦ ***Okay, your homework for tonight is to collect supporting documentation for just such an argument. Pair up with someone in the class, and each of you take opposite sides in the debate over a particular issue. Take some time now to decide on what that issue might be, and then tomorrow come to class prepared to support your side in the argument. Remember, the more "hard proof" you come to class with, the more likely you are to win the argument. . . .***

"Have a will to doubt. Accept premises as tentative to varying degrees, but be flexible and ready to question their credibility, and, if necessary, pry yourself loose of fixed convictions and reject them. Rejection may take the form of innovation, because to innovate is psychologically, at least, to overcome or discard the old if not always to reject it outright."

Moshe F. Rubinstein, *Patterns of Problem Solving*

Following Through: Problem Ideas from the Film

In each of the four ARGUMENT stories, there is the raw material to devise many problems to challenge students. Here are a few:

Boomerang: When Peter Ruhf and Holly English-Payne make their own boomerangs, they don't design from scratch. They choose a pattern to define the general shape, and then tailor it. In a way, this is how textbook problems are solved: The student reads the problem, identifies the basic relationship involved, chooses the appropriate equation and then tailors it by filling in the quantities from the problem.

Ask students to find the equations that "fit" the following problems.

- In a square ABCD with 16-inch sides, how long is the diagonal (AD)?
- What is the circumference of a circle with a diameter of 17 inches?
- If there are 18 teams in a football league, how many games will have to be scheduled in order to make sure every team plays every other team once?

Now have them find some similar problems from their own textbooks, where the task is to find the equation that the problem calls for.

Sun Daggers: The sun daggers discovered by Ann and Bob Preston on cave walls in America's southwest are formed by shadow lines.

Show students how to use shadows to measure height. Then ask them to solve some shadow problems. Here's one example:

- A tree 20 feet tall casts a shadow 12 feet long. A flagpole nearby casts a shadow 16 feet long. How high is the flagpole?

Discuss with students why this method is accurate?

The spiral petroglyphs etched in stone by the Anasazi are Archimedean Spirals, that is, their loops are spaced at equal intervals.

- Imagine that an insect in the center of a standard phonograph record turntable begins walking in a straight line toward the edge. As the turntable revolves, he will walk along an Archimedean Spiral with respect to the floor. It takes him 30 seconds to reach the edge. Draw a picture of the insect's route as he makes his way to the edge of the record.

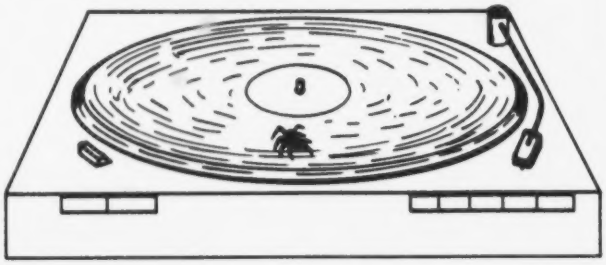


Fig. 1

SPIRAL JETTY (1970) by Robert Smithson is made of black rock, salt crystals and earth and projects into the Great Salt Lake, Utah. What arguments might future generations offer as to its purpose?

Blimp: In coming up with a new design for a lighter-than-air ship, Ferguson and his design team use models and prototypes to test the different functions and capabilities of the airship well in advance of building the "real thing."

- Two spheres have radii of 20 feet and 40 feet, respectively. Find the surface area and volume of each. On the basis of your answers, explain what happens to the surface area and volume when the radius of a sphere is doubled.

- Imagine a balloon is inflated at a steady rate. That is, the volume of air pumped into the balloon is the same each second. If it takes 5 seconds to inflate the balloon until the radius is 5 inches, how many more seconds will it take for the radius to become 15 inches?

Stonehenge: There are many theories about the purpose of Stonehenge, each based on patterns perceived by the theory-maker.

- Use number series problems for practice in pattern recognition. Rather than ask students to supply the next item in the series, ask them to explain the underlying pattern. For instance, give them the series 2,2,4,4, ...One student might see a repeating progression of even numbers. Another might see a progression of squares. Still another might see a double progression, one of squares, the other of even numbers. The class might also enjoy non-number progressions (Ann, Bart, Candy, Daniel, . . .). Or, have students devise their own progressions and challenge classmates to find the underlying pattern.

- The process of argument consolidates learning. Divide the class into pairs and have each team build the tallest structure it can, using only two sheets of $8^1/_2$-in $\times$ 11-in paper, scissors and tape. After the towers are finished, compare them and discuss the tallest. Then, play another round. The towers should get even higher as everyone learns not only from successful designs but from inspired mistakes.

"In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions, and they must be willing to subject these conclusions to scrutiny."

National Council of Supervisors of Mathematics, 1977

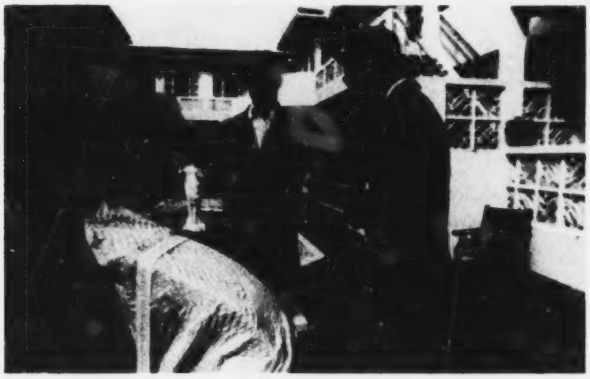
On the Lighter Side

In a rush to get their math problems finished as soon as possible, most students dispense with checking their work. The same is true of "Bill and Phil, the piano movers." They think they've got the answer, and they go ahead and solve the problem wrong, not once but *twice*. They don't make sure—they don't check their work—and they pay the price, lugging the piano up two flights of stairs and down and then up again.

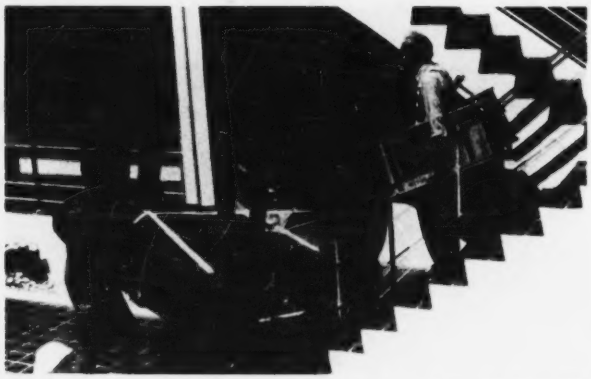
Bill and Phil disagree vociferously, but the point of the comedy segment in ARGUMENT is more than arguing. Asking yourself "Am I right?" may mean some disagreeing with answers, but more important, it means questioning the means by which the solutions are determined. It can save time and effort all along the way.

Solve these problems and then take another look at the problem *and* the solution, asking "What's wrong here?" or "What does my answer mean?"

A Comedy of Errors: Piano Moving



Bill: "I guess we'll take it right up there."
Phil: "No, I think we should take it over there, Bill."
Bill: "No, second floor, just take it right up there."
Phil: "Well, no, I think we should take it over there."



Bill: "Phil, just trust me, okay? We'll take it right up there. . . .Check the width. Hold it steady, steady as she goes. . . ."

Tuning Piano Problems

A good problem solver asks himself not only, "Does this answer make sense?" but also, "Does this *problem* make sense?"

- Bill and Phil are equally strong and equally hard workers. If together they can move $1\frac{1}{2}$ pianos in $1\frac{1}{2}$ days, how many pianos can one of them move in a day?
- Carlita Romanoff earns $250 for a 2-hour concert, consisting of 6 selections. How much does she earn per hour? Per selection?

Mrs. Romanoff: "Yes, you must be the gentlemen with the piano. . .excellent. Just take it off your truck and put it right down in the courtyard. It'll be right there, where the dear custodian is putting the bench. . . .Oh, my dear, the patrons are arriving. Please, quickly gentlemen, quickly."

Bill: "Don't you just love classical music, Phil?"

■ A piano weighs 500 pounds and has 88 keys. How many pounds does each key weigh?

■ Bill and Phil have 7 pianos to deliver. Each delivery takes on the average 45 minutes. How much time will they have to work to deliver all 7 pianos?

Many Happy Returns

"It's exciting because every year one of my friends, or myself, comes up with a slight variation. We've had guys hollow out the backs of boomerangs—and all of a sudden, you've got the fastest boomerang you've ever seen. It goes out and comes back in two and a half seconds."

There's a simple definition of a boomerang: a throwing device that returns. After that it gets more complicated. One baseball is pretty much like another, but there are many shapes and sizes of boomerangs. Each different type performs differently, flying farther or faster, tracing a teardrop or circular path, better suited to windy or calm conditions. Even the pedigree of the boomerang is a complex story. The earliest boomerangs have been found in America, Europe, India and, of course, Australia. Early boomerangs, elegantly weighted with gold tips, have even been found in the tomb of King Tutankhamen in Egypt.

And if you want to know why a boomerang boomerangs, you won't get a simple explanation, either. Bernoulli's principle, Newton's laws of motion, gyroscopic stability and gyroscopic precession all contribute to the round-trip flight of the boomerang.

Of course, when experienced boomerangers such as Peter Ruhf and Holly English-Payne take to the field, they aren't considering the abstract laws of physics. They are only concerned with the boomerang's performance. When they test-fly English-Payne's new boomerang, they find it isn't returning as accurately as she would like. Ruhf tries slightly reshaping the *dingle arm*.* A second test flight proves them right or wrong. After a little more reshaping and fine-tuning at the workbench and another test flight, the boomerang returns directly to the thrower.

*The *dingle arm* is the arm held in the thrower's hand; the other is called the *lifting arm*.

Ruhf developed his skill in making and throwing 'rangs (short for boomerangs) to the point that he is the world-distance champion. His 1982 record—125 yards out and 125 yards back—is listed in the *Guinness Book of World Records*.

Other boomerang competitions include juggling, by which the contestant throws one boomerang while another is in flight (the world record is 106 consecutive catches), the maximum-time-aloft event (the world record is 33.2 seconds), accuracy throwing and consecutive two-handed catches (the world record is 146).

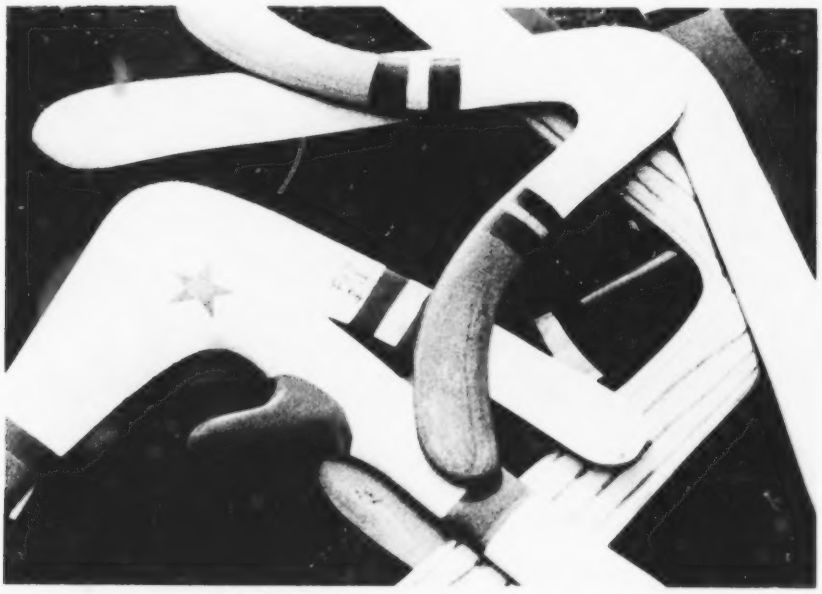
OUT FOR A SPIN: World distance champion Peter Ruhf and tournament thrower Holly English-Payne test-fly a new boomerang. The boomerang leaves the thrower's arm perpendicular to the ground and returns parallel to the ground.

Small Change, Big Difference

Different events require different kinds of boomerangs. "You can put weights—lead ballast—in certain parts of the wing and change the flight characteristics just like *that*," Ruhf says, snapping his fingers. The result, says English-Payne, is that the flight path "usually goes from a circular one on an unweighted boomerang to a teardrop-shaped flight with a weighted one. The distance boomerangs really go out in a teardrop shape."

In the fast-catch event, the thrower tries to launch and recover the boomerang as many times as he can within a given time limit. Success requires throwing skill and a boomerang that will follow a short flight path with great accuracy and high speed. "It's exciting," Ruhf says, "because every year one of my friends, or myself, comes up with a slight variation. We've had guys hollow out the backs of boomerangs—and all of a sudden you've got the fastest boomerang you've ever seen. It goes out and comes back in two and a half seconds."

As a result of this process of testing and retesting, boomerangs today come in many different shapes, sizes and weights, designed for many different purposes, including distance and trick throwing. There are even boomerangs in the shape of many letters of the alphabet—E-rangs, F-rangs and W-rangs, for example. But some alphabet throwing sticks can boomerang on you. For some reason, the shapes of the letters B, D, I, O, P and D don't return. As boomerangs go, they're dead letters. ■



FORM AND FUNCTION: The boomerang's basic form is two or more arms bent at an angle in the same plane and contoured like an airplane wing. Within this framework, there is great variety. Each boomerang's design performs differently—some fly higher, some further, some longer.

Further Readings

Cassidy, John. *The Boomerang Book*. Palo Alto, California: Klutz Press, 1985.

A light-hearted approach to boomeranging, with patterns to make your own.

Hess, F. "The Aerodynamics of Boomerangs." *Scientific American*, November 1968.

A thorough introduction to the scientific principles behind boomerang flight. Good line drawings; accessible to high school students.

***Many Happy Returns*. Washington, D.C.: Minner Press.**

This quarterly newsletter contains the most up-to-date information on boomeranging around the world, including new stores, tournaments, how to build and repair boomerangs and readers' notes and news. A nice subscription for a student who develops a real interest in the sport. (Address: 1882 Columbia Road, N.W., Washington DC 20009.)

Ruhe, Benjamin. *Boomerang*. Washington, D.C.: Minner Press, 1977.

One hundred and twenty pages of everything you always wanted to know about boomerangs, including why they return, how to make one, different shapes and sizes and the history of their development. This book also contains a listing of boomerang societies and clubs in the United States and a bibliography. Appropriate for junior and senior high-school students.

Thomas, Jacques. "Why Boomerangs Boomerang." *New Scientist*, 22 September 1983.

This article describes the difference between the Australian killing-stick (doesn't return) and the boomerang (it does), and also describes the flight path of a boomerang, complete with photos and illustrations. The aerodynamics of the boomerang are also discussed.

Walker, Jearl. "Boomerangs! How to Make Them and Also How They Fly." From "The Amateur Scientist," *Scientific American*, March 1979.

Excellent description of how to make a variety of boomerangs, including one fashioned simply from plastic rulers and rubber bands. Walker also gives throwing tips, as well as instructions on how to "fine tune" your creation.

Why Does a Boomerang Return?

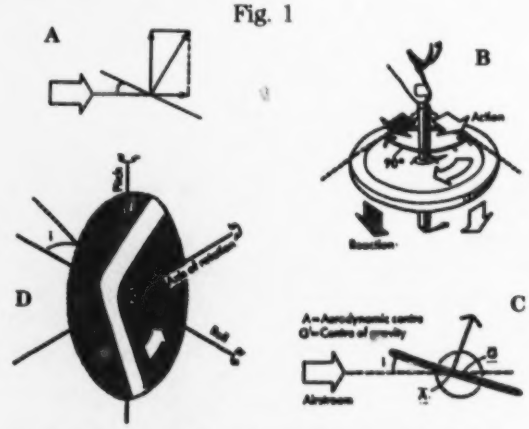
A good question! The answer lies in large part in its shape. The traditional boomerang is L-shaped with two flat arms. In cross-section, the boomerang is curved on top and flat on the bottom, like the wing of a plane, or *airfoil*. When the boomerang is thrown properly—the arms should be almost vertical when released—the airfoil design pushes it sideways instead of up, in much the same way that an airplane banks its wings to make a turn.

But this alone does not account for its "homing" instinct. *Gyroscopic precession* is largely responsible for that. According to the laws of precession, if a force is applied at any point on a spinning object, the object will respond not by turning in the direction of the force applied as expected, but will turn as if the force were exerted *perpendicular* to the line of original direction.

In the case of the boomerang, the force that operates to turn the boomerang is caused by the uneven lift of the two spinning arms. As the arms move through the air, the arm passing through the top of the arc in the direction of the boomerang's forward motion has a much higher speed than it has when the same arm passes through the bottom of the arc when its speed is counteracted by the forward motion of the boomerang.

The disparity in velocity of the two arms exerts pressure on the front edge of the boomerang causing the precession effect and causing the boomerang to turn. The boomerang begins its flight with arms perpendicular to the ground but returns flying parallel to the ground, gradually turning a full 180 degrees back on itself—to the place from which it was thrown.

Fig. 1



Clockwise, from top left: (A) vector diagram of the aerodynamic force created by an airfoil section; (B) the principle of *gyroscopic precession* gives the boomerang its circular flight path and makes it lie down; (C) the aerodynamic center of a boomerang is in front of its center of gravity; (D) the blades sweep a circle around the axis of rotation, which runs through the boomerang's center of gravity.

Teeing Off with a Boomerang

When Peter Ruhf cuts and shapes a boomerang, he uses precise measurements to ensure the correct form. Yet when he is out testing his product, he still uses trial and error to determine how to refine the shape.

This combination of mathematical precision and trial and error is important in mathematical problem solving. He uses a general law to modify his boomerang, but the degree to which he modifies it depends on an intuitive estimate.

The best argument for using this method for boomerangs is that *it works*. After a number of trial flights and subsequent refinements, the boomerang flies out and comes back to the same spot from which it was thrown.

The act of designing a boomerang is a good and fun way to gain awareness of this interplay of mathematics and intuition—an interplay which is the foundation of invention.

To start off, what are some of the characteristics that define a boomerang? Here are just a few you might think of:

1. A boomerang is a kind of "throwing stick" that returns to the point from which it is thrown.

2. A boomerang is thrown vertically but returns horizontally.

3. A boomerang's speed is not constant during flight.

4. A boomerang can be made to fly closer to the ground by the placement of a small amount of weight ballast in the boomerang's elbow.

5. A boomerang is thrown "overarm" like a baseball.

6. A boomerang's "homing" instinct is largely due to its airfoil shape and to the laws of gyroscopic precession.

7. A boomerang can be of any size and shape as long as it "returns."

8. A boomerang is described according to weight, angle between the arms, length of the arms and its range or radius of flight.

Exercises:
Figures 1–4 show the dimensions for four boomerangs.

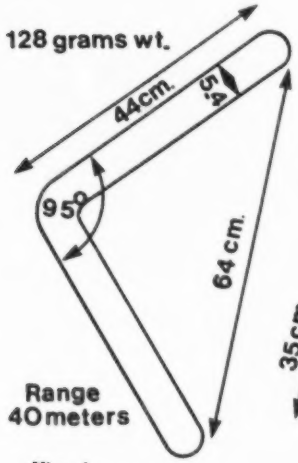


Fig. 1

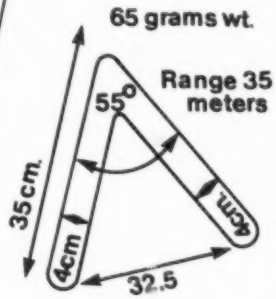


Fig. 2

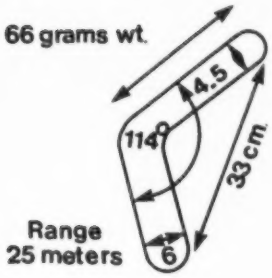


Fig. 3

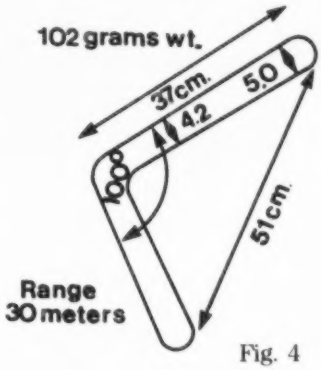


Fig. 4

Use the information in Figures 1–4 to support or refute each of the following arguments:

Arguments:
1. The range of a boomerang becomes greater as its angle becomes larger.

2. The distance between the arms determines the range of a boomerang.

3. The length of each arm determines the range of a boomerang. Adding ballast or weight to wing tips can add as much as 50 percent to the range of a boomerang.

4. How would adding ballast to wing tips affect boomerangs in Figures 1–4? Give the enhanced range if ballast were added to the rangs.

Answers to #4:
4. Fig. 1—60 m (enhanced range)
Fig. 2—48.78 m
Fig. 3—37.5 m
Fig. 4—45 m

Making Your Own Boomerangs

Here is a simple way to make a real working boomerang:

Quick-and-Easy Cross-Stick Boomerang

This boomerang, particularly suitable for younger students to make, requires two 1-foot plastic or wooden rulers, two strong rubber bands and very little time. (Note: *Never* use rulers with metal edges.) Fasten the rulers together at their centers with the two rubber bands as shown in Figure 1. That's all there is to it. The boomerang is ready for action.

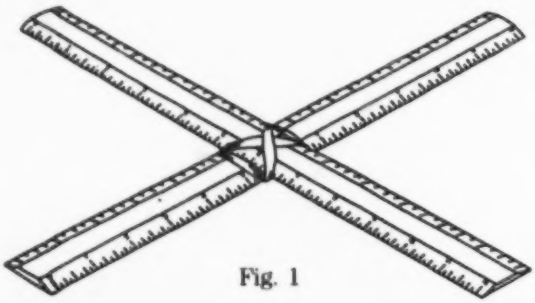


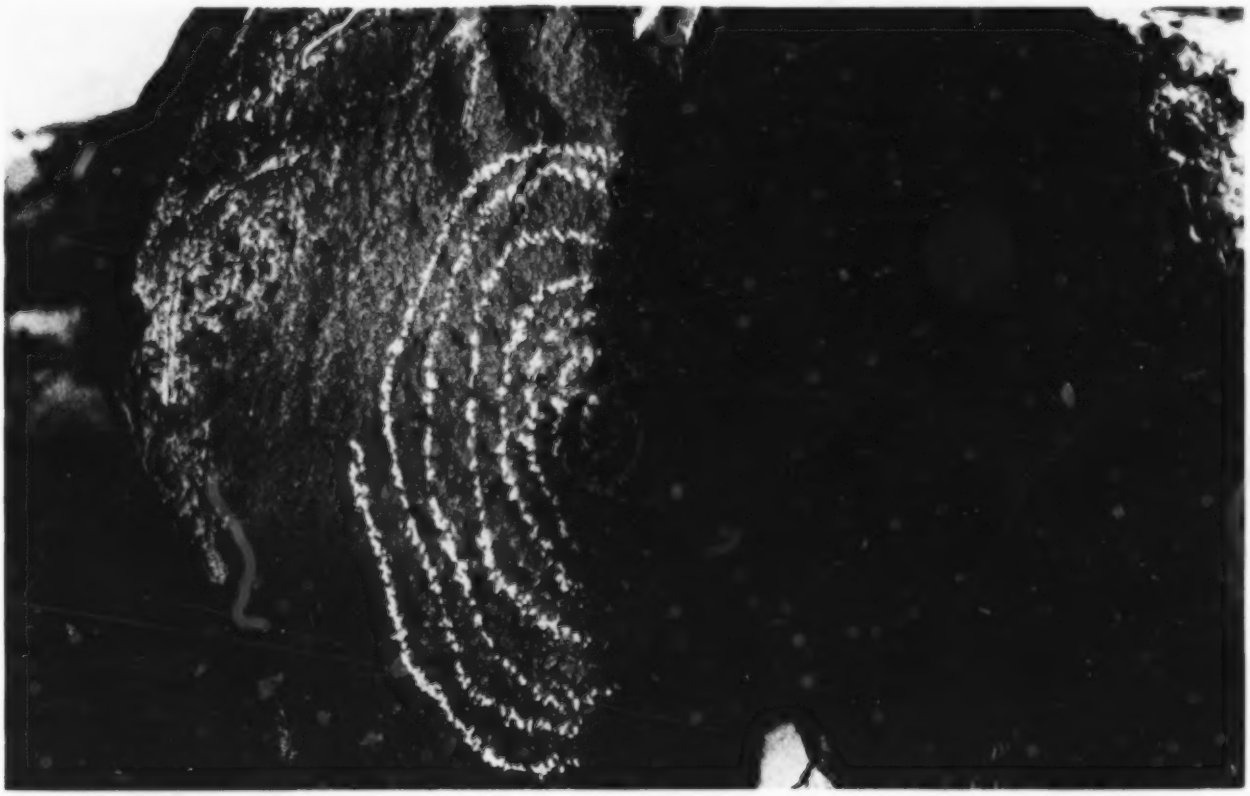
Fig. 1

Sun Daggers: A Little Light Reading

Had the Prestons unlocked a secret of the petroglyphs? In the cold, focused light of scientific investigation, what they *had* could be dismissed as inspired guesswork and mere coincidence. What they *needed* was enough corroborative data to yield a convincing argument.

Initially, Robert and Ann Preston had only stopped off in the Petrified Forest National Park in eastern Arizona to do some sightseeing. It was the first time Ann, an artist, or Bob, a radio astronomer, had had the opportunity to see the rock carvings, or *petroglyphs*, left in caves by prehistoric Indians. Neither was completely prepared for the beauty of the carvings or for the "daggers" of sunlight that appeared through cracks in the caves and at times intersected the petroglyphs. What Ann saw reminded her of the findings of another artist, Anna Sofaer, who had demonstrated that a similar petroglyph in New Mexico's Fajada Butte may have functioned as a seasonal marker—with a little help from the sun.

Were the Prestons gazing at a calendar or a coincidence? To try to answer this question, they changed their vacation plans. They returned to the Petrified Forest equipped to study the petroglyphs in more detail. They brought a compass, which they used to find the sun's position in the sky at the time when a sun dagger interacted with one of the petroglyphs.



LIGHT INTERACTION: At key points during the year, such as *solstices* or *equinoxes*, the sunlight interacts with the geometry of Anasazi petroglyphs, leading to speculation that the stone carvings may have been used as calendars.

"The first measurement turned out to be absolutely perfect," Bob Preston explains. "What we were looking at was a solar marker." During the rest of their vacation, the Prestons investigated other petroglyphs and found that they also appeared to mark the four major events of the solar year: the summer and winter solstice and the spring and autumn equinox.

Had the Prestons unlocked a secret of the petroglyphs? In the cold, focused light of scientific investigation, what they *had* could be dismissed as inspired guesswork and mere coincidence. What they *needed* was enough corroborative data to yield a convincing argument.

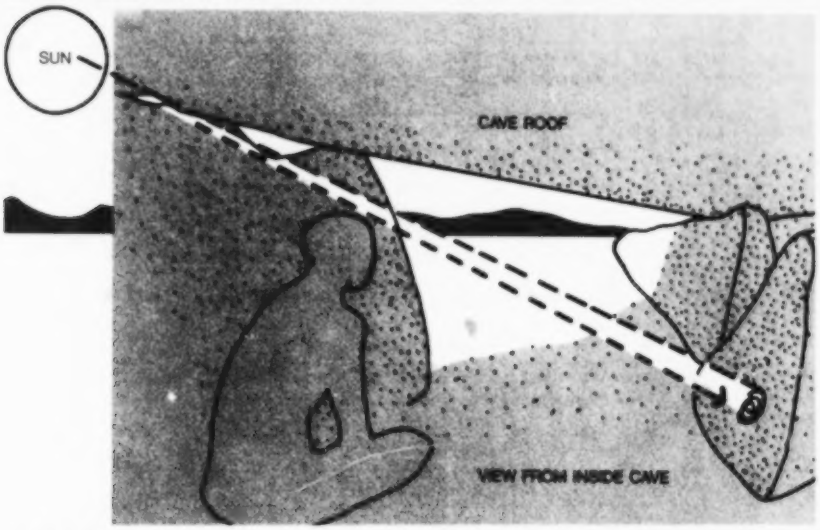
Shedding New Light on the Petroglyphs

One significant obstacle to their investigation was the fact that little is known about the petroglyphs and the people who made them. Rock carvings are scattered throughout the southwestern United States, but no knowledge about their historical use and meaning has survived.

The petroglyphs are estimated to be about 1,000 years old and are believed to have been etched into the sandstone by the Anasazi Indians.

These ancient people* flourished in the Southwest from 800 A.D. to 1250, only to disperse and eventually disappear. To this day, no one knows where they went or why they disappeared. From certain features of the civilization that *have* survived—including ruins of complex cliff dwellings (see story and film on the Anasazi Indians in the INFORMATION chapter) and evidence of a vast network of roads—it can be assumed that the Anasazi were a sophisticated enough culture to have developed a system of counting days from one solar event to the next.

Concentrating on the theory that the sun daggers are seasonal markers, the Prestons sought out more corroborative evidence over the course of the next few years. The more petroglyphs they found that worked the same way, the stronger their argument in support of their hypothesis.



SOLAR CALENDAR AT WORK? Forty-five days before the winter solstice, a dagger-like shaft of light from the afternoon sun shines through a crack in the roof of the Cave of Life (in the Petrified Forest in Arizona), illuminating a spiral petroglyph. As the sun lowers, the light moves further into the cave, finally coming to rest in a large cross-shaped petroglyph.

Robert Preston.

Site Searching

They asked ranchers, rangers or anyone else familiar with an area where carvings might be found. They ferreted out petroglyphs and searched for cracks in rocks through which sun daggers might appear. Using a compass with a level to measure the direction and elevation of the sun paths that might throw daggers against the carvings, Bob Preston was able to produce computer-generated graphs of the sun's track at specific locations on the days of solstice and equinox. If the graphs suggested that sun daggers would appear in relation to the petroglyphs on those days, the Prestons would return to the site and film the event as it took place. They continued their investigation for nearly two years, exploring 19 sites—some as far as 600 miles apart—and discovered about 60 petroglyphs that seemed to act as seasonal markers.

Having accumulated a large body of corroborating evidence, the Prestons have mounted a strong argument to support their theory, and some scholars now take it seriously as a possible—if ultimately unprovable—explanation of the meaning and purpose of the petroglyphs. ■

*The word *Anasazi* means "ancient people" in Navajo.

What's the Angle?

The earth is tilted at an angle of about 23° 27′ to the path of its orbit around the sun. This phenomenon causes the seasons.

During the summer, the Northern Hemisphere is tilted toward the sun. As a result, daylight hours are longer; nights are shorter; and the weather is generally warmer. On the summer solstice, about June 22, the sun is highest in the sky in the northern hemisphere. It appears 23° 27′ north of the equator. That is, to a person at latitude 23° 27′N (the Tropic of Cancer), the sun is directly overhead at noon.

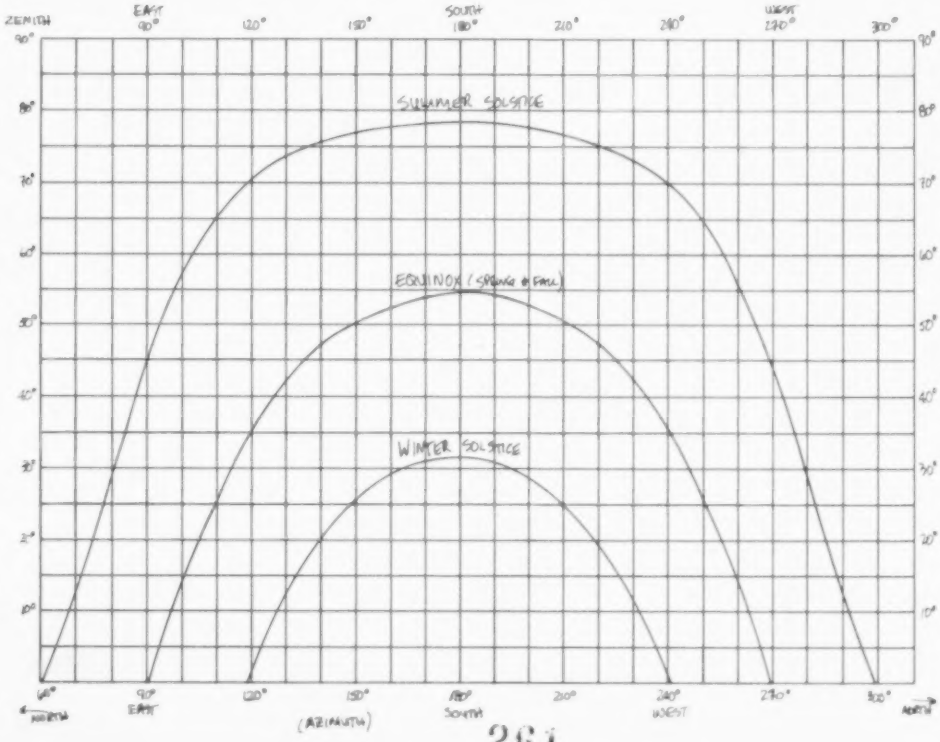
During the winter, the Northern Hemisphere is tilted away from the sun. The days are shorter; the nights longer; and the weather is generally colder. On the winter solstice, about December 22, the sun appears 23° 27′ south of the equator. In the Northern Hemisphere, it is the shortest day of the year and the sun appears lowest in the sky. (In the Southern Hemisphere, the seasons are reversed.)

Midway between the summer and winter solstices are the equinoxes, when the sun crosses the equator. On the equinoxes, about March 21 and September 23, every place on earth receives 12 hours of daylight and 12 hours of night.

If you were at latitude 23° 27′N at noon on the summer solstice, the sun would appear to be directly overhead. If you were at latitude 28° 27′N, the sun would appear to be 5° south of overhead. If you were 30°N, the sun would appear to be 6° 33′S, and so on. In fact, by measuring the sun's angle of elevation at noon on the summer solstice, you can determine your latitude. This can also be accomplished at winter solstice or on the equinoxes. For other days, you would need the sort of information about the sun that is available in a nautical almanac.

Shadowing Shadows

You can show the various paths of the sun from day to day by watching the shadow of some stationary object and marking the tip of that shadow several times in a day (say once every hour). The



FOLLOWING THE SUN: Throughout the year, the sun travels along different paths across the sky. The graph shows the paths followed by the sun at three important times of the year: (1) winter solstice, (2) summer solstice and (3) equinox. Elevation, or degrees above the horizon, is plotted on the vertical axis, while the azimuth compass position is plotted on the horizontal axis in degrees measured east from north. By plotting the measured elevation and azimuth of the shadow line joining the crack and the petroglyph on this graph, the Prestons could quickly determine if that particular alignment would make a sunlight image on the petroglyph at either solstice or equinox.

next day, if you mark the shadow at precisely the same times, you will find it traces a slightly different path. The shadow will be shortest on the summer solstice and longest on the winter solstice. From June 22 until December 22, the sun will appear to get lower in the sky each day, so the shadows will get longer. From December 23 until June 21, the sun will get higher and the shadows shorter.

If you were to mark the spot where a shadow fell on a given day at a given time, you'd find that the next year, at the same time and about the same day, the shadow would be in the same place.

The sun daggers appear to follow essentially this same principle, except that the dagger is made by the sun's rays going through an opening in the rocks, and the shadow could be thought of as that which surrounds the dagger.

Exercises:

1. If the noon sun appears to be directly overhead at 23° 27′N, what would it be at a latitude of 38° 27′N? Then, what angle would it be above the horizon for somebody at 38° 27′N?

2. Repeat problem 1 for 40°N.

3. Repeat problem 1 for 32°N.

4. Look at a map of your state. Approximately what is the latitude of the place where you live? Figure out what angle the sun will be at noon on the summer solstice where you live. What should be the approximate angle of the noon sun above the horizon at the time of the spring and autumn equinoxes, when the sun appears to be directly above the equator? What should be the approximate angle of the noon sun above the horizon on December 22, when the noon sun appears to be 23°27′ south of the equator?

5. How many degrees are there in a circle? Does the sun appear to make a complete circle around the earth every day? Then, how many degrees does the sun appear to travel in 1 day? How many degrees does the sun appear to travel in 1 hour (1/24th of a day)? Then, how many minutes will it take the sun to appear to move 1°?

6. Two hundred years ago, local time was determined by local astronomical noon, so your clock would have said 12:00 at precisely noon if it was telling time correctly. Why do you think people chose to change? Why didn't they change earlier?

Answers:

1. 15°; 75° (found by subtracting 15° from 90°, the number of degrees between horizontal and vertical).

2. 16°33′ (you may want to remind students that there are 60 minutes in a degree, that is, 1° = 60′); 73°27′.

3. 8°33′, 81°27′.

4. As an example, we will do the problem for Chicago: From a good road atlas we can see that 42°N goes through the northern section of Chicago. And, 41° is considerably south of Chicago (slightly south of Kankakee). So, the center of Chicago seems to be at about 41°50′.

The angle below vertical at noon on June 21 would be 41°50′ − 23°27′ or about 18°23′, so the sun would be 90° − 18°23′ = 71°37′ above the horizon at noon on June 21.

At the time of the equinoxes, the sun is directly above the equator, so from Chicago it would appear to be 41°50′ down from directly overhead, which is 48°10′ from the horizon at noon.

At noon on the winter solstice, the sun would appear 23°27′ + 41°50′, or 65°17′ south of directly overhead, or only 24°43′ above the horizon.

5. 360°; yes; 360°; 15°; 4 minutes.

6. As people began traveling greater distances in shorter periods of time (primarily with the advent of the railroad in the 1800s), it became inconvenient to reset clocks and watches to each local time. Communications, first with the telegraph and then with the telephone, radio, television, etc. made different times even more inconvenient.

Further Readings

Aveni, A.F., ed. *Archaeoastronomy in the New World*. New York: Cambridge University Press, 1982.

This book contains a variety of papers on archaeoastronomy in North, Central and South America.

"Graffiti with a Heavenly Message." *Time*, 31 January 1983.

This article describes Robert and Ann Preston's discovery of hundreds of functioning petroglyphs throughout the Southwest. A short, but complete, story encapsulating their initial discoveries.

Frazier, Kendrick. "The Anasazi Sun Dagger." *Science 80*, November/December 1979.

A well-illustrated article detailing artist Anna Sofaer's discovery of a "sun dagger" moving across an Anasazi petroglyph at Fajada Butte, New Mexico. Sofaer and her colleagues spent the next two years studying the phenomenon at this site and elsewhere, paving the way for the Prestons' explorations.

"Petrified Forest—Arizona's Other National Park." *Arizona Highways*, February, 1983.

An article describing new scientific findings in Petrified Forest National Park in Arizona, including a detailed account of the Preston's "sun dagger" discoveries. Many good color photographs and illustrations of the petroglyphs under investigation by the Prestons.

The Very Model of a Modern-Age Dirigible

The new airship will be able to carry a ton of cargo a mile at a cost of only 30 cents, and will be able to travel, fully loaded, over 500 miles without refueling.

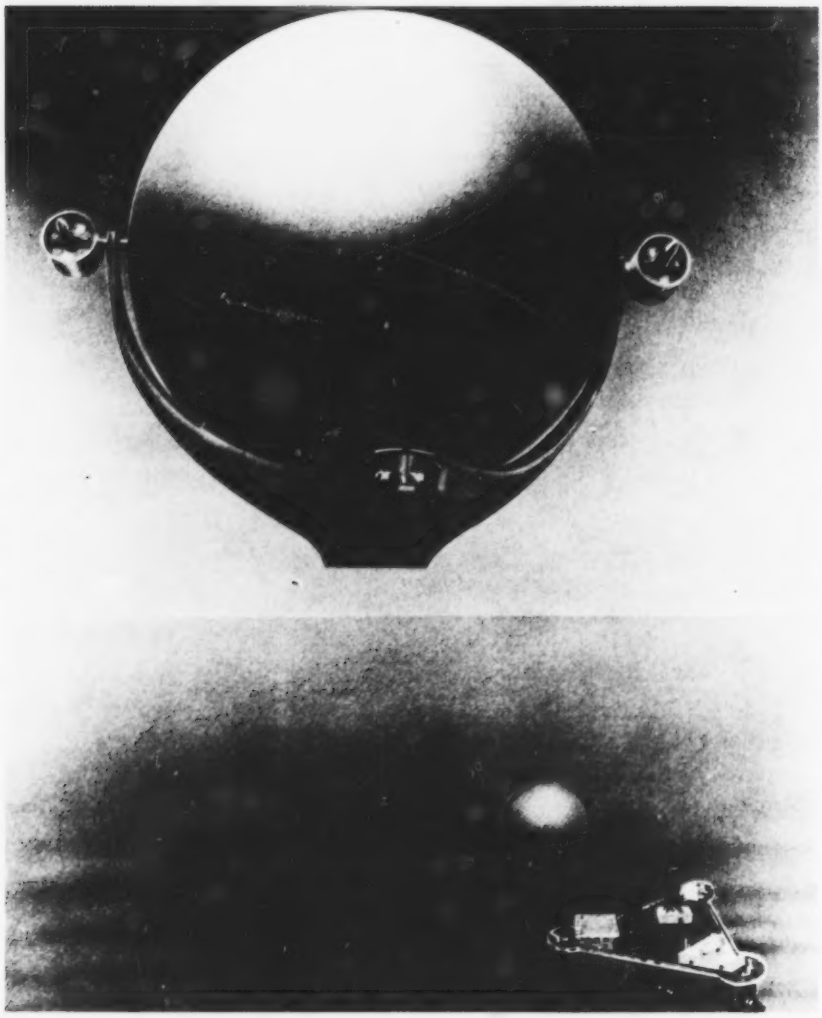
Since the fatal crash of the *Hindenburg* in 1937, lighter-than-air craft have been considered by many to be the dinosaurs of powered flight. Does this mean the entire idea of lighter-than-air flight is doomed to extinction? Frederick Ferguson and his design team at Magnus Aerospace Corporation, Ltd., in Ottawa, Canada, don't think so, and they are determined to prove their case.

Intrigued by the potential of lighter-than-air ships as "sky cranes," Ferguson and his team spent a year studying the history of airships and scrutinizing airship designs to develop an argument for bringing back this breed of flying machine.

"We isolated absolutely everything we could find that was wrong," Ferguson says. "The ellipsoid shape was causing problems." In the past, airships had been designed in an 8:1 length:width ratio, and currently the measurements of existing airships are more on the order of 4:1. Ferguson and his team decided to continue the trend and go to the optimum. They cut out the "unmanageable parts" from the ellipsoid shape and found themselves working with a shape in a 1:1 length:width ratio—a sphere.

Aerodynamically, the sphere has a number of advantages including the fact that it can hold the greatest volume of helium — and therefore has the greatest lifting power—for a fixed surface area of material. It is also structurally more sound, more stable and more maneuverable than the ellipsoid-shaped blimp. But spheres have one distinct drawback as well: They don't fly very well. That is, not unless they're spinning.

JUST RESTING: Familiar "cigar" shape of old-fashioned dirigibles makes them unwieldy and inefficient. (If you think a blimp moored on the top of the Empire State Building is an unlikely situation, you're right – this image is the product of a retoucher's skills.)



IN FLIGHT: Above, artist's rendering of two LTAs in flight over an oil rig. Below, Frederick Ferguson, designer of the new airship.

Backspin Breakthrough

Sports gave Ferguson and his team the clue they needed to solve this problem. "Where do the spherical golfball and baseball get their lift?" they asked. When a fastball is thrown forward, the ball spins backward on its horizontal axis. Backspin creates an area of high air pressure *under* the ball and low air pressure *above* the ball, as the air rushes to fill the vacuum behind the moving sphere. In flight, the ball rises, moving into the low pressure created by the swiftly moving air above it. This phenomenon is called the *Magnus effect*, for the German scientist Heinrich Magnus, who discovered it in 1853. (See box on Magnus effect, pg. 257.)

A baseball eventually stops spinning and falls, slowed by friction against the air and by its own weight and gravity. A small motor located in Ferguson's Lighter-Than-Air ship, the LTA 20-1, keeps the huge balloon spinning at the rate of about once every 17 seconds, elegantly creating its own lift.

At an estimated 70-mph maximum speed, the LTA 20-1 will never catch the supersonic Concorde, but it will have advantages over the heavy-duty lifting helicopters it is designed to replace. A helicopter depends for its lift on the energy it gets from its rotating blades. But a helicopter is an inefficient lifter because it puts a great deal of energy into lifting its own weight. Eighty percent of the lift in the LTA 20-1, on the other hand, comes from the helium contained in the sphere, so it requires much less energy to remain aloft.

Payload—Pay Dirt

Ferguson estimates the new airship will be able to carry a ton of cargo a mile at a cost of only 30 cents, and will be able to travel, fully loaded, over 500 miles without refueling. It is also estimated that the LTA 20-1 should be able to carry up to 60 tons of cargo—four times the carrying power of today's largest helicopters. In addition, it will carry a crew of two or three in an aerodynamically shaped gondola suspended from the axle.

Ferguson's team has worked with several scale models, including one 14 feet in diameter, which replicates the height and power distribution of the full-scale, 200-foot prototype yet to be developed. An axle pierces the sphere through its horizontal axis, and at each end of the axle is a ducted engine. Unlike the single engine, which is responsible for the rotation of the sphere itself, these engines actually maneuver the ship from the vertical takeoff position to the horizontal cruising position. With one engine in reverse and one in forward, the ship can easily make a full 360-degree turn in a matter of seconds—a maneuver that is impossible for ellipsoid airships.

The LTA 20-1 can also hover, almost effortlessly. Other airships have to tack into the wind to hold their position, and only *appear* to be hovering. What keeps the LTA 20-1 from floating off into the heavens like a helium balloon? The weight of air itself acts as ballast and is taken in and let out as needed and held in bladders within the sphere.

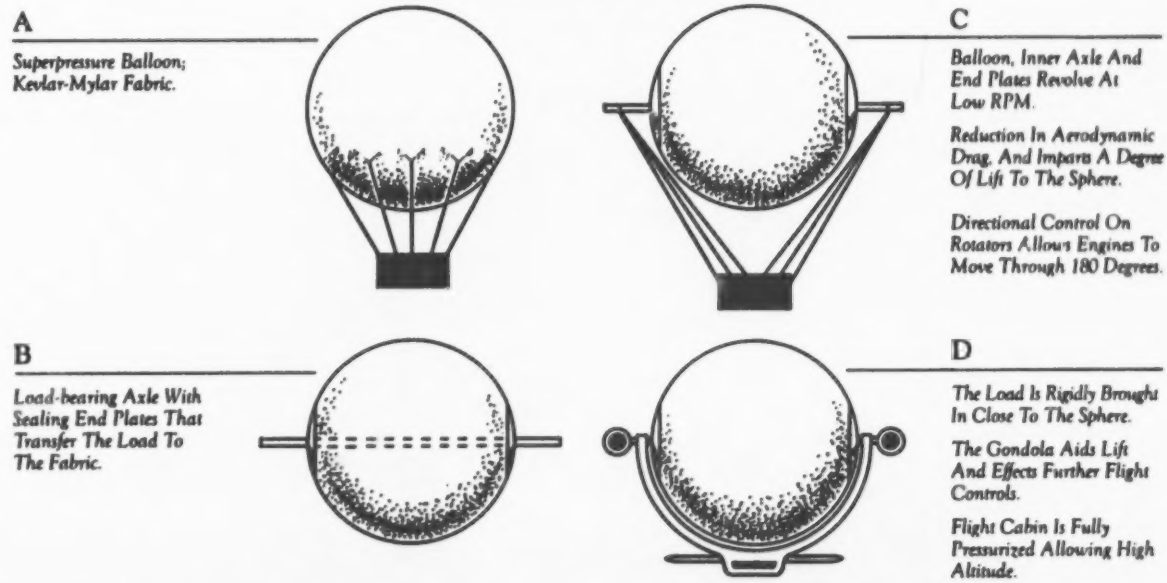
Because of its hovering ability, engine maneuverability and fuel efficiency, Ferguson claims that the LTA 20-1 has capabilities of no other existing aircraft. He believes it will be able to go where no other vehicle can go, and will be a boon to the more isolated and less developed countries of the world. Because of its unprecedented design and lifting capacity, he predicts the flying sphere could be used for the biggest lifting jobs: overweight, oversized, single-unit loads, for heavy construction, logging, offshore oil rigs and the movement of heavy equipment into remote areas.

The Real Thing

Inventors, such as Ferguson, argue with what *is* to discover what *can* be. But at each stage of research and development, the argument—and the invention—may change to accommodate new evidence discovered along the way.

The final proof of Ferguson's claims for the LTA 20-1 will come when Magnus Aerospace Corporation moves from the prototype stage to execution and testing of the "real thing." The first production vehicle—a 90-foot logging version—is slated for launching in late 1986. If all goes according to plan, there will be no room or need for argument then. ■

The Evolution of the LTA 20–1

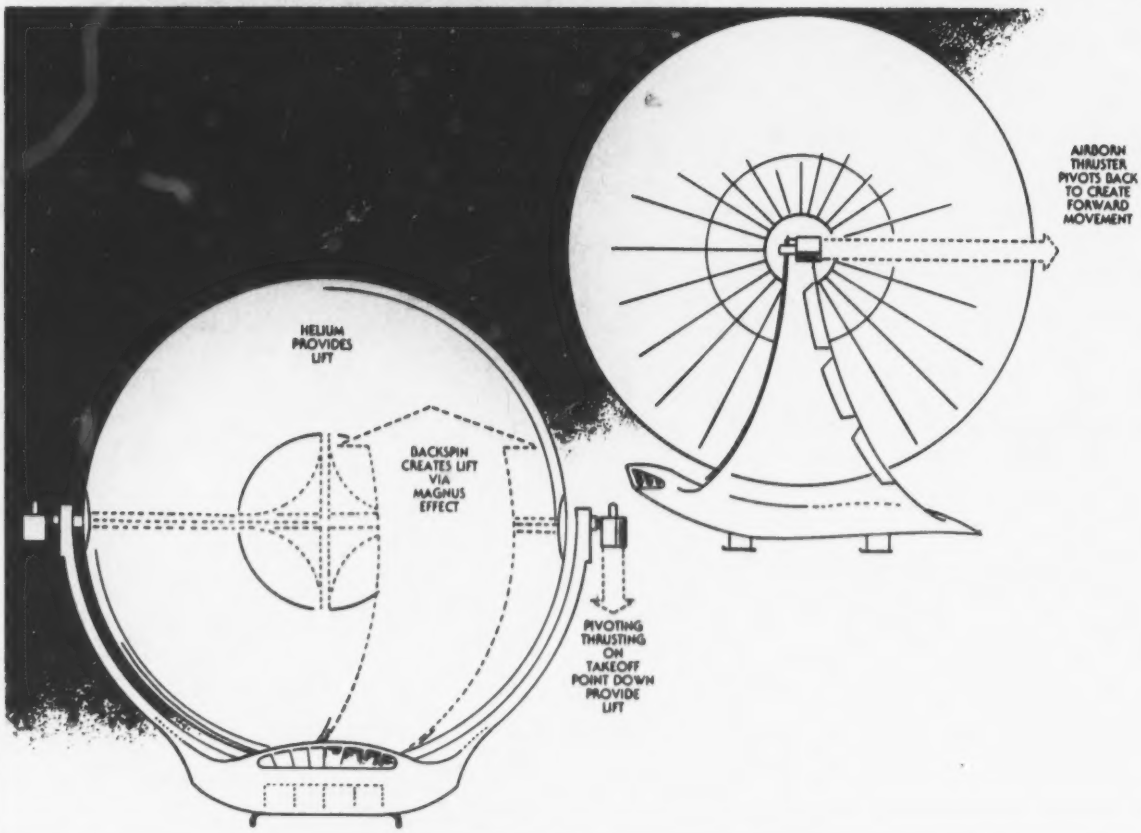


The Magnus Effect

If you've ever been fooled by a fastball that rises just in front of the plate, you've experienced the *Magnus effect*. Spheres do not generally fly very well. But give them a bit of backspin. . .and look out!

Normally air flows equally around the top and bottom of a forward-moving sphere—as long as it isn't spinning. But when a sphere is spinning backward, the air on top flows around the back to the bottom, interfering with the flow underneath. The air trying to go under the sphere follows the path of least resistance—over the top. The quicker flow means less air pressure on top—so the greater pressure underneath pushes the sphere up.

The fastball rises. . .and so does the LTA 20-1.



Further Readings

Carroll, Jack. "New Airship Era Dawning." *Science Mechanics*, January/February 1983.

Excellent description of the development of Ferguson's LTA 20-1, including a chart of the craft's technical specifications for detail-oriented students.

Magnus Aerospace Corporation, 200 First Avenue, Ottawa, Ontario Canada K1S 2G6.

For a kit containing black and white photos, reprints of articles about the LTA 20-1 and charts and specifications describing the vehicle, write directly to the company that produces the vehicle.

Pole, Ken. "Airships: Key to Canada's Frontiers?" *Canadian Geographic*, February/March 1983.

This article describes how the LTA might be useful in a variety of situations in Canada's wilderness areas, including its use for logging, equipment transportation to remote areas etc. Ferguson's LTA, as well as the more conventionally shaped Helio-Stat and Cyclo-Crane, are described. The article provides a thorough introduction to the history (successes and failures) of lighter-than-air airships.

Sizing Up a Situation

Building a 20-story blimp using a new and unproven technology is a risky and expensive proposition, so the designers of the LTA 20-1 try to eliminate as much uncertainty as possible through the use of models (smaller, nonworking versions sized to a certain scale) and, eventually, prototypes (working test models).

By using models, and small and then larger-scale prototypes, the designers can observe how a smaller version of the colossal blimp works and project their observations onto the future full-size version, which will be 200 feet in diameter.

Using scale models, the designers can calculate the LTA's *lifting capacity* (the maximum weight the blimp can lift by means of the helium gas that fills it) and *drag force* (the amount of power required from the engines to lift an object off the ground) with the same formulas they use for the full-size airship. To calculate lifting capacity, the LTA's designers use the formula:

$$LC = V(D_a - D_h)$$

where, LC = lifting capacity
V = volume of sphere
D_a = density of air
D_h = density of helium.

The densities of air and helium vary with the temperature of the air (see Table 1).

Temperature (F°)	Density of air (lb/cu ft)	Density of helium (lb/cu ft)
20°	0.0827	0.0114
30°	0.0811	0.0112
40°	0.0794	0.0110
50°	0.0779	0.0108
60°	0.0763	0.0106
70°	0.0750	0.0104
80°	0.0735	0.0102

Table 1

The volume of the blimp's sphere is calculated like the volume of any sphere:

$$V = 4/3 \times \pi R^3$$

where, π = 3.1416
R = radius of the sphere.

Use a calculator to find the lifting capacity of a sphere 50 feet in diameter in air 60°F. Substitute values from Table 1 in the formula, $LC = V(D_a - D_h)$.

diameter of sphere (ft)	=	50
volume of sphere (cu ft)	=	65,450.
density of air (lb/cu ft)	=	0.0763
density of helium (lb/cu ft)	=	0.0106

$LC = 65{,}450 \times (.0763 - .0106)$
$= 4{,}300.065$ lb
$\approx 4{,}300$ lb

The ratio of the final version to the diameter of this hypothetical model is 4:1, so the ratio of their volumes is 4^3:1. To project the lifting capacity of the full-size blimp under the same conditions, multiply 4,300 lb by 64. The helium in the 200-ft blimp should be able to lift 275,200 lb. You can

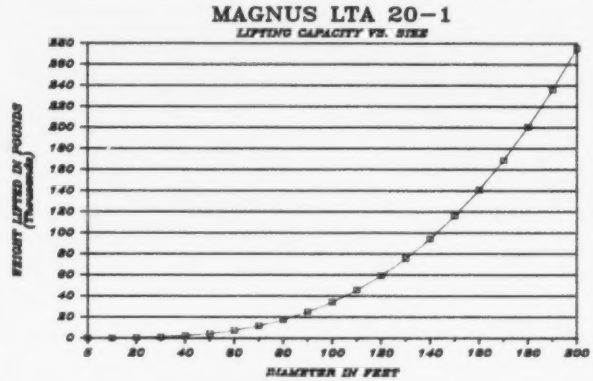


Fig. 1

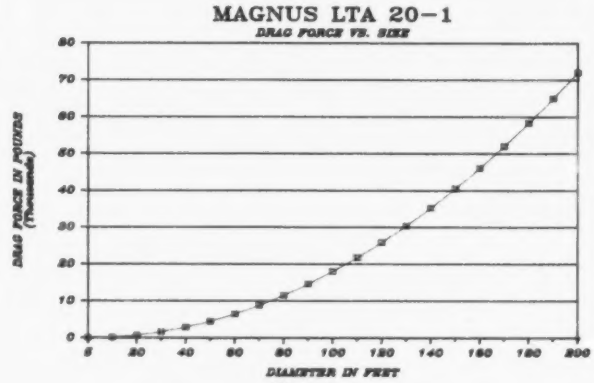


Fig. 2

Figure 1 shows how lifting capacity of LTA 20-1 increases with size of sphere. Figure 2 shows increase in drag force.

check by substituting the larger balloon's radius in the same formula. (Use a calculator.)

diameter of sphere (ft) = 200.
volume of sphere (cu ft) = 4,188,800.
density of air (lb/cu ft) = 0.0763
density of helium (lb/cu ft) = 0.0106

LC = 4,188,800. × (.0763 − .0106)
= 275,204.16 lb
≈ 275,200 lb

Exercises:
Fill in the missing information in Tables 2, 3 and 4; then answer the questions that follow.

Temperature (F°)	Density of air = D_a (lb/cu ft)	Density of helium = D_h (lb/cu ft)	$D_a - D_h$
20°	0.0827	0.0114	
30°	0.0811	0.0112	
40°	0.0794		0.0684
50°		0.0108	0.0671
60°		0.0106	0.0657
70°	0.0750		0.0646
80°	0.0735	0.0102	

Table 2

Diameter D	Radius R (ft)	R^3	$\frac{4}{3}\pi$	Volume (cu ft)
200		4.1888		
100		4.1888		
50		4.1888		
40		4.1888		
20		4.1888		

Volume of a blimp

Table 3

Temperature (F°)	$D_a - D_h$ (lb/cu ft)	Volume (cu ft)	Lifting capacity (lb)
20°		4,188,800	
40°		4,188,800	
60°		4,188,800	
80°		4,188,800	
60°		523,500	
60°		33,504	
60°		4,189	

Lifting Capacity of a Blimp

Table 4

1. What would you estimate the density of air to be at 10°F? At 90°F? What would you estimate the density of helium to be at 10°F? At 90°F?

2. Verify that 4.1888 is the best 5-digit approximation for $4/3\pi$. What is 4/3 (3.14)? What is 4/3 (22/7)? How do these two approximations of $4/3\pi$ change the volume of a 200-ft-diameter blimp?

3. What is the difference between the lifting capacity at 20°F and at 80°F for a 200-ft-diameter blimp?

4. Would you rather be heading toward warmer or cooler weather if you were a blimp pilot carrying a full load? Why?

5. If the diameter of a sphere is doubled, how is the volume changed? What if the diameter is multiplied by 5?

Answers:
1. .0843; .0721; .0116; .0100
2. 4.1867; 4.1905; 4,186,700; 4,190,500
3. 33,510
4. Toward cooler—increase not decrease in lifting capacity
5. It is multiplied by 8; it is multiplied by 125.

The Master Builders of Stonehenge

Concentrating on solstice estimates and radiocarbon dating may not be as enticing as dreaming about prehistoric flying saucers, but it takes Atkinson as close as he can get to at least one truth about Stonehenge.

On any day on a broad plain in southwestern England, you can see a number of people peering in amazement at a circle of gigantic stones.

The site is called Stonehenge, and it is, perhaps, the greatest engineering feat of prehistoric man. The riddles and mysteries surrounding this man-made monument have had amateurs and experts debating for thousands of years. How and why were these stones brought to this site? Why are they placed in this particular configuration? What could be their meaning?

To get closer to the answers to the questions surrounding Stonehenge, Richard Atkinson, professor of archaeology at University College, Cardiff, Wales, has spent three decades taking measurements and mapping the site. By learning more about the various peoples who built Stonehenge, Atkinson believes he can make a better case for how and why Stonehenge was built.

The archaeological evidence shows that Stonehenge evolved over three distinct periods, and was used by three separate prehistoric populations. The first people at the Stonehenge site grew wheat, kept cattle, dressed in leather, built log cabins and used tools made from stone and bone. For some reason, sometime after 2400 B.C., they decided to dig a circular ditch some 320 feet in diameter. They erected a bank of earth 6 feet high just inside the ditch; then they dug 56 regularly spaced holes in a circle inside the bank. They also erected a single, gigantic stone called the Heel Stone. It is believed that these early Stonehenge features were probably created for ceremonial purposes. But beyond that, little is known, save for the fact that the site was eventually abandoned. This leaves many questions about what was left behind.

STONY SILENCE: After 4,000 years, Stonehenge sits in cryptic silence. Why was it built? How was it built? It's doubtful that all—or even many—of the answers will ever be known.

The Beaker People

The next people to put their mark on Stonehenge were probably the Beakers (so called for the pottery style found in their graves) from northern Europe. These metalworkers, who found Stonehenge on their trade route between Ireland and the continent, made dramatic changes at the site about 500 years after the first emplacements, including positioning some 80 bluestones, each dressed or carved to shape and weighing between 3 and 4 tons. These form a double circle at the center of the site. It is believed that the bluestones were brought to the site from southern Wales, a land and sea journey up the Bristol Channel of some 240 miles.

The Beakers, it is also believed, were responsible for digging the broad avenue that leads to the site from the northeast. It was, perhaps, used as the access route for bringing the bluestones to the site.

Such "monumental" efforts must have had some purpose, but that purpose will never be known for certain. The Beakers didn't complete their work; like those before, they abandoned Stonehenge and moved on.

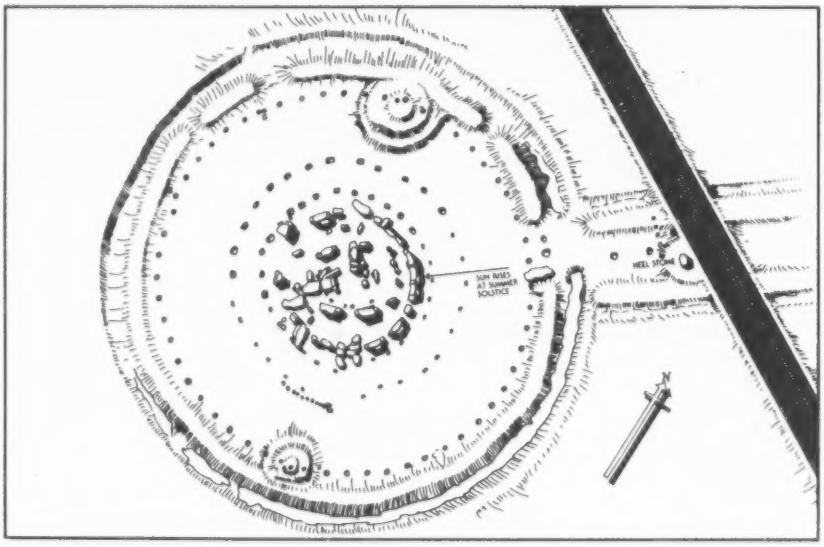
Sarsen Stones

A few centuries later, around 1500 B.C., construction resumed. This time, the master builders at Stonehenge were the Wessex people, who accomplished the seemingly impossible task of replacing the bluestones with huge sandstone boulders called *sarsen* stones. Brought to the site from Marlborough Downs, 20 miles north of Stonehenge, some of these sarsen stones measure over 13 feet tall and weigh up to 50 tons. How they were brought to the site, how they were raised and how the Wessex people placed the massive lintels spanning the tops of these stones were problems that the Wessex people seemed to have solved with great ingenuity. (See page 263.)

But what was the purpose of all these remarkable achievements? For centuries, people have speculated many roles for Stonehenge: a temple of sun worship, a burial ground, a cattle pen or even a launch pad for flying saucers. Some suggest that Stonehenge was a sophisticated observatory, a calculator or some sort of solar calendar, marking the interaction and alignments of the sun with a number of the site's prominent features.

Atkinson argues that almost all such tantalizing speculations are vulnerable to charges of coincidence or unanswerable questions of motive. He accepts as proven only one alignment: Someone standing at the center of Stonehenge facing northeast on a summer solstice morning in about 2000 B.C., would have seen the sun rise precisely in the center of the avenue.

Professor Richard Atkinson dismisses virtually all of the tantalizing speculations about the meaning of Stonehenge as unprovable.



IS STONEHENGE A SOLAR MARKER? Many believe the ancient structure is aligned to events in the solar calendar. According to Atkinson, only one alignment stands up to close scrutiny. When built, the Avenue was aligned to sunrise on the summer solstice. But, it's a tricky case to prove. The solution must be tested not only by observing the event today, but by computing where it would have taken place in prehistory.

Sun Spots

Every year, after the vernal equinox (about March 21), the sunrise in the northern hemisphere moves to a more northerly position every day until the summer solstice (about June 22), when it starts moving southward again. (See page 252 for more on solstice and equinox.)

"Of all the astronomical events that are easily observable with the naked eye," Atkinson says, "the sunrise at midsummer [summer solstice] is the best one to choose if you want a fixed point in the calendar, because this is simply a matter of observing where the sun rises each day, and finding the day when it rises furthest to the north."

Knowing the fixed rate of the earth's cycle around the sun, scientists like Atkinson can calculate where the sunrise would have appeared on the horizon at any date in the past. Or, looking at it the other way, by taking the bearing of the avenue, Atkinson can calculate precisely when in prehistory the summer solstice sunrise would have appeared in that position to an observer at Stonehenge. That date (allowing a 5 percent correction) falls somewhere between 2315 and 1895 B.C.

Confirming Dates

Fortunately, some prehistoric construction workers abandoned their deer-antler pickaxes in the ditch while the avenue was under construction. *Radiocarbon dating** places these tools within the same time frame—a very close match with the solstice estimates. This, Atkinson says, "appears to confirm that the alignment of the avenue on the rising sun at the solstice was both deliberate and precise."

*In geology, archaeology and paleontology, the dating of objects can be accomplished in a number of ways. The most widely used and accepted method is based on natural radioactivity. Radioactive material decays at a known rate, so the age of a specimen can be determined from the relative proportions of radioactive material and decay products. Radiocarbon dating, also called radioactive Carbon-14 dating, is useful because all organic material contains carbon.

Concentrating on solstice estimates and radiocarbon dating may not be as enticing as dreaming about prehistoric flying saucers, but it takes Atkinson as close as he can get to one truth about Stonehenge. So he continues to take measurements and test them against existing theories about the monument. "We can answer some questions which begin with 'who' and 'how,'" Atkinson cautions, "but we can't answer any of the questions which begin with 'why.'"

"One can put up all sorts of ideas about why, but there's nothing in the evidence itself to compel one to choose one idea rather than another," Atkinson continues. "If one does choose a particular reason why, it's simply because one prefers it." *He* prefers to keep an open mind. ■

Further Readings

Atkinson, R.J.C. *Stonehenge and Avebury*. London: Her Majesty's Stationery Office, 1976.

A thorough (63-page) introduction to Stonehenge and its neighboring monument at Avebury, complete with a map of how to get to the site and plenty of photos and drawings.

Chippindale, Christopher. "Life Around Stonehenge." *New Scientist*, 5 April 1984.

Excellent article on Stonehenge, covering its astronomical and religious possibilities, the people who built it and the different ways it has been studied. Good photographs and drawings.

Hadingham, Evan. "The Secrets of Stonehenge." *The World's Last Mysteries*. Pleasantville, New York: Reader's Digest Association, 1978.

Good article, with line drawings, showing possible ways the stones might have been raised, as well as the four different stages of construction that this ancient site is hypothesized to have gone through. Explains arguments and conflicting theories; in summary, agrees with Atkinson's supposition that there can be no final answer to the mystery.

Heggie, Douglas C. *Megalithic Science*. London: Thames and Hudson, 1981.

The Ancient Art of Weightlifting

Archaeologists believe that about 3500 years ago the Wessex people moved 50-ton sandstone rocks—called *sarsen*—some 20 miles from Marlborough Downs to the Stonehenge site. These stones were then raised upright and additional stones, called *lintels*, were raised more than 13 feet as spans between them.

Although the land journey from Marlborough Downs is only 20 miles, it is thought that 100 to 300 men were needed to move each stone, resting on a sledge which rolled over tree trunks.

What about erecting the stones once they got to the site? Assuming that one person can lift about 200 pounds, it would take about 500 people to lift one sarsen stone. Clearly, it would be an impossible task to get 500 people into position to lift a single stone.

It is believed that the raising of the stones began with the digging of a large hole, one side of which was vertical, the other sloped. The stone was rolled on the sledge to the edge of the hole (see Fig. 1). A long tree trunk was then used as a lever to lift the stone, angling it into the pit. When the slab was close to vertical, it was then pulled by ropes into its final position. The space left on the sloped side of the hole was filled with stones to secure and stabilize the massive sarsen stone.

Raising the lintels was another monumental undertaking. By levering up one side of the lintel at a time, it is thought workers built a timber platform beneath it. In this way, the stone was slowly raised to the level of the tops of the sarsens. (See Fig. 2.)

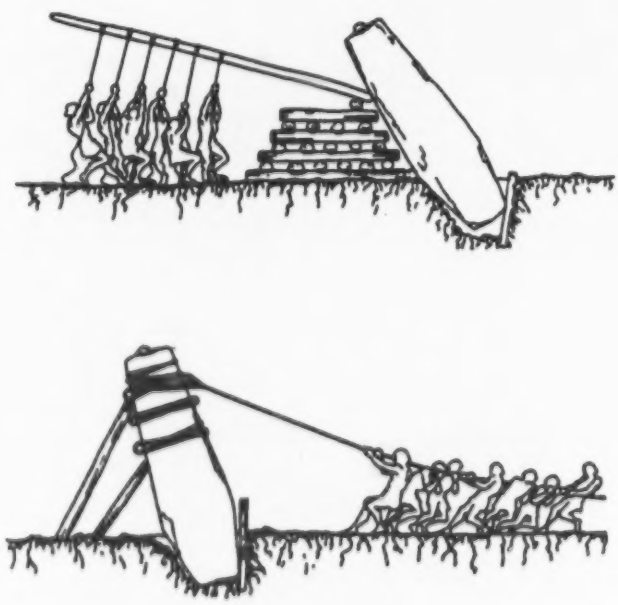


Fig. 1

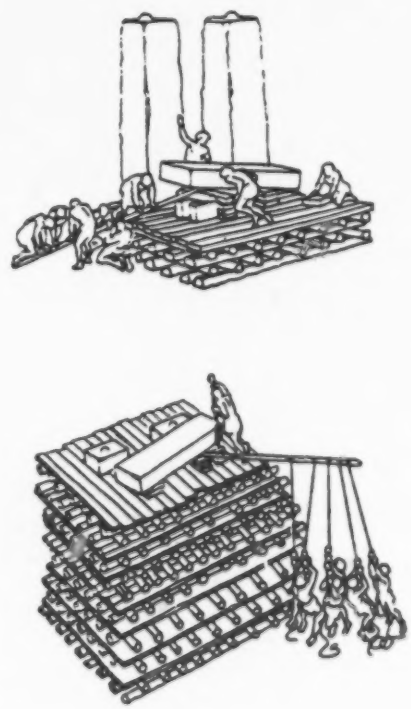


Fig. 2

Monumental Efforts

In physics, there are six so-called "simple machines:" the *lever*, the *pulley*, the *wheel and axle*, the *inclined plane*, the *screw* and the *wedge*. Each of these simple machines has the effect of spreading out the distance over which a force is applied to a given object, thus reducing the force needed at any given time to move that object.

So, for example, if you want to lift a rock that weighs 1,000 lb a distance of 1 foot, the lever lets you change the problem to the equivalent of lifting a 200-lb rock a distance of 5 ft, or a 100-lb rock a distance of 10 ft.

A child's seesaw is an example of a lever. If a 100-lb person sits 6 ft from the fulcrum (the point at which the seesaw is balanced), a 200-lb person will have to sit only 3 feet from the fulcrum on the other side to balance the first person. In general, the product of one person's weight and distance from the fulcrum will equal the product of the other person's weight and distance from the fulcrum.

Using Leverage

In Figure 1, the rock is a distance q from the fulcrum and the force is a distance p from the fulcrum. If the weight of the rock is W, and the force applied is F, then $pF = qW$. This can also be written as a proportion $p/q = W/F$.

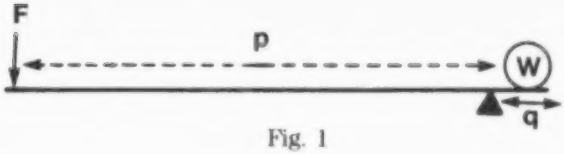


Fig. 1

So, if you wanted to lift one of the 4-ton (8,000-lb) Stonehenge stones, and you could only provide a force of 200 lb, you could figure out, using the above formula, that by putting the lever under the rock so there was a $^1/_2$-ft distance from the rock to the fulcrum, and by applying a 200-lb force 20 ft from the fulcrum, the two would be just balanced (as expressed in the formula: $20 \times 200 = {}^1/_2 \times 8{,}000$; where $p = 20$, $q = {}^1/_2$, $W = 8{,}000$ and $F = 200$). You would have to push a bit harder (say with a force of 201 lb) to make the rock move.

Similarly, an inclined plane can also be used to raise a rock. In Figure 2, the total vertical distance the rock is raised is q, but the force acts over the distance p. Again, if W is the weight of the rock and F is the force applied, then $pF = qW$. So, an 8,000-lb rock could be raised a distance of $^1/_2$ ft by pushing it up a 20-ft ramp with a force of 200 lb.

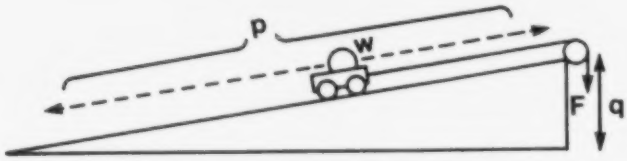


Fig. 2

In all of these cases, the results are "ideal." That means, we assume there is no friction, the lever has no weight and so on. In reality, there are always things that make the simple machine less efficient than the ideal. For the purposes of simplicity, let us assume the ideal situation for the exercises that follow.

Exercises:

1. If p is the distance between the fulcrum of a lever and the point where a force, F, is applied, and q is the distance between the fulcrum and an object of weight, W, then $pF = qW$ (see Figs. 1 and 2). This relationship can also be written $p/q = W/F$.

For each of the following, use one of the formulas to solve the problem:

a. To lift an object whose weight is 4 tons with a force of 200 lb, how far from the fulcrum must the force be if the object is 3 inches from the fulcrum?

b. If an object whose weight is 50 tons is to be lifted by a force of 1 ton, and the force cannot be more than 20 ft from the fulcrum, how close to the fulcrum must the object be?

c. With a lever 100 ft long and a force of 200 lb, how heavy an object could be moved if the weight were 1 inch from the fulcrum? Why would this not be a practical way to move the object?

2. If an object whose weight is moved a total distance p along an inclined plane by a force, F, and if q is the vertical displacement of the object (see Fig. 3), then $pF = qW$, or $P/q = W/F$.

Ignoring the effects of friction, use this relationship to solve the following:

a. A ramp that is 10 ft long will raise an object 1 ft vertically. How much force will be needed to raise an object whose weight is 1 ton to a height of 1 ft?

b. How long a ramp would be needed to raise an object whose weight is 4,000 lb 2 ft with a force of 200 lb?

c. How long a ramp would be needed to raise an object whose weight is 50 tons 13 ft with a force of 20,000 lb (about 100 people pushing and pulling)?

Answers:

1. a. 10 ft (or 120 in)
b. 0.4 ft (or 2/5 ft)
c. 240,000 lb or 120 tons. There is no material known that wouldn't bend under these conditions, and you would have to move the end of the lever 10 ft to move the weight 1/10 inch even if the lever didn't bend.

2. a. 200 lb
b. 40 ft
c. 32.5 ft

"A problem is not necessarily 'solved' because the correct response has been made. A problem is not truly solved unless the learner understands what he has done and knows why his actions were appropriate."

William A. Brownell, *Problem Solving in School Mathematics* (Suydam, ed.)

Teacher's Notes

(see opposite page)

Keeping a Horse

Leslie's argument must be a demonstration of the actual costs and time she expects a horse will demand. If she can show her parents that both the costs and time demanded by having a horse will be reasonable and that she will be able to meet them, she will have made a strong case for having a horse.

First, she should probably research the costs and present them to her parents:

Total cost per month

Hay	$50
Oats	24
Insurance	20
Vet	40
Horseshoes	30
Total:	$164 per month or $41 per week

Second, she should provide information on how she can defray these costs without having to rely on her parents. For example:

Total income per week

Salary for part-time work at the drugstore	$35 per week
Allowance	10 per week
Babysitting at $2.50 per hour	10 per week
Total:	$55 per week

A timetable would also help her case. Leslie might give her parents a schedule like this:

Daily timetable

6:45	get up
7:00	dress
7:15	leave for stable
8:00	return home for breakfast
8:30 – 2:00	attend school
2:00 – 4:30	work at drugstore
4:30 – 6:00	horse hours
6:00 – 9:00	dinner/homework

Graphic Persuasion

Both—and neither.

Graphs are particularly effective instruments of persuasion, and it's not unusual to see a graph that seems to be saying one thing when it's really saying something entirely different.

The graph on the left tells the story but omits the context. Its scale shows only the points roughly between 19,000 and 20,000—or approximately 5 percent of what the total graph from 0 to 20,000 would show.

The graph on the right exaggerates somewhat, too, but in the other direction. Its scale goes from 0 all the way to 30,000, so that the increase looks even smaller than it is.

For Further Discussion:

Using the data points plotted as a guide, have your students draw a graph that accurately reflects the increase in average system-wide daily attendance from September 1984 to March 1985. What kind of headline would fairly describe what's happening on the graph? Would it be news? What information is not provided by these graphs?

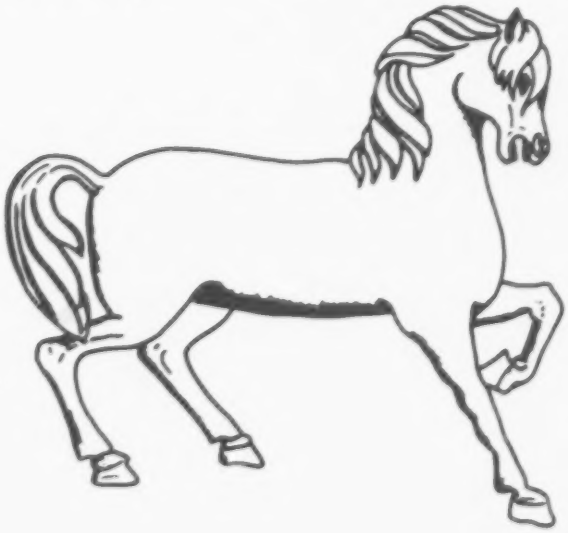
Keeping a Horse

Leslie, a high-school senior, loves horses and has learned to ride well. She helps sometimes at the local stables. She would like above all to have a horse of her own, but her parents are worried about the cost. They are also worried that the horse will take up too much of Leslie's time, which she should be spending on her schoolwork.

Leslie points out to her parents that she could keep the horse in the old barn behind their house, so there wouldn't be any stable fee. She says she could earn some money working part-time to pay for the horse's upkeep. She promises not to let the horse or the job interfere with her schoolwork.

Leslie's parents aren't convinced. They would like to say yes, but they say Leslie hasn't thought things through in enough detail.

Prepare a *quantitative argument* to support Leslie's case.



Graphic Persuasion

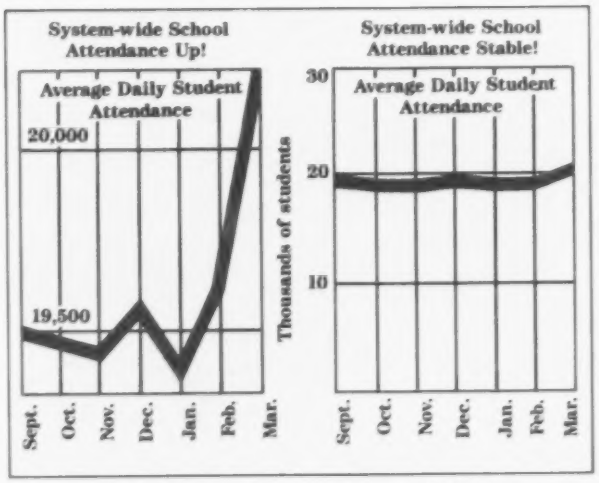


Fig. 1 Fig. 2

Which graph tells the true story?

Teacher's Notes

(see opposite page)

Twisted Logic

Mr. Vulcan must choose an action that will cancel out the difference between the two tribes.

The most useful answer would be a simple "yes" or "no," since the reply will be either 100 percent true or 100 percent false. There are several possible answers. This is only one of them.

Mr. Vulcan points to one of the roads and says to the native, "If I were to ask you if this road leads to the village, would you say 'yes'?" The native will give the right answer, no matter which tribe he belongs to.

Here's why: If the road leads to the village, the Liar would answer "no" to the direct question "Is this the road to the village?" But that's not what he would be asked. Mr. Vulcan wants to know what the tribesman would say *if he were asked* that question. So, since the Liar would say "no," he *must* answer "yes." A Truth-Teller would, of course, tell the truth and answer the same way. Thus, Mr. Vulcan can be certain that the road *does* lead to the village, regardless of whether the respondent is a Truth-Teller or a Liar.

On the other hand, if the road actually does not go to the village, the Liar would be forced to reply "no" while the Truth-Teller would answer "no" truthfully.

Making the Best for Bosco

Many decisions in life are attempts to make the best of a situation. The branch of mathematics that helps people choose the most desirable of several possibilities is called *optimization*.

Optimization is the process of achieving the largest values of some desirable quantity or the smallest of an undesirable one. It is the argument for the best path out of a number of possibilities.

One way to find the largest rectangular pen that can be surrounded by a total of 12 meters of wire fencing would be to try calculating the area of the pen for a variety of dimensions. The area of a rectangle is calculated $A = LW$.

The length and width will add up to half the perimeter, since the pen has four sides.

The table below shows dimensions and areas (integers only) possible with 12 meters of fencing.

Length	(meters)	6	5	4	3	2	1	0
Width	(meters)	0	1	2	3	4	5	6
Area	(meters^2)	0	5	8	9	8	5	0

This suggests that a square dog run with a width of 3 meters is the best.* (To check, try the values 2.9 and 3.1.)

*This problem not only helps students distinguish between perimeter and area, it also provides an opportunity to reinforce the concept that a square is a special type of rectangle.

Twisted Logic

While vacationing on a South Sea island, Mr. Vulcan decides one beautiful day to go for a long walk to a village he hasn't yet visited. After traveling about five miles, he comes to a fork in the road. He has forgotten the map. Which way to go?

A man stands by the road. Mr. Vulcan recognizes him as a member of one of the two local tribes: the proverbial Liars and Truth-Tellers. The two tribes dress alike. The rather significant difference between them is that the Truth-Tellers always tell the truth, while the Liars always lie.

Mr. Vulcan, aware of the customs of the local tribes, knows that he has no way of telling whether the native is a Truth-Teller or a Liar. But the man represents his only hope of getting directions to the village.

Mr. Vulcan, a computer programmer, thinks a moment, summoning up all his training in the workings of logic. Then he asks one question.

From the tribesman's one-word reply, he knows which road to take.

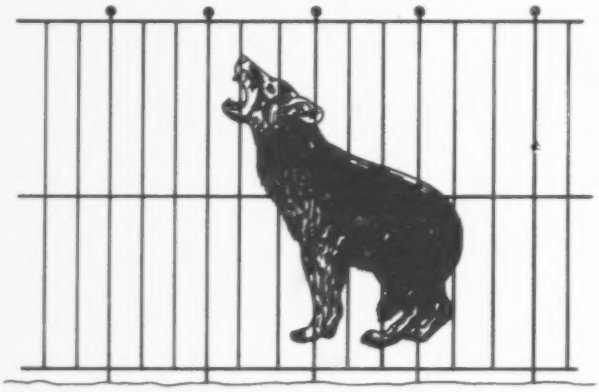
What question does Mr. Vulcan ask?

Making the Best for Bosco

Turner's dog, Bosco, tends to wander off when left to his own devices, so Turner can't let him run outside on his own. His father suggests that the best thing to do would be to build a small pen so that Bosco can run around all he wants without running away.

Together they go to the hardware store to buy wire fencing. Turner has saved from his allowance enough to buy 12 meters of fencing.

What dimensions will give him the largest area in a rectangular pen surrounded by 12 meters of fencing?



Teacher's Notes

(see opposite page)

Proof Positive

The tilted square in Figure 1 is drawn on the hypotenuse of each right triangle. The two squares in Figure 2 are drawn on each of the other two sides.

Let A = the area of the large square in Figure 1 and Figure 2. Let B = the combined areas of the four right triangles within each of these outer squares. Then let C = the area of the tilted square in Figure 1.

The area of the shaded square is what's left when you subtract the areas of the triangles from the area of the large square in Figure 1. So, $C = A - B$.

Now let D = the area of one of the two shaded squares in Figure 2, and let E = the area of the other.

Their combined areas are what is left when you subtract the areas of the four right triangles from the area of the large square in Figure 2. So,

$$D + E = A - B.$$

If $C = A - B$, and $D + E = A - B$, then $C = D + E$.

That is, the area of the shaded square in Figure 1 is equal to the sum of the areas of the two shaded squares in Figure 2. Proof positive!

So-o-o-o Big

No. Even if the print were close to rectangular in shape, his method would leave much to be desired.

Suppose the perimeter of the print was 36 inches. The following rectangles would all have the same perimeter:

3×15 (area: 45 sq in)
14×4 (area: 56 sq in)
9×9 (area: 81 sq in)
1×17 (area: 17 sq in)

By Foote's method, one would be as good as another, yet they yield very different areas.

"The essence of a good mathematical model is that it should embody bold ideas, unjustified assumptions and speculations which are our only means of interpreting nature. The good scientist then puts his model to the hazard of refutation."

Karl Popper

Proof Positive

The Pythagorean theorem states that in a right triangle "the square of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides." This can be stated another way: If squares are drawn on the three sides of a right triangle, the square on the longest side will be equal to the area of the sum of the areas of the other two squares.

The figures below can be used to make a general proof for the Pythagorean theorem. What is the proof? How does it work?

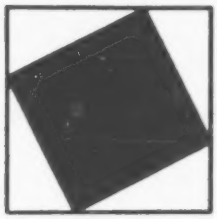


Fig. 1

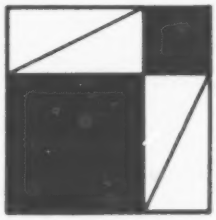


Fig. 2

Here are some questions/hints to get you started:

Figure 1 consists of four identical right triangles and a square.

1. On which side of the triangles is the square drawn?

Figure 2 consists of four triangles, identical with those in the first figure, and two squares.

2. On which sides of the triangles are these squares drawn?

3. Given that the two figures have the same area, explain why it follows that the large square must be equal in area to the other two squares put together.

So-o-o-o Big

Herbert Foote is an amateur ichnologist who stumbled on a spectacular dinosaur print. Not being terribly sure how to measure its size, he took a piece of string and laid it around the print to get its perimeter.

After cutting the string to this length, he used the string to make a square. Then he found the area enclosed by the square.

Is his answer reasonably accurate? Why?

Prove it.

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281

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283

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Math Matrix

	Angles	Circular figures	Data collection, analysis	Comparisons	Equations	Exponents, powers, large numbers	Fractions, decimals, percents	Graphs, coordinates, functions	Logic, reasoning	Maps, navigation, polar coordinates	Measurement	Models, drawing, blueprints	Physical science	Probability, odds	Projective geometry	Rates, d/r/t	Ratio and proportion	Rectangular figures	Scale	Sequence, ordering	Solid geometry	Statistics, error, sampling	Symmetry, patterns	Triangular figures
SITUATION																								
Orienteering	▼									●	▼				▼								●	
Wind Prospecting					▼					●	▼				●							●		
Sun Sculpture	▼										▼	▼		●	▼								▼	
Comedy: Revolving Door		●								●				●	●				▼					
Exercises	●	▼	▼		▼	▼		▼																
INFORMATION											▼		5											
Ice Climbing			▼								▼													
Anasazi Indians			●					●			▼	▼			▼	▼								
Speed Skiing			●	▼			●	●			▼		5		▼									
Fish Finding			▼	▼						●	▼								▼				▼	
Comedy: Lineup			▼							▼					●	▼				▼				
Exercises			●	▼		▼	▼	●																
RESTATEMENT											▼	●			●									
Dinosaurs					●	▼					▼	●					▼						●	
Locks											▼	●												
Okefenokee Swamp				▼							▼	●	2,4		▼								▼	
Ski Bus	▼																						●	
Comedy: Wallpaper												●		▼	▼		●							
Exercises				▼	●	●		●	▼														●	●
OUTCOMES											▼										●			
Vegreville Egg				▼							▼										●			
Cathedral							▼				▼		4		▼									
Demolition															●		▼		▼		●			▼
Comedy: Sandwich													4					▼		●	●			
Exercises		▼		▼														▼		●	▼		●	
MANAGEMENT											▼									●				
Canoe																				●	▼		●	
Comedy: Outing					▼							▼				▼	▼							
Exercises																	●							
ESTIMATION							▼				▼											●		
Sharks			●				▼				▼											●		
China Census			●	▼		●	▼				▼			●								▼		
Avalanche Forecast			▼													▼						●		
Comedy: Bakery									▼					●										
Exercises			●																					
ARGUMENT											▼		2									▼		
Boomerang	▼									●	▼	▼									●			
Sun Daggers	▼	▼										▼	1,3											
Blimp					▼					●		▼	3							▼				
Stonehenge										▼		▼											▼	▼
Comedy: Piano										●	▼		1,2,3			▼	▼	▼						
Exercises																								

● = Principal concept
▼ = Supplemental concept

1. air temperature and density
2. lift and drag
3. force, levels, inclined planes
4. vectors
5. water temperature and pressure
6. speed of sound

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This *Teaching Guide* accompanies **THE CHALLENGE OF THE UNKNOWN**, a seven-part film series about mathematics and problem solving. Combining documentary and comic elements, both the films and the *Teaching Guide* are designed for use in math and science classrooms throughout the United States, indeed wherever these subjects are taught in English.

It is hoped that by watching the films students will be stimulated to view math as a problem-solving tool of use in the real world. The *Guide* is a teacher-tool, a follow-up, designed to give teachers what they need to take students forward from the films toward the learning goals of the classroom. The *Guide* provides background material on each of the 29 film segments and explanations of the relevant mathematical concepts or themes in each. It includes more than 300 exercises and activities to give students hands-on experience in using the problem-solving techniques presented in the films. The board of 27 distinguished advisors, as well as 36 experienced contributing teachers from all over the United States, supervised the design and writing of this volume.

This *Teaching Guide* is only a first step. The advisors and teachers who helped compile this *Guide* have contributed a broad range of ideas and experience. Your's may well be different.

We're interested in how you and your students use THE CHALLENGE OF THE UNKNOWN. If you have developed new ways to use the films in math or other classes, additional activities or supplementary materials, please write to us:

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